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THE DISCRETE-TIME ANALYSIS OF THE LEAKY BUCKET SCHEME WITH DYNAMIC LEAKY RATE CONTROL

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ABSTRACT. The leaky bucket scheme is a promising method that regulates input traffics for preventive congestion control. In the ATM network, the input traffics are bursty and transmitted at high-speed. In order to get the low loss probability for bursty input traffics, it is known that the leaky bucket scheme with static leaky rate requires larger data buffer and token pool size. This causes the increase of the mean waiting time for an input traffic to pass the policing function, which would be inappropriate for real time traffics such as voice and video. We present the leaky bucket scheme with dynamic leaky rate in which the token generation period changes according to buffer occupancy. In the leaky bucket scheme with dynamic leaky rate, the cell loss probability and the mean waiting time are reduced in comparison with the leaky bucket scheme with static leaky rate. We analyze the performance of the proposed leaky bucket scheme in discrete-time case by assuming arrival process to be Markov-modulated Bernoulli process (MMBP).

1. Introduction

We present a queueing analysis for traffic control in ATM networks. ATM networks support the various kinds of traffic types with different traffic characteristics such as voice, data and video. All information in the ATM network is transmitted in a fixed-size packet called cell. Since input traffics are bursty and transmitted at high-speed, the network may easily get congested. Therefore, an appropriate preventive congestion control into and within the network is necessary.

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The leaky bucket (LB) scheme is a promising method[1] that regulates input traffics for preventive congestion control. The basic idea of LB scheme is that a cell, before entering the network, must obtain a token from the token pool. An arriving cell will consume one token and immediately depart from the LB scheme if there is at least one token available in the token pool. Tokens are generated at every constant period, and the tokens generated when the token pool is full are lost.

The original LB scheme[2] dose not have data buffer. In order to reduce the cell loss probability, data buffer is installed in the original LB scheme[3]. The data buffer size must be determined by considering the trade-off between the waiting time and the cell loss probability. In order to get the low loss probability for bursty input traffics, it is known that the LB scheme requires a larger data buffer and token pool size. This causes the increase of the mean waiting time for an input traffic to pass the policing function, which would be inappropriate for real time traffics such as voice and video.

We present the LB scheme with dynamic leaky rate in which the token generation period changes according to buffer occupancy. In the LB scheme with dynamic leaky rate, a token is generated at every constant period K_1 when the buffer occupancy is less than or equal to a threshold. As soon as the buffer occupancy exceeds the threshold, the token generation period is resetted without token generation, and tokens are generated by a token generation period $K_2 (< K_1)$ until the buffer occupancy drops to the threshold. Since token is generated with smaller period K_2 than the period K_1 when there are many cells waiting in the data buffer, we expect that the cell loss probability and the waiting time are reduced in comparison with the LB scheme with static leaky rate. Therefore, the required data buffer and token pool size in the LB scheme with dynamic leaky rate can be reduced to satisfy the same Quality of service (QoS) as in the LB scheme with static leaky rate. The token generation periods and the token pool size can be determined by considering the trade-off between the shaping function and the waiting time.

Lee and En[4] analyzed the LB scheme with different dynamic leaky rate for on-off data input in which the token generation period in the on period is somewhat smaller than that in the off period. They obtained the cell loss probability and the mean waiting time by using the uniform arrival and service (UAS) model in which the queue length and the arrival process of input traffic and token are treated as continuous variables.

To model bursty input traffics, the Markov-modulated Poisson process (MMPP) is used in continuous-time case. The Markov-modulated Bernoulli process (MMBP) is the discrete-time version of MMPP[5].

This paper is concerned with the discrete-time analysis of the LB scheme with dynamic leaky rate when the arrival process is Bernoulli process and MMBP. We obtain the buffer occupancy distribution by using the supplementary variable method. Furthermore, we obtain the interdeparture time distribution by using 'the state-splitting method'[6].

In the case of continuous-time, the LB scheme is analyzed by many authors (see [7, 8] and references therein). On the other hand, there are few papers about the performance analysis of the LB scheme with static leaky rate in the discrete-time case. Ahmadi et al[9] considered a batch Bernoulli arrival process as an input process and obtained the buffer occupancy distribution by using the matrix analytic techniques.

MMBP. We also give numerical example in Section 5. Finally, conclusions are give in Section 6.

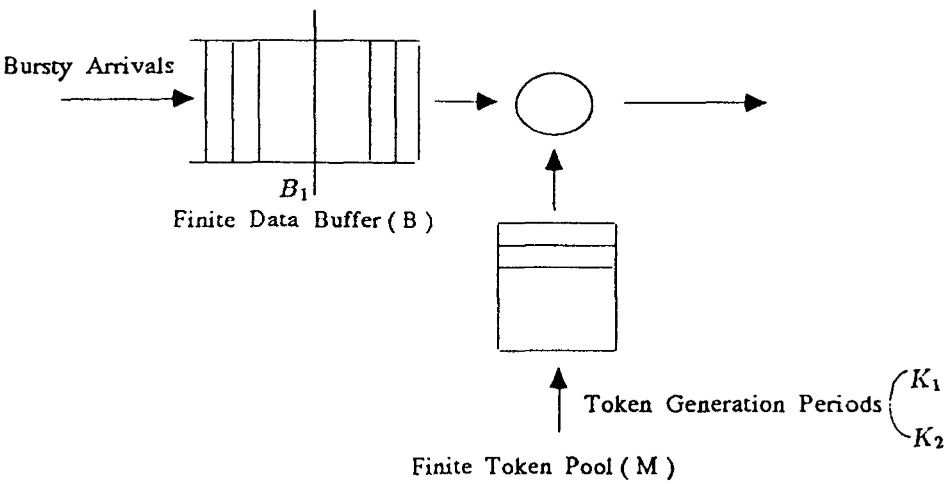

FIGURE 1. Model of the leaky bucket scheme with dynamic leaky rate control

K. Sohraby and M. Sidi[10] analyzed the LB scheme with infinite data buffer and obtained the buffer occupancy distribution for Markovian arrival process. Recently, Guo-Liang Wu et al[6] obtained the performance measures such as the cell loss probability, the waiting time distribution and the interdeparture time distribution by assuming the arrivals to be processes with constant and time-varying arrival rate.

We describe the LB scheme with dynamic leaky rate in Section 2. In Section 3 we analyze the performance of the LB scheme when the arrival process is Bernoulli. In Section 4 we analyze the performance of the LB scheme when the arrival process is MMBP. We also give numerical examples in Section 5. Finally, conclusions are given in Section 6.

2. Model description

We consider the discrete-time system in which the time axis is segmented into a sequence of equal intervals of unit duration, called slots. It is always assumed that cell arrival, departure and token generation occur only at slot boundary. Arriving cells are stored in the data buffer of finite capacity B, and those cells queued in the data buffer are served on the first-come first-service basis. Cells arriving when the data buffer is full are blocked and lost. The transmission right into the network is given by a token in the token pool. The token pool has a finite capacity M, so that the newly generated tokens are discarded when the token pool is full. Each token allows a single cell to be transmitted in a slot, and a token following a transmission is removed from the token pool. A transmission in a slot takes place only if a token in the token pool is available at that slot.

As shown in Fig. 1, the token generation period in the proposed LB scheme changes according to buffer occupancy as follows: The token generation period is K_1 unit time when the buffer occupancy is less than or equal to the threshold B_1. As soon as the buffer occupancy exceeds the threshold B_1, the token generation period is resetted without token generation, and tokens are generated by the token generation period $K_2(< K_1)$ until the buffer occupancy drops to the threshold B_1.

3. The LB scheme with Bernoulli arrival process

In this section we investigate the LB scheme with Bernoulli arrival process. Let λ be the probability that a cell arrives at arbitrary slot boundary, and we introduce the following notations:

$N_b(n)$ = the number of cells in the data buffer at time $n+$,

$N_p(n)$ = the number of tokens in the token pool at time $n+$,

$R(n)$ = the remaining token generation interval present at time $n+$.

Since cells wait in the data buffer only if there is no token in the token pool, we can represent the state of the data buffer and the token pool by one random variable $X(n)$ as following:

$$X(n) = N_b(n) + M - N_p(n).$$

Then $\{(X(n), R(n)), n \geq 0\}$ forms a Markov chain, and its transition probability matrix is given by

$$Q = \left| \begin{array}{ccccccccccccc}
B_1' & C_1 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
A_1 & B_1 & C_1 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & A_1 & B_1 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & B_1 & C_1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & A_1 & B_1 & C_1' & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & A_2' & B_2 & C_2 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & A_2 & B_2 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & B_2 & C_2 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & A_2 & B_2 & C_2 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & A_2 & B_2'
\end{array} \right|$$

The blocks A_1, B_1, B_1', C_1 of order K_1, A_2, B_2, B_2', C_2 of order K_2, C_1' of dimension $K_1 \times K_2$ and A_2' of dimension $K_2 \times K_1$ are as following:

$$A_1 = \left| \begin{array}{cccc}
0 & \ldots & 0 & (I-\Lambda)H \\
0 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
0 & \ldots & 0 & 0
\end{array} \right|, \quad
B_1 = \left| \begin{array}{ccccc}
0 & \ldots\ldots & & 0 & \Lambda H \\
(I-\Lambda)H & \ldots\ldots & & 0 & 0 \\
0 & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots \\
\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots \\
0 & \ldots & 0 & (I-\Lambda)H & 0
\end{array} \right|,$$

$$B_1' = \begin{vmatrix} 0 & \cdots\cdots\cdots & & 0 & H \\ (I-\Lambda)H & 0 & \cdots & 0 & 0 \\ 0 & \cdots\cdots\cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & (I-\Lambda)H & 0 \end{vmatrix}, \quad C_1 = \begin{vmatrix} 0 & \cdots\cdots\cdots & & 0 & 0 \\ \Lambda H & 0 & \cdots & 0 & 0 \\ 0 & \cdots\cdots\cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \Lambda H & 0 \end{vmatrix},$$

$$A_2 = \begin{vmatrix} 0 & \cdots & 0 & (I-\Lambda)H \\ 0 & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 \end{vmatrix}, \quad B_2 = \begin{vmatrix} 0 & \cdots\cdots\cdots & & 0 & \Lambda H \\ (I-\Lambda)H & 0 & \cdots & 0 & 0 \\ 0 & \cdots\cdots\cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & (I-\Lambda)H & 0 \end{vmatrix},$$

$$B_2' = \begin{vmatrix} 0 & \cdots\cdots & 0 & \Lambda H \\ H & 0 & \cdots\cdots & 0 \\ 0 & \cdots\cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \; H & 0 \end{vmatrix}, \quad C_2 = \begin{vmatrix} 0 & \cdots\cdots\cdots & & 0 & 0 \\ \Lambda H & 0 & \cdots & 0 & 0 \\ 0 & \cdots\cdots\cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \Lambda H & 0 \end{vmatrix},$$

$$C_1' = \begin{vmatrix} 0 & \cdots & \cdots & 0 & 0 \\ 0 & \cdots & \cdots & 0 & \Lambda H \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & \Lambda H \end{vmatrix}, \quad A_2' = \begin{vmatrix} 0 & \cdots & 0 & (I-\Lambda)H \\ 0 & \cdots\cdots & & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots\cdots & & 0 \end{vmatrix}.$$

The ΛH and $(I-\Lambda)H$ in the above blocks will appear in section 4 when the arrival process is MMBP. In this section, the ΛH and $(I-\Lambda)H$ are replaced by λ and $(1-\lambda)$ respectively.

Throughout the paper we assume that the following order occurs at slot boundary: departure if any, arrival of cell if any, and token generation if any.

3.1. The buffer occupancy distribution

Define the limiting probabilities

$$p_{i,j} \triangleq \lim_{n\to\infty} Pr\{X(n)=i, R(n)=j\},$$
$$0 \le i \le M+B, \quad 1 \le j \le K_1(\text{ or } K_2).$$

By the Chapman-Kolmogorov's forward equation we obtain the following equations for the limiting probabilities $p_{i,j}$:

(1–1) $$p_{0,j} = (1-\lambda)p_{0,j+1}, \qquad 1 \le j < K_1,$$

(1–2) $$p_{0,K_1} = (1-\lambda)p_{1,1} + p_{0,1},$$

(1–3) $$p_{i,j} = (1-\lambda)p_{i,j+1} + \lambda p_{i-1,j+1}, \quad 0 < i < M+B_1+1, \quad 1 \le j < K_1$$

(1–4) $$p_{i,K_1} = (1-\lambda)p_{i+1,1} + \lambda p_{i,1}, \quad 0 < i < M+B_1+1,$$

(1–5) $$p_{M+B_1+1,j} = (1-\lambda)p_{M+B_1+1,j+1}, \quad 1 \le j < K_2$$

(1–6) $$p_{M+B_1+1,K_2} = (1-\lambda)p_{M+B_1+2,1} + \lambda p_{M+B_1+1,1} + \lambda \sum_{j\ge 2} p_{M+B_1,j},$$

(1–7) $$p_{i,j} = (1-\lambda)p_{i,j+1} + \lambda p_{i-1,j+1}, \quad M+B_1+1 < i < M+B, \quad 1 \le j < K_2,$$

(1–8) $$p_{i,K_2} = (1-\lambda)p_{i+1,1} + \lambda p_{i,1}, \quad M+B_1+1 < i < M+B,$$

(1–9) $$p_{M+B,j} = p_{M+B,j+1} + \lambda p_{M+B-1,j+1}, \quad 1 \le j < K_2,$$

(1-10) $$p_{M+B,K_2} = \lambda p_{M+B,1}.$$

We obtain from $(1-1)$ and $(1-2)$ that

(1–11) $$p_{0,j+1} = \frac{1}{(1-\lambda)^j} p_{0,1}, \qquad 1 \le j < K_1,$$

(1–12) $$p_{1,1} = \frac{1}{1-\lambda}\left(\frac{1}{(1-\lambda)^{K_1-1}} - 1\right)p_{0,1}.$$

The equations $(1-3)$ and $(1-4)$ are rewritten into following equations:

(1–13) $$p_{i,j+1} = \frac{1}{1-\lambda}p_{i,j} - \frac{\lambda}{1-\lambda}p_{i-1,j+1}, \quad 0 < i < M+B_1+1, \quad 1 \le j < K_1.$$

(1–14) $$p_{i+1,1} = \frac{1}{1-\lambda}p_{i,K_1} - \frac{\lambda}{1-\lambda}p_{i,1}, \quad 0 < i < M+B_1+1.$$

By applying $(1-13)$ and $(1-14)$ recursively, $p_{i,j}(i = 1, 2, \cdots, M+B_1, j = 1, \cdots, K_1)$ and $p_{M+B_1+1,1}$ can be expressed in terms of $p_{0,1}$. By

$(1-5) \sim (1-9)$ we can express all $p_{i,j} (i = M + B_1 + 1, \cdots, M + B, j = 1, \cdots, K_2)$ in terms of $p_{0,1}$.
The unknown number $p_{0,1}$ can be obtained by the normalization condition

$$\sum_{i=0}^{M+B_1} \sum_{j=1}^{K_1} p_{i,j} + \sum_{i=M+B_1+1}^{M+B} \sum_{j=1}^{K_2} p_{i,j} = 1$$

Using the probabilities $p_{i,j}$, we obtain the following performance measures:

a. Cell loss probability:

$$P_B = \sum_{j \geq 2} p_{M+B,j}.$$

b. Token loss probability:

$$P_T = p_{0,1}(1 - \lambda).$$

c. Buffer occupancy distribution:

$$Pr\{\text{ buffer occupancy} = i\} = \begin{cases} \sum_{i=0}^{M} \sum_{j=1}^{K_1} p_{i,j}, & i = 0, \\ \sum_{j=1}^{K_1} p_{M+i,j}, & 1 \leq i \leq B_1, \\ \sum_{j=1}^{K_2} p_{M+i,j}, & B_1 + 1 \leq i \leq B. \end{cases}$$

d. Mean buffer occupancy:

$$M_b = \sum_{i=0}^{B_1} \sum_{j=1}^{K_1} i p_{M+i,j} + \sum_{i=B_1+1}^{B} \sum_{j=1}^{K_2} i p_{M+i,j}.$$

By Little's Law, we have the mean waiting time in the data buffer

$$W = \frac{M_b}{(1 - P_B)\lambda}.$$

When $B_1 = B$, our model is reduced to the LB scheme with static leaky rate, and our result coincides with known ones[6].

3.2. The interdeparture time distribution

In order to find the interdeparture time (T), we use the state splitting method[6]. Each state is split into two substates based on whether there is a departure.

Introduce the notation

$$D(n) = \begin{cases} 0 & \text{if there is no departure at time } n, \\ 1 & \text{if there is a departure at time } n, \end{cases}$$

and define the limiting probabilities

$$r_{i,j} \triangleq \lim_{n\to\infty} Pr\{X(n) = i, R(n) = j, D(n) = 1\},$$
$$\overline{r}_{i,j} \triangleq \lim_{n\to\infty} Pr\{X(n) = i, R(n) = j, D(n) = 0\},$$
$$0 \le i \le M + B, \quad 1 \le j \le K_1(\text{ or } K_2).$$

The probabilities $r_{i,j}$ and $\overline{r}_{i,j}$ can be obtained by the limiting probabilities $p_{i,j}$:

- $i = 0, \quad 1 \le j < K_1$

$$r_{0,j} = 0, \qquad \overline{r}_{0,j} = (1-\lambda)p_{0,j+1},$$
$$r_{0,K_1} = \lambda p_{0,1}, \qquad \overline{r}_{0,K_1} = (1-\lambda)p_{0,1} + (1-\lambda)p_{1,1},$$

- $1 \le i \le M-1, \quad 1 \le j < K_1,$

$$r_{i,j} = \lambda p_{i-1,j+1}, \qquad \overline{r}_{i,j} = (1-\lambda)p_{i,j+1},$$
$$r_{i,K_1} = \lambda p_{i,1}, \qquad \overline{r}_{i,K_1} = (1-\lambda)p_{i+1,1},$$

- $i = M, \quad 1 \le j < K_1$

$$r_{M,j} = \lambda p_{M-1,j+1}, \qquad \overline{r}_{M,j} = (1-\lambda)p_{M,j+1},$$
$$r_{M,K_1} = \lambda p_{M,1} + (1-\lambda)p_{M+1,1}, \qquad \overline{r}_{M,K_1} = 0,$$

- $M+1 \le i \le M+B_1, \quad 1 \le j < K_1,$

$$r_{i,j} = 0, \qquad \overline{r}_{i,j} = (1-\lambda)p_{i,j+1} + \lambda p_{i-1,j+1},$$
$$r_{i,K_1} = \lambda p_{i,1} + (1-\lambda)p_{i+1,1}, \qquad \overline{r}_{i,K_1} = 0,$$

- $i = M + B_1 + 1, \quad 1 \le j < K_2,$

$$r_{M+B_1+1,j} = 0, \qquad \overline{r}_{M+B_1+1,j} = (1-\lambda)p_{M+B_1+1,j+1},$$
$$r_{M+B_1+1,K_2} = \lambda p_{M+B_1+1,1} + (1-\lambda)p_{M+B_1+2,1}, \qquad \overline{r}_{M+B_1+1,K_2} = \lambda \sum_{j=2}^{K_1} p_{M+B_1,j},$$

- $M+B_1+1<i<M+B, \quad 1\le j<K_2,$

$r_{i,j}=0,$ $\quad \bar r_{i,j}=(1-\lambda)p_{i,j+1}+\lambda p_{i-1,j+1},$

$r_{i,K_2}=\lambda p_{i,1}+(1-\lambda)p_{i+1,1},$ $\quad \bar r_{i,K_2}=0,$

- $i=M+B, \quad 1\le j<K_2,$

$r_{M+B,j}=0,$ $\quad \bar r_{M+B,j}=p_{M+B,j+1},$

$r_{M+B,K_2}=\lambda p_{M+B,1},$ $\quad \bar r_{M+B,K_2}=0.$

The probability P_D that a cell departs at an arbitrary time is given by

$$P_D=\sum_{i=0}^{M}\sum_{j=1}^{K_1}r_{i,j}+\sum_{i=M+1}^{M+B_1}r_{i,K_1}+\sum_{i=M+B_1+1}^{M+B}r_{i,K_2}.$$

Suppose that a cell departs at an arbitrary time τ. The system state at time $\tau+$ is one of the following 4 cases:

- There are $i(1\le i\le M)$ tokens in the token pool. In this case, the next cell departure will occur as soon as a cell arrives. So, the interdeparture time is the time interval from instant τ to instant when the first cell after τ arrives.
- There is no token in the token pool and no cell in the data buffer. In this case, if there is at least one cell arrival during $(\tau,\tau+R(\tau))$, the next cell departure will occur at $\tau+R(\tau)$. So, the interdeparture time is $R(\tau)$. If there is no cell arrival during $(\tau,\tau+R(\tau))$, the next cell departure will occur at instant when the first cell after $\tau+R(\tau)$ arrives. So, the interdeparture time is $R(\tau)$+(the time interval from $\tau+R(\tau)$ to instant when the first cell after $\tau+R(\tau)$ arrives).
- There are $i(1\le i\le B_1)$ cells in the data buffer. In this case, if the buffer occupancy exceeds the threshold B_1 during $(\tau,\tau+K_1)$, the next cell departure will occur at time $\tau+Y_{i,K_1}+K_2$, where

$$\begin{aligned}Y_{i,K_1}\triangleq inf\{n\ge 1;(X(n),R(n))&\\ &=(M+B_1+1,K_2)|\,(X(0),R(0))\\ &=(M+i,K_1)\},\quad 1\le i\le B_1.\end{aligned}$$

Here Y_{i,K_1} is the time which it takes the buffer occupancy to hit the level B_1+1. So, the interdeparture time is $Y_{i,K_1}+K_2$. Otherwise, the interdeparture time is K_1.

– There are $i(B_1+1 \le i \le B)$ cells in the data buffer. In this case, the next cell departure will occur at time $\tau + K_2$. So, the interdeparture time is K_2.

In order to derive the probability distribution of Y_{i,K_1} in the third case, we introduce the square matrices P and P' of order $((M+B_1+1)K_1+K_2)$:

The matrix P consists of the first $M+B_1+1$ rows of the transition probability matrix Q of the Markov chain $\{(X(n),R(n)),n \ge 0\}$ and the blocks 0 in the final row. The matrix P' is the same as the matrix P except that the $(M+B_1, M+B_1+1)$-block C_1' of dimension $K_1 \times K_2$ in the matrix P is replaced by the block 0.

Event $\{Y_{i,K_1}=l\}$ means that the Markov chain $\{(X(n),R(n)),n \ge 0\}$ starting at the state $(M+i, K_1)$ stays in the level less than the level B_1+1 during $l-1$ transitions , and at the lth transition the Markov chain hits the state $(M+B_1+1, K_2)$. Therefore, we have

$$\begin{aligned} &Pr\{Y_{i,K_1}=l\} \\ &= [P'^{(l-1)}P](M+i,K_1;M+B_1+1,K_2) \\ &\triangleq f^l_{i,K_1}, \qquad l \ge 1, \quad 1 \le i \le B_1. \end{aligned}$$

where $[X](i_1,k_1;i_2,k_2)$ is the (k_1,k_2)-element of the (i_1,i_2)-block of the matrix X.

By above 4 cases, we can obtain the conditional probability generating function of the interdeparture time:

(1) The state of $X(\tau)$ is $i, \quad 0 \le i \le M-1$,

$$\begin{aligned} E[z^T|X(\tau)=i, D(\tau)=1] &= \frac{\lambda z}{1-(1-\lambda)z} \\ &\triangleq T_-(z). \end{aligned}$$

(2) The state of $X(\tau)$ is M, and $R(\tau)=j, \quad 1 \le j \le K_1$,

$$\begin{aligned} &E[z^T|X(\tau)=M, R(\tau)=j, D(\tau)=1] \\ &= [1-(1-\lambda)^j]z^j + [(1-\lambda)z]^j \frac{\lambda z}{1-(1-\lambda)z} \\ &\triangleq T_{0,j}(z). \end{aligned}$$

(3) The state of $X(\tau)$ is $M+i(1 \le i \le B_1)$, and $R(\tau)=K_1$,

$$\begin{aligned} E[z^T|X(\tau) = M+i, R(\tau)=K_1, D(\tau)=1] \\ = [\sum_{k=1}^{K_1-1} f_{i,K_1}^k z^{k+K_2} + (1-\sum_{k=1}^{K_1-1} f_{i,K_1}^k) z^{K_1}] \\ \triangleq T_{i,K_1}(z). \end{aligned}$$

(4) The state of $X(\tau)$ is $M+i(i \ge B_1+1)$, and $R(\tau)=K_2$,

$$\begin{aligned} E[z^T|X(\tau) = M+i, R(\tau)=K_2, D(\tau)=1] = z^{K_2} \\ \triangleq T_+(z). \end{aligned}$$

Finally, we obtain the probability generating function of the interdeparture time

$$\begin{aligned} T(z) = \frac{1}{P_D}[(\sum_{i=0}^{M-1}\sum_{j=1}^{K_1} r_{i,j})T_-(z) + \sum_{j=1}^{K_1} r_{M,j}T_{0,j}(z) \\ + \sum_{i=M+1}^{M+B_1} r_{i,K_1}T_{i-M,K_1}(z) + \sum_{i=M+B_1+1}^{M+B} r_{i,K_2}T_+(z)]. \end{aligned}$$

The mean and the variance of the interdeparture time is given by differentiating this probability generating function as

$$\begin{aligned} E[T] &= \frac{d}{dz}T(z)|_{z=1}, \\ Var(T) &= \frac{d^2}{dz^2}T(z)|_{z=1} + E[T](1-E[T]). \end{aligned}$$

4. The LB scheme with MMBP arrival

The cell arrival process at each slot boundary is governed by an irreducible, aperiodic discrete-time Markov chain $\{J(n), n \ge 0\}$ with finite state space $\{1, 2, \cdots, N\}$, called the modulating Markov chain. The transition between states of the Markov chain$\{J(n), n \ge 0\}$ takes place only at the slot boundary, and the transition probability matrix of the Markov chain $\{J(n), n \geq 0\}$ is $H = (h_{i,j})_{i,j=1,\cdots,N}$. We assume that H is invertible. When the modulating Markov chain $\{J(n), n \geq 0\}$ is in state i at time $n+$, the cell arrival at time $n+1$ is Bernoulli with probability λ_i. Let Λ be a diagonal matrix represented by $\Lambda =$ diag$(\lambda_1, \lambda_2, \cdots, \lambda_N)$. Then, a Markov-modulated Bernoulli process (MMBP) is characterized by the transition probability matrix H and the arrival probability matrix Λ. Let $\pi \triangleq (\pi_1, \pi_2, \cdots, \pi_N)$ be the stationary probability vector of the Markov chain $\{J(n), n \geq 0\}$. The effective arrival probability λ^*, which is defined as the inverse of the expected length of the cell interarrival time, is given by $\pi\Lambda e$, where $e = (1, 1, \cdots, 1)^T$. We assume that transitions of the Markov chain $\{J(n), n \geq 0\}$ are independent from any other events in the system.

The evolution of the system can be described by a 3-dimensional Markov chain $\{(X(n), R(n), J(n)), n \geq 0\}$, where $X(n)$ and $R(n)$ are the same as ones in the previous section and $J(n)$ is the state of the modulating Markov chain at time $n+$.

4.1. The buffer occupancy distribution

Define the limiting probabilities and the probability vectors

$$p^l_{i,j} \triangleq \lim_{n\to\infty} Pr\{X(n) = i, R(n) = j, J(n) = l\},$$
$$p_{i,j} \triangleq (p^1_{i.j}, p^2_{i,j}, \cdots, p^N_{i,j}),$$
$$0 \leq i \leq M + B, \quad 1 \leq j \leq K_1(\text{ or } K_2), \quad 1 \leq l \leq N.$$

By the Chapman-Kolmogorov's forward equation we obtain the following equations for the limiting probabilities $p_{i,j}$:

(2–1) $$p_{0,j} = p_{0,j+1}D, \qquad 1 \leq j < K_1,$$

(2–2) $$p_{0,K_1} = p_{1,1}D + p_{0,1}H,$$

(2–3) $$p_{i,j} = p_{i,j+1}D + p_{i-1,j+1}D_1, \quad 0 < i < M + B_1 + 1, \quad 1 \leq j < K_1,$$

(2–4) $$p_{i,K_1} = p_{i+1,1}D + p_{i,1}D_1, \qquad 0 < i < M + B_1 + 1,$$

(2–5) $$p_{M+B_1+1,j} = p_{M+B_1+1,j+1}D, \qquad 1 \leq j < K_2,$$

(2–6) $$p_{M+B_1+1,K_2} = p_{M+B_1+2,1}D + p_{M+B_1+1,1}D_1 + \sum_{j\geq 2} p_{M+B_1,j}D_1,$$

(2–7) $$p_{i,j} = p_{i,j+1}D + p_{i-1,j+1}D_1, \quad M+B_1+1 < i < M+B, \quad 1 \leq j < K_2,$$

(2–8) $$p_{i,K_2} = p_{i+1,1}D + p_{i,1}D_1, \quad M+B_1+1 < i < M+B,$$

(2–9) $$p_{M+B,j} = p_{M+B,j+1}H + p_{M+B-1,j+1}D_1, \quad 1 \leq j < K_2,$$

(2–10) $$p_{M+B,K_2} = p_{M+B,1}D_1.$$

where $D \triangleq (I - \Lambda)H$ and $D_1 \triangleq \Lambda H$ are $N \times N$ matrices.
Since H is invertible, the matrices D and D_1 are invertible. From $(2-1)$, we have

(2–11) $$p_{0,j+1} = p_{0,1}(D^{-1})^j, \quad 1 \leq j < K_1.$$

By $(2-2)$ and $(2-11)$, we can express $p_{1,1}$ in terms of $p_{0,1}$. The equations $(2-3)$ and $(2-4)$ can be rewritten as the following equations:

(2–12) $$p_{i,j+1} = p_{i,j}D^{-1} - p_{i-1,j+1}D_1D^{-1}, \quad 0 < i < M+B_1+1, \quad 1 \leq j < K_1.$$

(2–13) $$p_{i+1,1} = p_{i,K_1}D^{-1} - p_{i,1}D_1D^{-1}, \quad 0 < i < M+B_1+1.$$

By applying $(2-12)$ and $(2-13)$ recursively, $p_{i,j}(i = 1,\cdots,M+B_1, j = 1,2,\cdots,K_1)$ and $p_{M+B_1+1,1}$ can be expressed in terms of $p_{0,1}$. By $(2-5) \sim (2-9)$ we can express all $p_{i,j}(i = M+B_1+1,\cdots,M+B, j = 1,2,\cdots,K_2)$ in terms of $p_{0,1}$.
The unknown number $p_{0,1}$ can be obtained by the following boundary conditions:

$$\sum_{i=0}^{M+B_1}\sum_{j=1}^{K_1} p_{i,j}^l + \sum_{i=M+B_1+1}^{M+B}\sum_{j=1}^{K_2} p_{i,j}^l = \pi_l, \qquad l = 1,2,\cdots,N-1.$$

$$\left[\sum_{i=0}^{M+B_1}\sum_{j=1}^{K_1} p_{i,j} + \sum_{i=M+B_1+1}^{M+B}\sum_{j=1}^{K_2} p_{i,j}\right]e = 1.$$

Using the probability vectors $p_{i,j}$, we obtain the following performance measures:

a. Cell loss probability:

$$P_B = \frac{\sum_{j=2}^{K_2} p_{M+B,j}\Lambda e}{[\sum_{i=0}^{M+B_1}\sum_{j=1}^{K_1} p_{i,j} + \sum_{i=M+B_1+1}^{M+B}\sum_{j=1}^{K_2} p_{i,j}]\Lambda e}$$

b. Token loss probability:

$$P_T = p_{0,1}(I - \Lambda)e.$$

c. Buffer occupancy distribution:

$$Pr\{\text{ buffer occupancy} = i\} = \begin{cases} \sum_{i=0}^{M}\sum_{j=1}^{K_1} p_{i,j}e, & i = 0, \\ \sum_{j=1}^{K_1} p_{M+i,j}e, & 1 \le i \le B_1. \\ \sum_{j=1}^{K_2} p_{M+i,j}e, & B_1 + 1 \le i \le B. \end{cases}$$

d. Mean buffer occupancy:

$$M_b = [\sum_{i=0}^{B_1}\sum_{j=1}^{K_1} i p_{M+i,j} + \sum_{i=B_1+1}^{B}\sum_{j=1}^{K_2} i p_{M+i,j}]e$$

Using the Little's Law, we have the mean waiting time in the data buffer

$$W = \frac{M_b}{(1 - P_B)\lambda^*}.$$

If we set all $\lambda_i (i = 1, \cdots, N)$ equal to λ, the model is reduced to the LB scheme with dynamic leaky rate when the arrival process is Bernoulli process.

4.2. The interdeparture time distribution

In this subsection we analyze the interdeparture time when the arrival process is MMBP. Define the limiting probabilities and the probability vectors

$$r^l_{i,j} \triangleq \lim_{n\to\infty} Pr\{X(n)=i, R(n)=j, J(n)=l, D(n)=1\},$$
$$\overline{r}^l_{i,j} \triangleq \lim_{n\to\infty} Pr\{X(n)=i, R(n)=j, J(n)=l, D(n)=0\},$$
$$r_{i,j} \triangleq (r^1_{i,j},\cdots,r^N_{i,j}), \qquad \overline{r}_{i,j} \triangleq (\overline{r}^1_{i,j},\cdots,\overline{r}^N_{i,j}),$$
$$0 \le i \le M+B, \quad 1 \le j \le K_1(\text{ or } K_2), \quad 1 \le l \le N.$$

The probability vectors $r_{i,j}$ and $\overline{r}_{i,j}$ can be obtained by the limiting probability vectors $p_{i,j}$:

- $i=0, \quad 1 \le j < K_1,$

  $r_{0,j} = 0,$ $\qquad \overline{r}_{0,j} = p_{0,j+1}D,$

  $r_{0,K_1} = p_{0,1}D_1,$ $\qquad \overline{r}_{0,K_1} = p_{0,1}D + p_{1,1}D,$

- $1 \le i \le M-1, \quad 1 \le j < K_1,$

  $r_{i,j} = p_{i-1,j+1}D_1,$ $\qquad \overline{r}_{i,j} = p_{i,j+1}D,$

  $r_{i,K_1} = p_{i,1}D_1,$ $\qquad \overline{r}_{i,K_1} = p_{i+1,1}D,$

- $i=M, \quad 1 \le j < K_1,$

  $r_{M,j} = p_{M-1,j+1}D_1,$ $\qquad \overline{r}_{M,j} = p_{M,j+1}D,$

  $r_{M,K_1} = p_{M,1}D_1 + p_{M+1,1}D,$ $\qquad \overline{r}_{M,K_1} = 0,$

- $M+1 \le i \le M+B_1, \quad 1 \le j < K_1,$

  $r_{i,j} = 0,$ $\qquad \overline{r}_{i,j} = p_{i,j+1}D + p_{i-1,j+1}D_1,$

  $r_{i,K_1} = p_{i,1}D_1 + p_{i+1,1}D,$ $\qquad \overline{r}_{i,K_1} = 0,$

- $i = M+B_1+1, \quad 1 \le j < K_2,$

  $r_{M+B_1+1,j} = 0,$ $\qquad \overline{r}_{M+B_1+1,j} = p_{M+B_1+1,j+1}D,$

  $r_{M+B_1+1,K_2} = p_{M+B_1+1,1}D_1 + p_{M+B_1+2,1}D,$ $\qquad \overline{r}_{M+B_1+1,K_2} = \sum_{j=2}^{K_1} p_{M+B_1,j}D_1,$

- $M+B_1+1 < i < M+B, \quad 1 \le j < K_2,$

  $r_{i,j} = 0,$ $\qquad \overline{r}_{i,j} = p_{i,j+1}D + p_{i-1,j+1}D_1,$

  $r_{i,K_2} = p_{i,1}D_1 + p_{i+1,1}D,$ $\qquad \overline{r}_{i,K_2} = 0,$

- $i = M+B, \quad 1 \le j < K_2,$

  $r_{M+B,j} = 0,$ $\qquad \overline{r}_{M+B,j} = p_{M+B,j+1},$

  $r_{M+B,K_2} = p_{M+B,1}D_1,$ $\qquad \overline{r}_{M+B,K_2} = 0.$

The probability P_D that a cell departs at an arbitrary time is given by

$$P_D = [\sum_{i=0}^{M}\sum_{j=1}^{K_1} r_{i,j} + \sum_{i=M+1}^{M+B_1} r_{i,K_1} + \sum_{i=M+B_1+1}^{M+B} r_{i,K_2}]e$$

As in the subsection 3.2, we define the hitting time for the level B_1+1

$$\begin{aligned} Y_{i,K_1}(k,m) \triangleq inf\{n \geq 1; (X(n), R(n), J(n)) = (M+B_1+1, K_2, m) \\ \mid (X(0), R(0), J(0)) = (M+i, K_1, k)\}, \\ 1 \leq i \leq B_1, \qquad 1 \leq k,m \leq N. \end{aligned}$$

Then, the probability distribution of $Y_{i,K_1}(k,m)$ is given by

$$\begin{aligned} &Pr\{Y_{i,K_1}(k,m) = l\} \\ &= [\overline{P}^{'(l-1)}\overline{P}](M+i, K_1, k; M+B_1+1, K_2, m) \\ &\triangleq f^l_{i,K_1}(k,m), \quad l \geq 1, \quad 1 \leq i \leq B_1, \quad 1 \leq k,m \leq N. \end{aligned}$$

where $[X](i_1,k_1,m_1;i_2,k_2,m_2)$ is the $((k_1-1)N+m_1,(k_2-1)N+m_2)$ -element of the (i_1,i_2)-block of the matrix X.
Here the matrix $\overline{P}$ is similar in form with the matrix P with blocks substituted the elements λ and $1-\lambda$ by ΛH and $(I-\Lambda)H$ respectively, and the matrix $\overline{P}^{'}$ is the same as the matrix $\overline{P}$ except that the block $C_1^{'}$ of dimension $K_1N \times K_2N$ is replaced by the block 0.
Let f^l_{i,K_1} be the probability matrix with the (k,m)-element $f^l_{i,K_1}(k,m)$. Suppose that a cell departs at an arbitrary time τ. By considering the system state at time τ as in the subsection 3.2, we can obtain the conditional probability generating function of the interdeparture time:

(1) The state of $X(\tau)$ is i, and $J(\tau) = l$, $0 \leq i \leq M-1$,

$$\begin{aligned} &E[z^T|X(\tau) = i, J(\tau) = l, D(\tau) = 1] \\ &= l - \text{th row of } [\{I-(I-\Lambda)Hz\}^{-1}\Lambda ez] \\ &\triangleq T^l_{-}(z). \end{aligned}$$

(2) The state of $X(\tau)$ is M, $R(\tau)=j$, and $J(\tau)=l$, $\quad 1 \le j \le K_1$,

$$\begin{aligned}
&E[z^T|X(\tau)=M, R(\tau)=j, J(\tau)=l, D(\tau)=1] \\
&= l-\text{th row of } \{\sum_{k=1}^{j}[(I-\Lambda)H]^{k-1}\Lambda e z^j \\
&\quad + [(I-\Lambda)Hz]^j[I-(I-\Lambda)Hz]^{-1}\Lambda e z\} \\
&\triangleq T^l_{0,j}(z).
\end{aligned}$$

(3) The state of $X(\tau)$ is $M+i(1\le i\le B_1)$, $R(\tau)=K_1$, and $J(\tau)=l$,

$$\begin{aligned}
&E[z^T|X(\tau)=M+i, R(\tau)=K_1, J(\tau)=l, D(\tau)=1] \\
&= l-\text{th row of } [\sum_{k=1}^{K_1-1} f^k_{i,K_1} e z^{k+K_2} + (e-\sum_{k=1}^{K_1-1} f^k_{i,K_1} e) z^{K_1}] \\
&\triangleq T^l_{i,K_1}(z).
\end{aligned}$$

(4) The state of $X(\tau)$ is $M+i(i\ge B_1+1)$, $R(\tau)=K_2$ and $J(\tau)=l$,

$$\begin{aligned}
&E[z^T|X(\tau)=M+i, R(\tau)=K_2, J(\tau)=l, D(\tau)=1] \\
&= l-\text{th row of } [e z^{K_2}] \\
&\triangleq T^l_{+}(z).
\end{aligned}$$

Combining the above 4-cases, we obtain the probability generating function of the interdeparture time

$$\begin{aligned}
T(z) = \frac{1}{P_D}[(\sum_{i=0}^{M-1}\sum_{j=1}^{K_1} r_{i,j})T_{-}(z) + \sum_{j=1}^{K_1} r_{M,j}T_{0,j}(z) \\
+ \sum_{i=M+1}^{M+B_1} r_{i,K_1}T_{i-M,K_1}(z) + \sum_{i=M+B_1+1}^{M+B} r_{i,K_2}T_{+}(z)],
\end{aligned}$$

where $T_{\cdot}(z) = (T^1_{\cdot}(z), \cdots, T^N_{\cdot}(z))^T$.

The mean and the variance of the interdeparture time are given by differentiating this probability generating function as

$$\begin{aligned}
E[T] &= \frac{d}{dz}T(z)|_{z=1}, \\
Var(T) &= \frac{d^2}{dz^2}T(z)|_{z=1} + E[T](1-E[T]).
\end{aligned}$$

5. Numerical examples

In this section, we give some numerical examples to compare the performances of the LB scheme with dynamic leaky rate and the LB scheme with static leaky rate. As an input process for numerical examples, we use a two-state MMBP

$$H = \begin{vmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{vmatrix} \qquad \Lambda = \begin{vmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{vmatrix}.$$

The effective arrival probability λ^* for this MMBP is given by

$$\lambda^* = \frac{h_{1,2}\lambda_2 + h_{2,1}\lambda_1}{h_{1,2} + h_{2,1}}$$

In all figures we use the parameters

the transition probabilities $h_{1,2} = 0.6$ and $h_{2,1} = 0.2$,
the effective arrival probability $\lambda^* = 0.096$ (except for Fig. 8),
the threshold $B_1 = 3$,
the token generation periods $K_1 = 11, K_2 = 9($ or $8)$.

and the notations

S: the LB scheme with the token generation period 10,
D_1: the LB scheme with the token generation periods $K_1 = 11$, $K_2 = 9$,
D_2: the LB scheme with the token generation periods $K_1 = 11$, $K_2 = 8$.

For the effective arrival probability $\lambda^* = 0.096$, the weighted average token generation period in the proposed LB scheme is about 10, we compare the proposed LB scheme with the LB scheme with the token generation period 10.

Plotted in Fig. 2 is the cell loss probability of the proposed LB scheme as a function of the data buffer size plus the token pool size. From the figure we see that the cell loss probability in the proposed LB scheme is less than that of the LB scheme with token generation period 10. In Fig. 3 and 4 we show the cell loss probability and the mean waiting time of the proposed LB scheme as a function of the data buffer size. In the figures we keep the token pool size at constant 10. From the Fig.

2 and 3 we see that the cell loss probability is mainly affected by the data buffer size. We also see from Fig. 4 that the mean waiting time is reduced considerably in comparison with the LB scheme with token

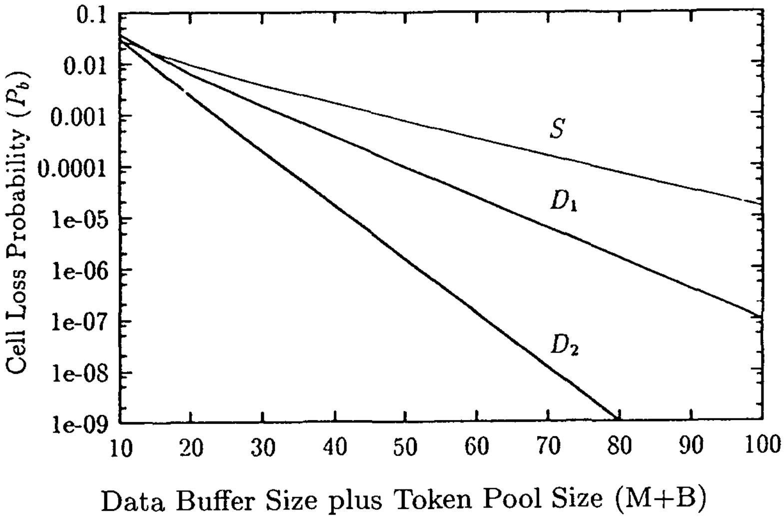

FIGURE 2. The cell loss probability vs. the data buffer size plus the token pool size, $\lambda^* = 0.096$, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $B_1 = 3$

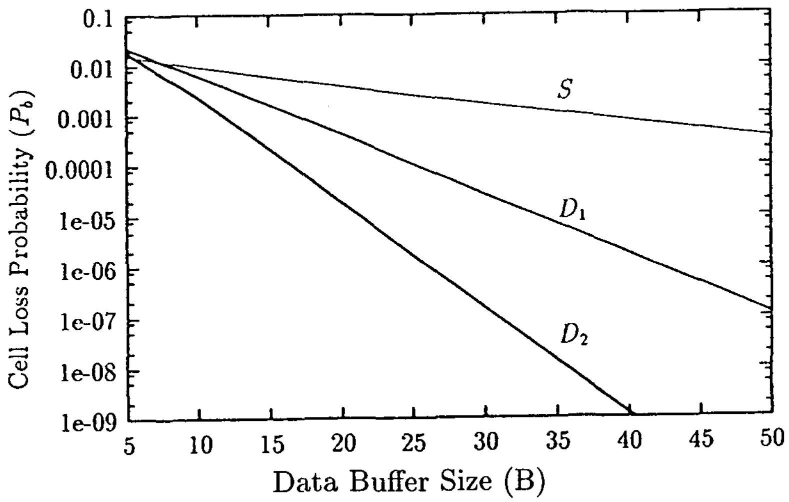

FIGURE 3. The cell loss probability vs. data buffer size $\lambda^* = 0.096$, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $M = 10$

generation period 10 as the data buffer size increases. Therefore, the required data buffer and token pool size in the proposed LB scheme can be reduced to satisfy the same QoS as in the LB scheme with static leaky rate.

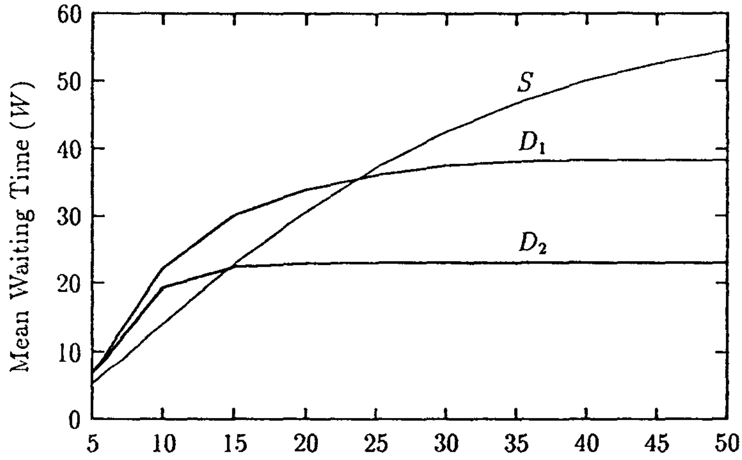

FIGURE 4. The mean waiting time in the buffer vs. the data buffer size $\lambda^* = 0.096$, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $B = 10$

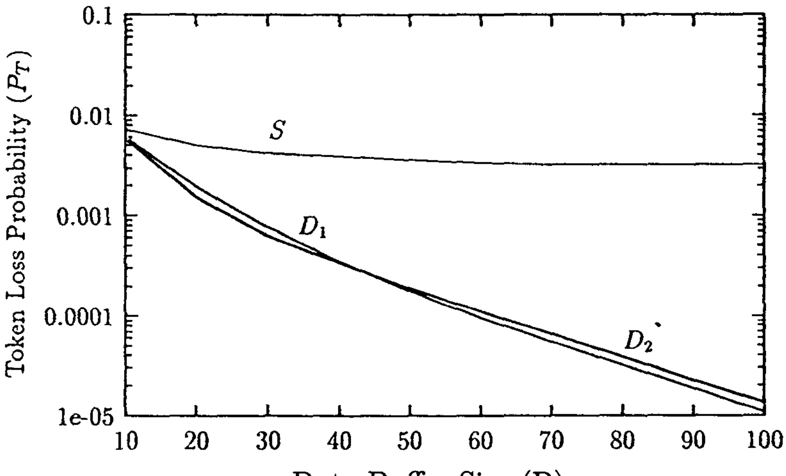

FIGURE 5. The token probability vs. data buffer size plus the token pool size, $\lambda^* = 0.096$, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $B_1 = 3$

Fig. 5 and Fig. 6 show the token loss probability as a function of the data buffer size plus the token pool size and the token pool size respectively. From the figures we see that the token loss probability in the LB scheme with dynamic leaky rate is less than that of the LB scheme with static leaky rate.

We also investigate the squared coefficient of variation $C^2(= Var(T)/E[T]^2)$ of the interdeparture time as a measure for the smoothness of the output process. Fig. 7 shows the squared coefficient of variation of the interdeparture time as a function of the data buffer size when the token pool size is 10. From the figure we see that the squared coefficient of variations of the interdeparture time in the proposed LB scheme are larger than that of the LB scheme with token generation period 10. We note from the figure that the token generation periods in the LB scheme with dynamic leaky rate must be determined by considering the shaping function. Fig. 8 shows the squared coefficient of variation of the interdeparture time as a function of the effective arrival probability when the data buffer size and the token pool size are 15 and 10 respectively. From the figure we see that the squared coefficient of variations of the interdeparture time decrease to zero as the effective arrival probability increases.

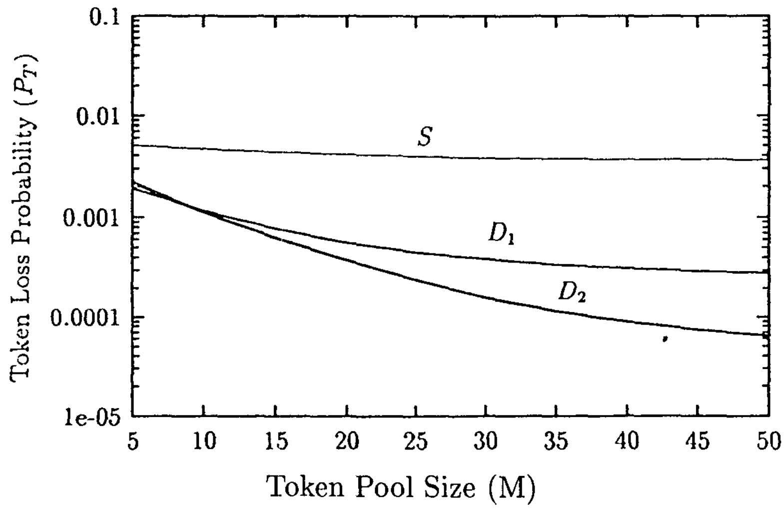

FIGURE 6. The token loss probability vs. the token pool size
$\lambda^* = 0.096$, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $B = 15$

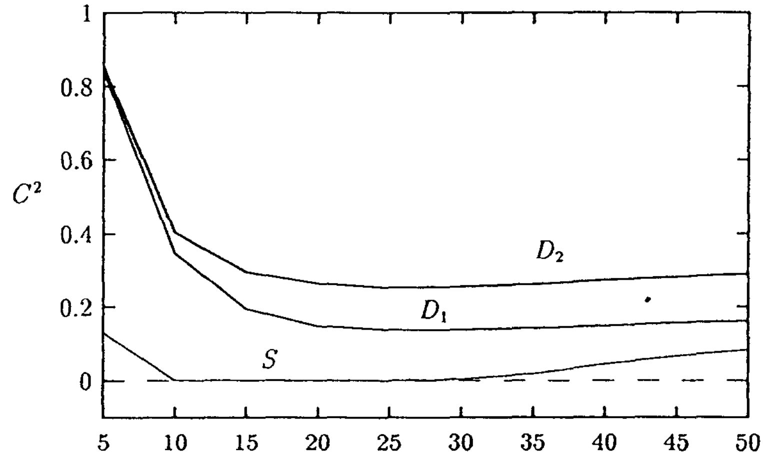

FIGURE 7. The squared coefficient of variation of the interdeparture time vs. the data buffer size, $\lambda^* = 0.096$, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $B_1 = 3$, $M = 10$

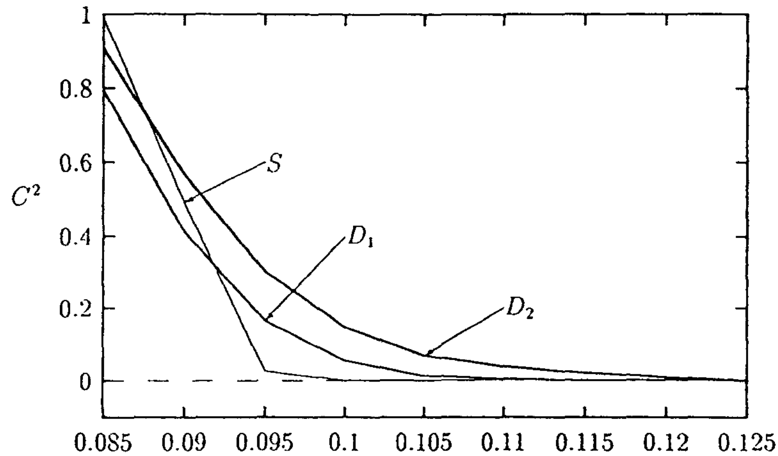

FIGURE 8. The squared coefficient of variation of the interdeparture time vs. the effective arrival probability, $h_{1,2} = 0.6$, $h_{2,1} = 0.2$, $B = 15$, $M = 10$

6. Conclusion

In this paper, we analyze the performance of the LB scheme with dynamic leaky rate control. We use an MMBP to model the bursty input traffics and obtain the cell loss probability, the token loss probability, the mean waiting time and the interdeparture time distribution. We also compare the performance measures of the LB scheme with dynamic leaky rate and the LB scheme with static leaky rate through the numerical examples. From these examples, we note that the cell loss probability, the token loss probability and the mean waiting time in the LB scheme with dynamic leaky rate are reduced in comparison with the LB scheme with static leaky rate. Therefore, the required data buffer and token pool size in the LB scheme with dynamic leaky rate can be reduced to satisfy the same QoS as in the LB scheme with static leaky rate. We also note that the output process of the LB scheme with dynamic leaky rate must be adjusted with proper token generation periods, taking into account the shaping function.

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