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CATEGORIES OF NEARNESS FRAMES

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ABSTRACT. We investigate categorical properties of the category NFrm of nearness frames and uniform homomorphisms. We introduce a concept of weakly strong nearness frames and study its permanence properties.

1. Introduction

Bénabou noticed that frames(=locales=complete Heyting algebras) can be used to study topological structure in the setting of order structure([5]) and Herrlich has introduced the concept of nearness which generalize topological structure, uniform structure, proximal structure and contiguity structure([6], [7]).

Combining these two concepts, Banaschewski and Pultr have introduced the concept of nearness frames and then constructed completions of nearness frames([2], [4]) which give rise to coreflection for strong nearness frames([3]).

Hong and Kim constructed Cauchy completions of nearness frames and show that the category of strong Cauchy complete nearness frames is coreflective in the category **SNFrm** of strong nearness frames and uniform homomorphisms([8]).

The purpose of this paper is to study categorical properties of nearness frames.

We show that the forgetful functor G of the category **NFrm** of nearness frames and uniform homomorphisms to the category **Frm** of frames and frame homomorphisms has the right adjoint. Moreover, we show

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that **NFrm** is complete, cocomplete and study permanence properties of important subcategories of **NFrm**.

Furthermore, generalizing strong nearness frames, we introduce a concept of weakly strong nearness frames and study its categorical properties.

We include some basic definitions for the completeness of the paper.

DEFINITION 1.1. A complete lattice L is said to be a *frame* if for any element x and any subset S of L,

$$x \wedge \left(\bigvee S\right) = \bigvee\{x \wedge y \mid y \in S\}.$$

In the following, top(bottom,resp) of a frame L will be denoted by $e(0, \text{resp})$, and for any $a \in L$, it has the pseudo complement $a^* = \bigvee\{x \in L \mid a \wedge x = 0\}$.

Furthermore, we write $a \prec b$ if $a^* \vee b = e$ and L is called a *regular frame* if for any $x \in L,\ x = \bigvee\{a \in L \mid a \prec x\}$.

DEFINITION 1.2. A map $h : L \to M$ between frames is called a *frame homomorphism* if h preserves all finite meets and arbitrary joins.

A subset C of a frame L is called a *cover* of L if $\bigvee C = e$ and the set of covers of L will be denoted by $CovL$. For $C, D \in CovL$, we say that C refines D if for any $c \in C$, there is $d \in D$ with $c \leq d$. In this case, we write $C \leq D$. Moreover, $(CovL, \leq)$ is a quasi-ordered set.

For $x \in L$ and $C \in CovL$, $C_x = \bigvee\{c \in C \mid x \wedge c \neq 0\}$ and CC denotes the set $\{C_c \mid c \in C\}$. For $C, D \in CovL$, we write $C \leq^* D$ if $CC \leq D$.

NOTATION 1.3. For a subset $\mathcal{N}$ of $CovL$ and $x, y \in L$, we write $x \lhd_{\mathcal{N}} y$ or simply $x \lhd y$ if there is $C \in \mathcal{N}$ with $C_x \leq y$.

The following is due to Banaschewski and Pultr ([4]).

DEFINITION 1.4. (1) Let L be a frame and $\mathcal{N}$ a subset of $CovL$. Then $\mathcal{N}$ is called a *nearness* on L if $\mathcal{N}$ is a filter in $(CovL, \leq)$ and for any $x \in L,\ x = \bigvee\{a \in L \mid a \lhd x\}$ and $(L, \mathcal{N})$ is called a *nearness frame*.

(2) A map $h : (L, \mathcal{N}) \to (M, \mathcal{M})$ between nearness frames is called a *uniform homomorphism* if h is a frame homomorphism and for any $C \in \mathcal{N},\ h(C) \in \mathcal{M}$.

For any frame L, there is a nearness on L if and only if L is a regular frame, because $x \lhd_{\mathcal{N}} y$ implies $x \prec y$ for a nearness frame $(L, \mathcal{N})$ and for $\mathcal{N} = CovL$, $x \prec y$ implies $x \lhd_{\mathcal{N}} y$ (see [2] for the details).

DEFINITION 1.5. A nearness frame $(L, \mathcal{N})$ is said to be :

(1) *uniform* if for any $C \in \mathcal{N}$, there exists $D \in \mathcal{N}$ with $D \leq^* C$.
(2) *strong* if for any $C \in \mathcal{N}$, $\check{C} = \{x \in L \mid x \lhd y \text{ for some } y \in C\}$ belongs to $\mathcal{N}$.
(3) *almost uniform* if it is strong and $\lhd$ interpolates, i.e., if $x \lhd y$, there is $z \in L$ such that $x \lhd z \lhd y$.
(4) *totally bounded* if $\{C \in \mathcal{N} \mid C \text{ is finite }\}$ generates $\mathcal{N}$.

For the terminology not introduced in the paper, we refer to [1] for the category theory and [9][11] for frames.

2. The category of nearness frames

In what follows, the category of frames (regular frames, resp.) and frame homomorphisms will be denoted by **Frm** (**RFrm**, resp.) and **NFrm** denotes the category of nearness frames and uniform homomorphisms. One has immediately the forgetful functor U : **NFrm** $\to$ **RFrm**.

We define F : **RFrm** $\to$ **NFrm** as follows : for any $L \in$ **RFrm**, $F(L) = (L, CovL)$ and $f \in$ **RFrm**, $F(f) = f$. Then it is clear that F is a functor, for any frame homomorphism preserves covers.

PROPOSITION 2.1. *The functor F :* **RFrm** $\to$ **NFrm** *is embedding and F is a right adjoint of U :* **NFrm** $\to$ **RFrm**.

PROOF. The first half is immediate from $UF = 1_{\mathbf{RFrm}}$. For any $L \in$ **RFrm**, let $\varepsilon_L : UFL \to L$ be the identity map of L. For any $(M, \mathcal{M}) \in$ **NFrm** and a frame homomorphism $h : U(M, \mathcal{M}) \to L$, the map $\overline{h} : (M, \mathcal{M}) \to FL\big(\overline{h}(x) = h(x)\big)$ is clearly a uniform homomorphism, because for any $C \in \mathcal{M}$, $h(C) \in CovL$. Furthermore, $\varepsilon_L \circ U(\overline{h}) = h$ and such an $\overline{h}$ is unique. Thus F is the right adjoint of U. □

COROLLARY 2.2. *The functor U :* **NFrm** $\to$ **RFrm** *preserves colimits and F :* **RFrm** $\to$ **NFrm** *preserves limits.*

Let $G : \mathbf{NFrm} \to \mathbf{Frm}$ denote the forgetful functor and $E : \mathbf{RFrm} \to \mathbf{Frm}$ the embedding functor, then $G = E \circ U$. For any frame L, it contains the largest regular subframe γL and since the homomorphic image of a regular frame is again a regular frame, the inclusion map $\gamma : \gamma L \to L$ gives rise to the $\mathbf{RFrm}$-coreflection of L (see [2] for The details) and hence E has the right adjoint. Thus we have the following by Proposition 2.1.

COROLLARY 2.3. *The functor $G : \mathbf{NFrm} \to \mathbf{Frm}$ has the right adjoint and therefore preserves colimits.*

Proof of the following lemma can be found in [10].

LEMMA 2.4. *Suppose that $(h_i : L_i \to M)_{i\in I}$ is an extremal epi sink in $\mathbf{Frm}$ such that $(L_i, \mathcal{N}_i)$ is a nearness frame for each $i \in I$. If $\mathcal{M}$ is a filter generated by $\mathcal{B} = \{\wedge_{j\in J} h_j(A_j) \mid J$ is a finite subset of $I, A_j \in \mathcal{N}_j\}$, then $\mathcal{M}$ is the G-final nearness on M with respect to $(h_i)_{i\in I}$.*

THEOREM 2.5. *The category $\mathbf{NFrm}$ is cocomplete.*

PROOF. Let $D : \mathbf{I} \to \mathbf{NFrm}$ be any diagram and let $\big(l_i : GD(i) \to L\big)_{i\in\mathbf{I}}$ be the colimit of $G \circ D$ in $\mathbf{Frm}$. Since $(l_i)_{i\in\mathbf{I}}$ is an extremal epi sink in $\mathbf{Frm}$, there is the G-final nearness $\mathcal{N}$ on L with respect to $(l_i)_{i\in\mathbf{I}}$ by the above lemma. We now show that $\big(l_i : D(i) \to (L, \mathcal{N})\big)_{i\in\mathbf{I}}$ is the colimit for D. Take any natural sink $\big(f_i : D(i) \to (M, \mathcal{M})\big)_{i\in\mathbf{I}}$ for D, then $\big(G(f_i)\big)_{i\in\mathbf{I}}$ is a natural sink for $G \circ D$ and hence there is a unique frame homomorphism $f : L \to M$ such that for any $i \in \mathbf{I}$, $f \circ l_i = G(f_i)$. Since $(l_i)_{i\in\mathbf{I}}$ is G-final, there is a unique uniform homomorphism $g : (L, \mathcal{N}) \to (M, \mathcal{M})$ with $G(g) = f$ and $g \circ l_i = f_i$. Since G is faithful, such a $g : (L, \mathcal{N}) \to (M, \mathcal{M})$ is unique. This completes the proof. □

THEOREM 2.6. *The category $\mathbf{NFrm}$ is complete.*

PROOF. It is enough to show that $\mathbf{NFrm}$ has equalizers and products. First we prove that $\mathbf{NFrm}$ has equalizers. Let $(L, \mathcal{N}) \underset{h_2}{\overset{h_1}{\rightrightarrows}} (M, \mathcal{M})$ be a pair of uniform homomorphisms and let $j : E \to L$ be the equalizer for h_1, h_2 in $\mathbf{Frm}$. In fact, E is the subframe $\{x \in L \mid h_1(x) = h_2(x)\}$ of L. Let $\big((E_i, \mathcal{N}_i)\big)_{i\in I}$ be the family of nearness frames such that E_i is a subframe of E and the inclusion frame homomorphism $j_i : E_i \to (L, \mathcal{N})$ is uniform for $i \in I$. Let E_o be the subframe of L generated by $\bigcup\{E_i \mid i \in I\}$, which is clearly a subframe of E. Let $h_i : E_i \to E_o$ be the corestriction of j_i to E_o, then $(h_i : E_i \to E_o)_{i\in I}$ is an extremal epi sink in **Frm** ; therefore there is the G-final nearness $\mathcal{N}_o$ on E_o with respect to $\big(h_i : (E_i, \mathcal{N}_i) \to E_o\big)_{i\in I}$. Let $m : E_o \to L$ be the inclusion homomorphism. Since $m \circ h_i = j_i$ and j_i is uniform for all $i \in I$, $m : (E_o, \mathcal{N}_o) \to (L, \mathcal{N})$ is also a uniform homomorphism. Clearly $h_1 \circ m = h_2 \circ m$. Take any uniform homomorphism $h : (H, \mathcal{H}) \to (L, \mathcal{N})$ with $h_1 \circ h = h_2 \circ h$. Then there is a unique frame homomorphism $k : H \to E$ with $j \circ k = h$. Let $k^o : H \to k(H)$ be the corestriction of k to $k(H)$, which is an onto frame homomorphism. Thus there is the G-final nearness $\mathcal{H}'$ on $k(H)$ with respect to $k^o : (H, \mathcal{H}) \to k(H)$. Let $n : k(H) \to L$ be the inclusion homomorphism, then $n \circ k^o = h$ implies that $n : \big(k(H), \mathcal{H}'\big) \to (L, \mathcal{N})$ is uniform, because $k^o : (H, \mathcal{H}) \to (k(H), \mathcal{H}')$ is G-final. Thus there is $\lambda \in I$ such that $n = j_\lambda$, i.e., $\big(k(H), \mathcal{H}'\big) = (E_\lambda, \mathcal{N}_\lambda)$. Thus $k(H) \subseteq E_o$. Let $\overline{h} : H \to E_o$ be the corestriction of k to E_o, then $\overline{h} = (H, \mathcal{H}) \overset{k^o}{\to} (E_\lambda, \mathcal{N}_\lambda) \overset{j_\lambda}{\to} (E_o, \mathcal{N}_o)$ is a uniform homomorphism and $m \circ \overline{h} = h$. Since m is $1-1$, such an $\overline{h}$ is unique. Thus $m = equ(h_1, h_2)$ in **NFrm**.

Regarding products in **NFrm**, let $\big((L_i, \mathcal{N}_i)\big)_{i\in I}$ be a family of nearness frames. Let $L = \prod_{i\in I} L_i$ be the product of $(L_i)_{i\in I}$ in **Frm**. For each $\lambda \in I$ and $C_\lambda \in \mathcal{N}_\lambda$, define $\tilde{C}_\lambda = \{(x_i)_{i\in I} \in L \mid x_\lambda \in C_\lambda,\ x_i = 0 \text{ for } i \neq \lambda\}$. Take $C = (C_i)_{i\in I} \in \prod_{i\in I} \mathcal{N}_i$ and let $\tilde{C} = \bigcup_{i\in I} \tilde{C}_i$, then $\bigvee \tilde{C} = e_L$. Let $\mathcal{B} = \{\tilde{C} \mid C = (C_i)_{i\in I} \in \prod_{i\in I} \mathcal{N}_i\}$. Choose $\tilde{C}$ and $\tilde{D}$ in $\mathcal{B}$ with $C = (C_i)_{i\in I}$ and $D = (D_i)_{i\in I}$. For each $i \in I$, $E_i = C_i \wedge D_i \in \mathcal{N}_i$ and $E = (C_i \wedge D_i)_{i\in I} \in \prod_{i\in I} \mathcal{N}_i$. Since $\tilde{E}_\lambda \leq \tilde{C}_\lambda \wedge \tilde{D}_\lambda (\lambda \in I)$, $\tilde{E} \leq \tilde{C} \wedge \tilde{D}$. Then $\tilde{E} \in \mathcal{B}$ for $E \in \prod_{i\in I} \mathcal{N}_i$. Hence $\mathcal{B}$ is a filterbase on $\big(CovL, \leq\big)$. Let $\mathcal{N}$ be the filter on $(CovL, \leq)$ generated by $\mathcal{B}$. We claim that for $x = (x_i)_{i\in I}$, $y = (y_i)_{i\in I} \in L$, $x \lhd_{\mathcal{N}} y$ if $x_i \lhd y_i$ for all $i \in I$. Indeed, let $C_i \in \mathcal{N}_i$ with $C_{ix_i} \leq y_i$ and $C = (C_i)_{i\in I}$. Take any $(c_i)_{i\in I} \in \tilde{C}$ with $(c_i)_{i\in I} \wedge (x_i)_{i\in I} \neq 0$, then there is $\lambda \in I$ with $c_\lambda \wedge x_\lambda \neq 0$; hence $c_i = 0$ for all $i \neq \lambda$. Thus $c_\lambda \leq y_\lambda$ which implies $(c_i)_{i\in I} \leq (y_i)_{i\in I}$. Take any $x = (x_i)_{i\in I} \in L$, then $x_\lambda = \bigvee\{y_\lambda \in L_\lambda \mid y_\lambda \lhd x_\lambda\}$ for each $\lambda \in I$. For any $a_\lambda \in L_\lambda$, we define

$$\tilde{a}_\lambda = \begin{cases} 0, & i \neq \lambda \\ a_\lambda & i = \lambda \end{cases}$$

in L and $\tilde{x}_\lambda = \bigvee\{\tilde{y}_\lambda \mid y_\lambda \lhd x_\lambda\}$, then $x = \bigvee_{\lambda \in I} \tilde{x}_\lambda \leq \bigvee\{y \mid y \lhd x\} \leq x$. Thus $(L, \mathcal{N})$ is a nearness frame.

For each $\lambda \in I$, the λ-th projection $Pr_\lambda : L \to L_\lambda$ is a frame homomorphism. For any $\tilde{C} \in \mathcal{B}$, $Pr_\lambda(\tilde{C}) = \bigcup_{i \in I} Pr_\lambda(\tilde{C}_i) = \{0\} \cup C_\lambda \in \mathcal{N}_\lambda$ ($\lambda \in I$). Hence each Pr_λ is uniform. Take any source $\big(f_i : (M, \mathcal{M}) \to (L_i, \mathcal{N}_i)\big)_{i \in I}$ in **NFrm**, let $f = \sqcap_{i \in I} f_i : M \to L$ be the frame homomorphism with $Pr_i \circ f = f_i (i \in I)$. It remains to show that f is uniform. Take any $D \in \mathcal{M}$, then $f_i(D) \in \mathcal{N}_i$ for all $i \in I$. Put $C_i = f_i(D)$, and let $C = (C_i)_{i \in I} \in \prod_{i \in I} \mathcal{N}_i$, then $\tilde{C} \in \mathcal{N}$. Take any $x = (x_i)_{i \in I} \in \tilde{C}$, then there is $\lambda \in I$ with $x \in \tilde{C}_\lambda$; hence $x_\lambda \in C_\lambda$ and $x_i = 0$ for $i \neq \lambda$. Let $d \in D$ with $f_\lambda(d) = x_\lambda$, then $x \leq f(d)$. Thus $\tilde{C} \leq f(D)$, so that $f(D) \in \mathcal{N}$. Such an f is unique, for $(Pr_i)_{i \in I}$ is a mono-source. In all, $\big(Pr_i : (L, \mathcal{N}) \to (L_i, \mathcal{N}_i)\big)_{i \in I}$ is a product in **NFrm**. □

PROPOSITION 2.7. *The following subcategories of* **NFrm** *is closed under the formation of products in* **NFrm***:*

(1) *The category* **UFrm** *of uniform frames,*
(2) *The category* **SNFrm** *of strong nearness frames,*
(3) *The category* **AUFrm** *of almost uniform frames.*

PROOF. Let $(L_i, \mathcal{N}_i)_{i \in I}$ be a family in **NFrm** and $(L, \mathcal{N})$ the product of the family. Using the notions in the proof of Theorem 2.6, $\mathcal{B} = \{\tilde{C} | C = (C_i)_{i \in I} \in \prod_{i \in I} \mathcal{N}_i\}$ generates $\mathcal{N}$.

1) Suppose that each $(L_i, \mathcal{N}_i)$ is a uniform frame for all $i \in I$. Take any $\tilde{C} \in \mathcal{B}$, i.e., $\tilde{C} = \bigcup_{i \in I} \tilde{C}_i$, where $C = (C_i)_{i \in I} \in \prod_{i \in I} \mathcal{N}_i$. Then there is $D_i \in \mathcal{N}_i (i \in I)$ with $D_i \leq^* C_i$ and hence $D = (D_i)_{i \in I} \in \prod_{i \in I} \mathcal{N}_i$ and $\tilde{D} \in \mathcal{B}$. Take any $(d_i)_{i \in I} \in \tilde{D}$, then there is $\lambda \in I$ with $d_\lambda \in D_\lambda$ and $d_i = 0$ for all $i \neq \lambda$ so that there is $c_\lambda \in C_\lambda$ with $D_{\lambda_{d_\lambda}} \leq c_\lambda$. Put

$$a_i = \begin{cases} c_\lambda & \text{if } i = \lambda \\ 0 & \text{if } i \neq \lambda, \end{cases}$$

then $(a_i)_{i \in I} \in \tilde{C}$ and $\tilde{D}_{(d_i)_{i \in I}} \leq (a_i)_{i \in I}$, because for $(x_i)_{i \in I} \in \tilde{D}$, $(x_i)_{i \in I} \wedge (d_i)_{i \in I} \neq 0$ implies $x_\lambda \wedge d_\lambda \neq 0$ and hence $x_i = 0$ for $i \neq \lambda$. Thus $\tilde{D} \leq^* \tilde{C}$, so that $(L, \mathcal{N})$ is a uniform frame.

2) Suppose that each $(L_i, \mathcal{N}_i)$ is strong for all $i \in I$. For any $\tilde{C} \in \mathcal{B}(C = (C_i)_{i\in I} \in \prod_{i\in I} \mathcal{N}_i)$, $\check{C}_i \in \mathcal{N}_i (i \in I)$. Let $D = (\check{C}_i)_{i\in I} \in \prod_{i\in I} \mathcal{N}_i$ and take any $(d_i)_{i\in I} \in \tilde{D}$, then there is $(c_i)_{i\in I} \in \tilde{C}$ with $(d_i)_{i\in I} \triangleleft (c_i)_{i\in I}$, because $0 \triangleleft 0$ in any frame. Thus $\tilde{D} \leq \check{\tilde{C}}$ and therefore $\check{\tilde{C}} \in \mathcal{N}$.

3) Suppose that each $(L_i, \mathcal{N}_i)$ is almost uniform. By 2), it remains to show that $\triangleleft$ in $(L, \mathcal{N})$ interpolates, which is immediate from the fact that $(x_i)_{i\in I} \triangleleft (y_i)_{i\in I}$ in $(L, \mathcal{N})$ iff for all $i \in I$, $x_i \triangleleft y_i$. □

Generalizing strong nearness frames, we introduce a concept of weakly strong nearness frames.

DEFINITION 2.8. *A nearness frame $(L, \mathcal{N})$ is said to be weakly strong if for any $C \in \mathcal{N}$, $\overline{C} = \{y \in L | y \prec x$ for some $x \in C\}$ belongs to $\mathcal{N}$.*

Since $x \triangleleft y$ in a nearness frame $(L, \mathcal{N})$ implies $x \prec y$, for any $C \in \mathcal{N}$, $\check{C} \leq \overline{C} \leq C$; hence every strong nearness frame is weakly strong.

For a regular frame L, let $FCov(L)$ denote the nearness on L generated by the set of finite covers of L, then for any $x, y \in L$, $x \prec y$ iff $x \triangleleft y$ in $(L, FCov(L))$. Thus $(L, FCov(L))$ is weakly strong iff it is strong and hence a uniform frame, because a totally bounded frame is uniform iff it is strong.

Let **WSNFrm** denote the subcategory of **NFrm** determined by weakly strong nearness frames.

THEOREM 2.9. *The subcategory* **WSNFrm** *is closed under the G-final extremal epi sinks in* **NFrm** *and therefore closed under colimits in* **WSNFrm**.

PROOF. Let $(h_i : (L_i, \mathcal{N}_i) \longrightarrow (L, \mathcal{N}))_{i\in I}$ be a G-final extremal epi sink such that each $(L_i, \mathcal{N}_i)(i \in I)$ is weakly strong. By Lemma 2.4, $\mathcal{B} = \{\wedge_{j\in J} h_j(A_j) | J$ is a finite subset of I and $A_j \in \mathcal{N}_j\}$ generates $\mathcal{N}$. Take any $C = \wedge_{j\in J} h_j(A_j) \in \mathcal{B}$, then $\overline{A_j} \in \mathcal{N}_j$ for each $j \in J$. Since each $h_j (j \in J)$ is a frame homomorphism, $h_j(\overline{A_j}) \leq \overline{h_j(A_j)}$; hence $\wedge_{j\in J} h_j(\overline{A_j}) \leq \wedge_{j\in J} \overline{h_j(A_j)} \leq \overline{C}$, because $\prec$ is a sublattice of $L \times L$. Thus $\overline{C} \in \mathcal{N}$, so that $(L, \mathcal{N})$ is weakly strong. The second half follows from the proof of Theorem 2.5. □

COROLLARY 2.10. *The category* **WSNFrm** *is cocomplete.*

PROPOSITION 2.11. *The subcategory* **WSNFrm** *is productive in* **NFrm**.

PROOF. For any family $\big((L_i,\mathcal{N}_i)\big)_{i\in I}$ in **WSNFrm**, take any basic cover $\tilde{C}$ in the product $(L,\mathcal{N})$ of the family, i.e., $\tilde{C} = \cup_{i\in I}\tilde{C}_i$, where $(C_i)_{i\in I} \in \prod \mathcal{N}_i$. Since each $(L_i,\mathcal{N}_i)(i \in I)$ is weakly strong, $\overline{C_i} \in \mathcal{N}_i(i \in I)$; hence $\tilde{D} = \cup_{i\in I}\tilde{\overline{C_i}} \in \mathcal{N}$.

Since $(x_i)_{i\in I} \prec (y_i)_{i\in I}$ iff $x_i \prec y_i$ for all $i \in I$, $\tilde{D} \subseteq \overline{\tilde{C}}$ and therefore $\overline{\tilde{C}} \in \mathcal{N}$. Thus $(L,\mathcal{N})$ is weakly strong. □

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