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Tornado-Induced Extreme Waves in an Offshore Basin Revisited
토네이도가 유발한 막대한 파에 대한 재고

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Abstract □ The present study was initiated to protect floating nuclear power plants from the tornado. The solution shows that a tornado induces extreme waves of 27 ft (8.2 m) in height if it crosses the basin with a speed close to the critical speed. Waves generated by wind stress are ignored.

Keywords : tornado, waves, offshore basin, nuclear power plant

要　旨 : 본 연구는 인공항 속에 떠 있는 원자로를 토네이도로부터 보호하기 위하여 시작되었다. 연구결과에 의하면 토네이도가 유발할 수 있는 파고는 8.2 m이다. 본 연구에 있어 바람의 마찰 전단력에 의하여 발생하는 파는 작다고 가정하여 무시하였다.

핵심용어 : 회오리 바람, 파랑, 인공항, 원자로

1. INTRODUCTION

The probable severest tornado is one of the severest environmental loads to be considered for nuclear power plants located in the U.S. East and Gulf Coasts. Two floating nuclear power plants were planned in the Atlantic Ocean off the state of New Jersey of U.S.A. in 1973. The power plants were to be protected by semi-circular breakwaters with two small openings for the traffic of ships.

The floating power plants inside the basin should resist the possible severe forces if a tornado happens to cross the basin. Hence, the present study was undertaken to find extreme waves for the safety of the planned nuclear power plants (Chung, 1973).

The semi-circular basin has a constant depth of 45 ft (13.7 m) and a radius of 800 ft (244 m). The tornado, characterized by the pressure distribution and the tangential velocity of wind about the center of the tornado, crosses the basin with a constant forward speed on an arbitrary path. The forward speed of the tornado is in range of 5-70 MPH (8.05-112.7 km/h). By applying linear transient wave theory, the solution of tornado-induced waves is obtained as an eigenfunction expansion. Time-dependent wave displacements for various directions and forward speeds of tornado are computed and results are shown. Surface waves induced by wind stress are ignored. See Discussions and Conclusions.

2. TORNADO CHARACTERISTICS

Tornado over water, while rare, is one of the extreme environmental conditions imposed by the US Nuclear Regulatory Agency in 1973. The tornado to be considered in the East and Gulf Coasts has a forward design speed of 70 MPH and a tangential design speed of 290 MPH. The tornado pressure distribution is

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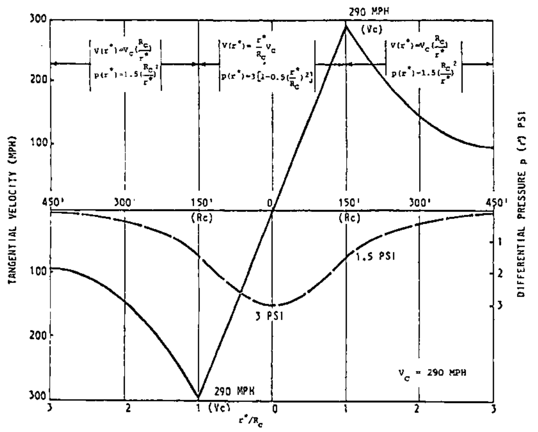

Fig. 1. Tangential velocity and differential pressure distribution about tornado center.

described by

$$p=\begin{cases} p_o\,[1-\dfrac{1}{2}(\dfrac{r^*}{R_C})^2] & \text{for } r^*\le R_C \\ \dfrac{1}{2}p_o\,(\dfrac{R_C}{r^*})^2 & \text{for } r^*> R_C \end{cases} \tag{1}$$

where p_o is the pressure drop at the center of the tornado, R_C is the distance from the center of the tornado to the boundary of the rotational zone and r* denotes the distance to a selected point from the center of the tornado as shown in Fig. 1. The values of p_o and R_C are -432 lb/ft (0.0064 kg/m) and 150 ft (45.75 m), respectively, at the condition of the tornado to be considered.

3. FORMULATION OF THE PROBLEM

We seek the time-dependent transient potential for the motion of the water induced by a passing tornado. We assume that the basin is semi-circular and closed. Furthermore, we assume that the water inside the basin is initially at rest and that a tornado moves forward to cross the basin on an arbitrary path from the point $(x_o,\ y_o)$ where the tornado is assumed to be generated at t=0. The coordinate system is shown in Fig. 2 in which the x- and y-coordinates lie on the surface of the calm water and the z-axis is directed upward.

The transient potential ϕ $(x,\ y,\ z;\ t)$ satisfies the following equations:

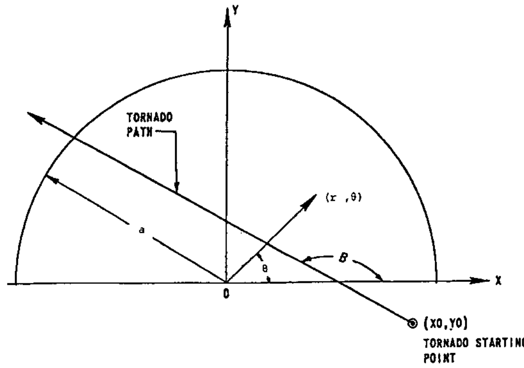

Fig. 2. Definition sketch.

$$\nabla^2\phi=0 \qquad \text{for } t>0 \tag{2}$$

$$g\eta+\phi_t+\frac{p}{\rho}=0 \qquad z=0 \tag{3}$$

$$\phi_z-\eta_t=0 \qquad \text{at } z=0 \tag{4}$$

$$\phi_n=0 \qquad \text{along basin wall and at bottom} \tag{5}$$

$$\phi=\phi_t=0 \qquad \text{at } t=0 \tag{6}$$

The pressure p in (3) is the tornado pressure on the surface of water. If the solution ϕ is found, the wave displacement $\eta(x,\ y;\ t)$ is given by

$$\eta=-\frac{1}{g}\,\phi_t-\frac{p-p_m}{\rho g} \qquad at \quad z=0 \tag{7}$$

where p_m is the mean pressure acting on the surface of water inside the basin. p_m is given by

$$p_m=\frac{2}{\pi a^2}\int_s p\ r\ dr d\theta \tag{8}$$

where a is the radius of the basin and s is the surface of water inside the basin.

4. WAVES INSIDE BASIN

We apply the finite Fourier transform with respect to the polar angle θ and the Hankel transform with respect to r (Sneddon, 1966). Then the solution to (2) through (6) is given by

$$\phi(r,\theta,z;t)=-\frac{1}{\pi\rho a^2}\sum_{i=1}^{\infty}\frac{I_i^oJ_o(\xi_i^o r)}{[J_o(\xi_i^o a)]^2}\,\frac{\cosh\,\xi_i^o(h+z)}{\cosh\,\xi_i^o h}$$

$$-\frac{4}{\pi\rho}\sum_{n=1}^{\infty}\sum_{i=1}^{\infty}\frac{(\xi_i^n)^2I_i^n}{(\xi_i^n a)^2-n^2}\,\frac{J_n(\xi_i^n r)}{[J_n(\xi_i^n a)]^2}$$

$$\frac{\cosh \xi_i^n (h+z)}{\cosh \xi_i^n h} \cos n\theta \tag{9}$$

where $J_n(x)$ is the Bessel function of order n, ξ_i^n is given by $\xi_i^n = \eta_i^n / a$ from $J'_n(\eta_i^n) = 0$,

$$\alpha_i^n = \sqrt{g\,\xi_i^n \tanh \xi_i^n h}$$

$$I_i^n = \int_o^t \bar{p}_i^n \cos \alpha_i^n (t-\eta)\, d\eta$$

$$\bar{p}_i^n = \int_o^a \int_o^\pi p J_n(\xi_i^n r) \cos n\theta \, dr d\theta$$

We now consider the numerical scheme for computing η in (7). Let the origin of the tornado be the point (x_o, y_o). Then the tornado moves on a straight path with a constant speed U_o. The center of the tornado at time t is given by

$$\bar{x} = x_o + \cos \beta\, U_o t, \quad \bar{y} = y_o + \sin \beta\, U_o t \tag{10}$$

The tornado pressure at a point (x, y) is given by

$$p(x, y; t) = \begin{cases} -432\,[1 - (\frac{r^*}{150})^2] & \text{for } r^* \leq 150 \\ -216(\frac{150}{r^*})^2 & \text{for } r^* > 150 \end{cases}$$

where $r^* = \sqrt{(x-\bar{x})^2 + (y-\bar{y})^2}$. The water surface inside the basin is divided by several elements as shown in Fig. 3 and the center of each area element is used for the mean value of the tornado pressure.

We apply the shallow water theory in order to check the accuracy of the results. The computed results from the two theories are shown for comparison. The shallow water theory is an approximation from the linear wave theory based on the assumption that the depth of the water is sufficiently small, compared with other significant dimensions (Stoker, 1996). See Appendix. In the present study, the depth of the water is considered sufficiently small compared with the diameter of the basin and R_C in (1).

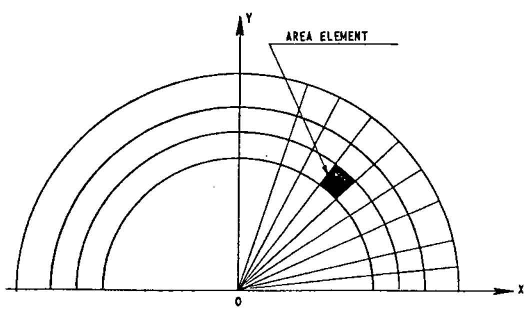

Fig. 3. Water surface segmentation.

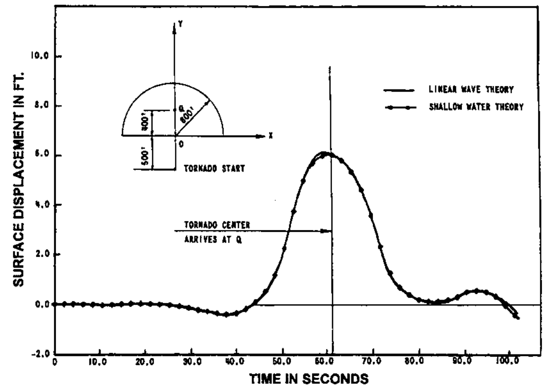

Fig. 4. Time history of displacement at Q for forward speed of 10 MPH.

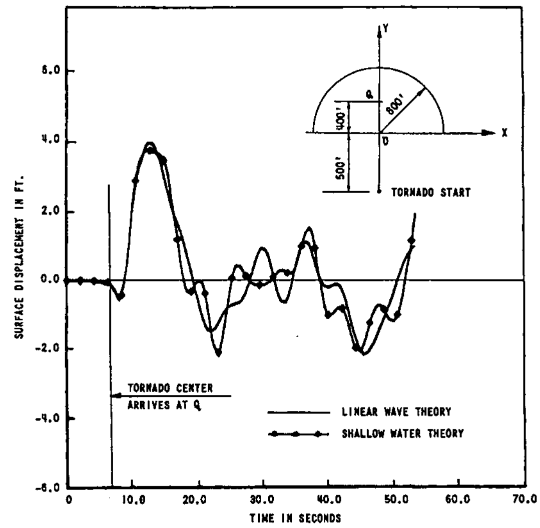

Fig. 5. Time history of surface displacement at Q for forward speed of 70 MPH.

5. DISCUSSIONS AND CONCLUSIONS

The wind stress excites a still water along the free surface by friction so that surface waves are generated. The tornado is transient in nature and stays inside the basin for a short period of time. Hence, the fetch of tornado inside the basin is short. Further, the height of

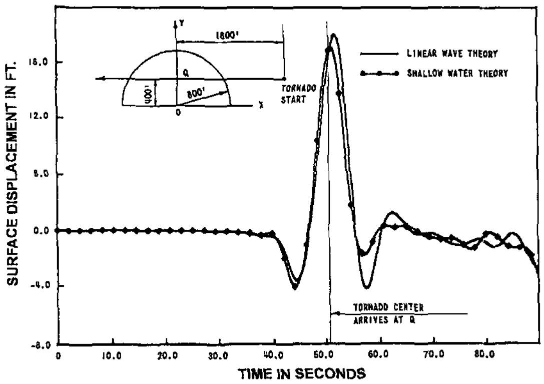

Fig. 6. Time history of surface displacement at Q for forward speed of 25.1 MPH.

the walls of the breakwaters above the still water level provides some shielding. It takes some time for the wind stress to build up large waves. But surface waves due to the free surface condition (3) and (7) are almost instant. Hence, the effect of wind stress on the generation of waves are ignored in the present study.

The surface displacements at selected points for various speeds and directions of the tornado are computed. Near the critical speed $U_o=\sqrt{gh}=1$, the surface displacement appears unstable as shown in Figs. 7, 8, and 9. Since the potential theory would fail to give a solution if the speed of a moving pressure point is critical, the critical speed is avoided. The limited search shows that the maximum surface displacement occurs at a speed close to the critical value. The maximum surface displacemnt is 27 ft (8.2 m) in

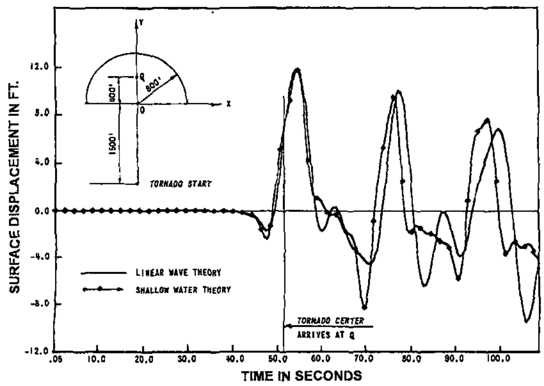

Fig. 7. Time history of surface displacement at Q for forward speed of 25.1 MPH.

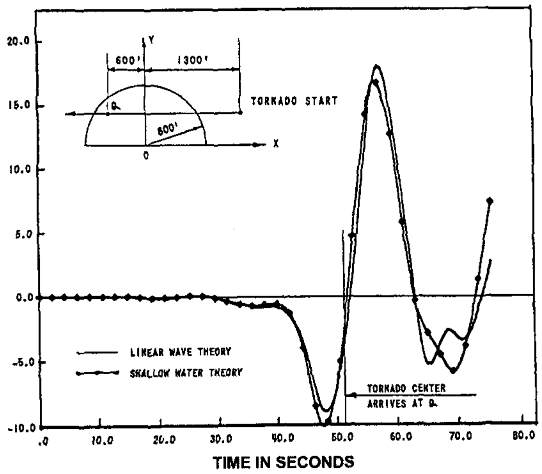

Fig. 8. Time history of surface displacement at Q for forward speed of 25.1 MPH.

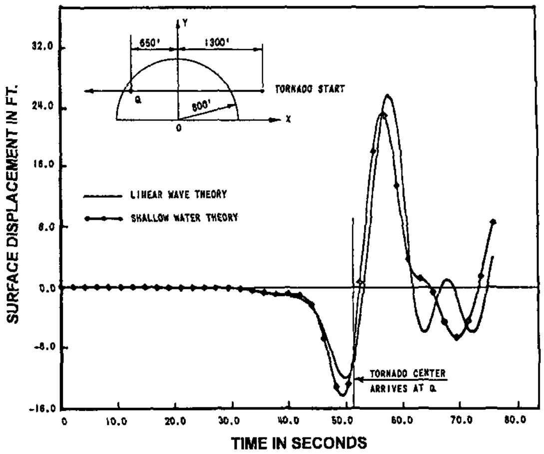

Fig. 9. Time history of surface displacement at Q for forward speed of 25.1 MPH.

height near the boundary in Fig. 9. If the forward speed of the tornado is subcritical and very small, the surface displacements obtained by the linear and linear shallow water theories agree well. The reason is that ϕ_t in the both theories is neglegibly small, and mainly the pressure p induces the surface displacement. See Appendix.

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APPENDIX. SHALLOW WATER THEORY

Let ψ $(r,\ \theta;\ t)$ be the transient potential for linear shallow water theory (Stoker, 1966). Then

$$\nabla^2_{r,\theta}\,\psi - \frac{1}{gh}\,\psi_{tt} = \frac{p_t}{\rho g h} \qquad \text{(A1)}$$

for which

$$\psi_n = 0 \qquad \text{along semi-circular boundary} \qquad \text{(A2)}$$

$$\psi = \psi_t = 0 \qquad \text{at } t=0 \qquad \text{(A3)}$$

The solution to (A1), (A2), and (A3) is given by

$$\psi(r,\ \theta;\ t) = -\frac{2}{\pi\rho a^2}\sum_{i=1}^{\infty}\frac{H_i^o J_o(\xi_i^o r)}{[J_o(\xi_i^o a)]^2} - \frac{4}{\pi\rho}\sum_{n=1}^{\infty}\sum_{i=1}^{\infty}\frac{(\xi_i^n)^2 H_i^n}{(\xi_i^n a)^2 - n^2}\,\frac{J_n(\xi_i^n r)}{[J_n(\xi_i^n a)]^2}\cos n\theta \qquad \text{(A4)}$$

where

$$H_i^n = \int_o^t \bar{p}_i^n\, \xi_i^n \sqrt{gh}\ (t-\eta)\ d\eta$$