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공액경사법을 이용한 정칙화 반복 복원 방법

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Regularized Iterative Image Restoration by using Method of Conjugate Gradient

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요 약

비촛점흐려짐과 가산잡음에 의해 훼손된 영상을 정칙화 공액경사법을 이용하여 복원하는 방법을 제안하였다. 기존의 반복복원방법에 비하여 공액경사법은 초선형적인 수렴속도로 해에 수렴할 뿐 아니라 한정된 반복횟수내에 원래의 해에 수렴할 수 있다는 장점을 지닌다. 본 논문은 반복복원시 잡음의 증폭을 억제하기 위하여 정칙화 공액 경사법에 구속조건을 적용함으로써 적응적으로 윤곽부분과 평면부분의 복원을 행하는 정칙화 공액경사법을 제안하였다. 제안한 방법은 기존의 방법에 비해, 윤곽부분에서는 1에 근사한 구속조건의 값을 적용함으로서 윤곽의 복원을 행하고, 평면부분에서는 0에 근사하는 구속조건을 정칙화 공액경사법에 적용함으로써 잡음을 증폭을 억제할 수 있다는 장점을 지닌다.

Abstract

This paper proposes a regularized iterative image restoration using method of conjugate gradient considering a priori information. Compared with conventional regularized method of conjugate gradient, this method has merits to prevent the artifacts by ringing effects and the partial magnification of the noise in the course of restoring the image degraded by blur and additive noise. Proposed method applies the constraints to accelerate the convergence ratio near the edge portions, and the regularized parameter suppresses the magnification of the noise. As experimental results, I show the superior convergence ratio and the suppression by the artifacts of the proposed method compared with conventional methods.

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I. Introduction

Iterative processing methods can restore the images degraded by blur and additive noise. Compared with conventional restoration methods, Iterative methods have a merit which does not necessary to estimate the inverse matrix of the point spread function in the course of restoration procedure. By observing the restoration procedure, it is possible to obtain the optimal restoration result, and to apply the constraints to suppress the restoration error creating in the course of restoration, or magnification of the additive noise by applying a priori information of the original image. But, conventional iterative methods bring serious degradation of the restoration result or the magnification of the noise when those methods face with the noise, the ill-conditions or the singularities, which created by point spread function. It is possible to suppress the error cause by ill-conditions or the singular points by applying the regularization theorem into iterative restoration methods.

Regularization theorem was proposed by Tikhonov[11] and Miller[12] et al., and Hunt[4] proposed the constrained least-squares estimate algorithm to restore the degradation image by applying constraints. Biemond et al[6] proposed the method to decrease the ringing effect by applying the regularization theorem into Jacobi methods. Katsaggelos et al[7] proposed the method to enhance the restoration results by applying the regularization parameter which applies the human visual system into the regularized iterative restoration method.

But these methods have a problem to bring the magnification of the noise by the influence of wrong determined regularization parameter or have slow convergence ratio when ill-conditions are processed by conventional methods in the course of iterative restoration. Therefore, Biemond et al.[3] proposed the method of steepest-descent or the method of conjugate gradient applying regularization theorem in order to accelerate the convergence ratio. Among these methods, regularized method of conjugate gradient has a merit to have a super-linear convergence ratio, but if direction vectors do not have a conjugate property or the noise is partially amplified, this method estimates the corrupted restoration solutions. Also, the method of conjugate gradient brings serious ringing effects.

In this paper, I propose the regularized iterative method of conjugate gradient by using constraint. This method has a merit to enhance the restoration effects near the edges and to suppress the magnification of the noise near the flat regions by estimating and applying the constraints from each pixels of the restored image. Also, this method can increase the restoration effects by estimating constraints satisfying global minimization of the restoration error in each pixel and applying into the restored image.

This paper consists that regularized iterative restoration method of conjugate gradient is at section 2, regularized iterative restoration method of conjugate gradient with constraints is at section 3, and the experimental results are at section 4.

II. Regularized Iterative Restoration Method of Conjugate Gradient

Generally, images are corrupted by linear blurring like motion, out-of-focusing, and atmospheric turbulence, and the additive noise, these degraded images can be expressed as linear space-invariant model as follows :

$$g(i,j)=h(i,j)*f(i,j)+n(i,j) \tag{1}$$

where, i, j are spatial coordinates, and $h(i,j), f(i,j), g(i,j)$ are point spread function, original image, and observed image, respectively. And $n(i,j)$ is additive noise. By applying inverse filter, it is possible to restore (1) as follows :

$$\begin{aligned}\hat{f}(\omega_1,\omega_2) &= \frac{G(\omega_1,\omega_2)}{H(\omega_1,\omega_2)} \\ &= F(\omega_1,\omega_2)+\frac{N(\omega_1,\omega_2)}{H(\omega_1,\omega_2)}\end{aligned} \tag{2}$$

The restoration error at (2) can be expressed

$$\begin{aligned}\| \hat{F}(\omega_1,\omega_2)- F(\omega_1,\omega_2)\| \\ &= \| \frac{N(\omega_1,\omega_2)}{H(\omega_1,\omega_2)} \| \\ &= \sqrt{\sum_{\omega_1,\omega_2}\left|\frac{N(\omega_1,\omega_2)}{H(\omega_1,\omega_2)}\right|^2}\end{aligned} \tag{3}$$

From above equation, we know that if image is degraded by ill-conditions or singularities, restored image can be degraded by huge noise error. Specially, restored image is seriously degraded by noise in high frequency portions.

Iterative methods can be used to restore the image corrupted by ill-conditions and singularities. Among them, it is able to apply the method of conjugate gradient, one of the optimization methods in order to accelerate the convergence ratio. One of the advantages in this method is to have the super-linear convergence ratio, this method guarantees to converge into the original solution within a finite numbers of iteration.

When the regularization theorem proposed by Miller et al.[12] is applied into the method of conjugate gradient in order to reduce the restoration error

$$\begin{aligned}\Phi(\hat{f}_k) &= \|g-H\cdot \hat{f}_k\|^2 \\ &+\alpha\|C\cdot \hat{f}_k\|^2\le 2\varepsilon^2\end{aligned} \tag{4}$$

algorithm of iterative method of conjugate gradient is as follows :

$$\begin{aligned}r_k &= -\frac{1}{2}\nabla_{f_k}\Phi(\hat{f}_k) \\ &= -(H^T\cdot H+\gamma C^T\cdot C)\hat{f}_k \\ &+H^T\cdot g\end{aligned} \tag{5}$$

$$p_k=r_k+\beta_k p_{k-1}$$

$$\hat{f}_{k+1}=\hat{f}_k+\alpha_k p_k$$

Method of conjugate gradient can enhance the convergence ratio by optimizing β_k in each number of iteration. The optimized value of α_k can be estimated by minimizing the error function at (4), and the values of α_k and β_k are expressed as follows :

$$\beta_k=\frac{\|r_k\|^2}{\|r_{k-1}\|^2} \tag{6}$$

$$\alpha_k=\frac{r_k p_k}{\|H\cdot p_k\|^2+\gamma\|C\cdot p_k\|^2} \tag{7}$$

If we apply constraint P into (5), the restoration solution is that

$$\hat{f}_{k+1}=P[\hat{f}_k+\alpha_k p_k] \tag{8}$$

and from (8), the value of α_k is obtained as follows :

$$\alpha_k=\frac{(p_k, r_k)}{\|H^T\cdot P\cdot p_k\|^2+\gamma\|C\cdot P\cdot p_k\|^2} \tag{9}$$

By using optimized value of α_k in each iterations, it is possible to reduce the error function $\Phi(\hat{f}_k)$ monotonically, and the iteratively restored results converge to the original solution.

III. Regularized Iterative Restoration Meth-od of Conjugate Gradient with Con-straints.

When we apply the regularization theorem into the conventional method of conjugate gradient, the direction vector is not satisfied with conjugate property, or the magnification of the noise or the ringing effects are created, the quality of the restored result is degraded than that of the conventional methods.

In the course of iterative restoration by using the regularized method of conjugate gradient, estimation and application of the constraints are possible to suppress the restoration error. So, in order to suppress the restoration error which can be created by the conventional methods, we propose the regularized method of conjugate gradient considering constraints as follows :

$$\begin{aligned}\hat{f}_{k+1}(i,j) &= (1-P_k(i,j))m_{\hat{f}_k}(i,j) \\ &+ P_k(i,j)\,\hat{f}_k(i,j) \\ &+ P_k(i,j)\,\alpha_k p_k(i,j)\end{aligned} \tag{10}$$

where $m_{\hat{f}_k}(i,j)$ is a local mean estimated from the local window of the restored image $\hat{f}_k$.

$P_k(i,j)$ is a constraint to be applied to suppress the magnification of the noise around the edge and flat regions at k th iterative restoration. This constraint has the value near to 0 at the flat region having small variance, and 1 at the variable edge regions. So, by applying this constraint it is possible to reduce artifacts created in the course of iterative restoration. In (10), $P_k(i,j)\simeq 0$ at the flat regions, so iteratively restored solutions become

$$\hat{f}_{k+1}(i,j) \simeq m_{\hat{f}_{k+1}(i,j)} \tag{11}$$

so, it is possible to suppress the magnification of the noise at flat regions, and $P_k(i,j)\simeq 1$ at edge regions, so restored solution at k+1th iteration

$$\hat{f}_{k+1}(i,j) = \hat{f}_k(i,j) + \alpha_k p_k(i,j) \tag{12}$$

converges to the original solution as super-linear convergence ratio.

In order to obtain enhanced solutions by using proposed method, it is necessary to estimate optimal value of constraint $P_k(i,j)$ and the regularization parameter $\gamma_k(i,j)$ in each iterations.

There is a local linear minimum mean square error algorithm to satisfy the property of $P_k(i,j)$. We assume that the observed image is only degraded by additive noise as follows

$$g = f + n \tag{13}$$

the restored solution by local linear minimum mean square error algorithm becomes

$$\begin{aligned}\hat{f}_{LLMMSE}(i,j) &= m_{f(i,j)} \\ &+ \frac{\sigma_g^2(i,j)-\sigma_n^2}{\sigma_g^2(i,j)}(g(i,j)-m_g(i,j))\end{aligned} \tag{14}$$

Where $\sigma_g^2(i,j), \sigma_n^2, m_f, m_g$ are variance of observed image, white noise, and the mean of original and observed image, respectively. The constraint in (14) has the characteristics to preserve the edge portions and to smooth the noise in the flat regions simultaneously in the course of restoration. When the local mean and variance are estimated as follows:

$$\begin{aligned}m_g(i,j) &= \frac{1}{(2m+1)(2n+1)}\cdot \sum_{k=i-m}^{i+m}\sum_{l=j-n}^{j+n} g(k,l) \\ \sigma_g^2(i,j) &= \frac{1}{(2m+1)(2n+1)}\cdot \sum_{k=i-m}^{i+m}\sum_{l=j-n}^{j+n}[g(k,l)-m_g(i,j)]^2\end{aligned} \tag{15}$$

from (13), we knows that the variance of observed image is the sum of the variance of original image and the variance of the noise. So, the variance of the observed image is larger or equal to the one of the noise. Therefore, we set the variance of observed image as the maximum value between the variance of noise and estimated one :

$$\sigma_g'^2(i,j) = \max[\sigma_n^2, \sigma_g^2(i,j)] \tag{16}$$

By applying (16) into (14), it is possible to restore the noisy image.

In this paper, we apply the transfer function in (14) as a constraint $P_k(i,j)$ in order to obtain to suppress the magnification of the noise in the flat regions as follows :

$$P_k(i,j) = \frac{\sigma^2_{f_k(i,j)} - \sigma_n^2}{\sigma^2_{f_k(i,j)}} \tag{17}$$

Also, the regularization parameters[11][13]

$$\gamma(\hat{f}_k) = \frac{\|g - H\cdot \hat{f}_k\|^2}{2\|g\|^2 - \|C\cdot \hat{f}_k\|^2} \tag{18}$$

are estimated in each iteration steps to minimize the restoration error and the magnification of the noise.

IV. Experimental Results.

We use the 8 bits "bridge" and "sbar" images having 8 bits, 256×256 pixels as standard images. And we assume that original images are degraded by out-of-focusing and the additive noise. Through the experiments, we compare the restoration results processed by proposed method with the conventional methods proposed by Jan Biemond et al., and the conventional regularization method of conjugate gradient.

Fig.1 shows the original images and the degraded ones. Observed images are degraded by out-of-focus having length $L = 7$ as follows

$$h(i,j) = \frac{1}{L^2}, 0 \le i,j < L \tag{19}$$

and the additive noise having $20dB$ BSNR.

We use the Mean Square Error to measure the restoration results.

We use 2-D Laplacian filter as a regularization operator and the mean and variance of the restored image are estimated using local window having 7×7 window size.

Fig.2 shows the variation of the constraint $P_k(i,j)$ estimated from degraded "sbar" image. The value of $P_k(i,j)$ is nearly 1 around the edge regions, so propose method enhanced the blurred edges to sharped one. And at flat regions, the variance of the restored image is similar with noise variance, therefore the value of $P_k(i,j)$ became 0, it is possible to smooth the noise in flat regions.

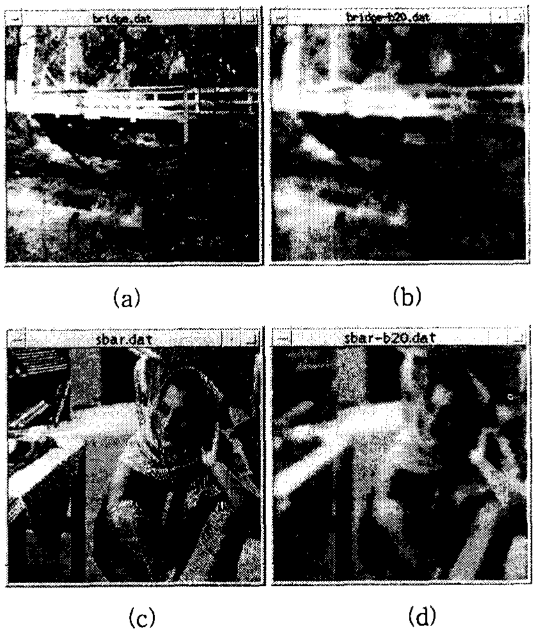


Fig.1 Original and degraded images having 8 bits, 256×256 size.
(a) Original "bridge" image
(b) Observed "bridge" image degraded by 7×7 out-of-focus blur and noise having 20 BSNR
(c) Original "sbar" image
(d) Observed "sbar" image degraded by 7×7 out-of-focus blur and noise having 20 BSNR

α_k in(7) is applied to accelerate the convergence ratio. So, α_k is estimated as a optimal value to satisfy the global minimization of the restoration error, proposed method converges to the solution as a super-linear convergence ratio.

The restoration results processed by the proposed method are compared with conventional method of conjugate gradient[4], regularized method of steepest descent[4], and the conventional regularized method of conjugate gradient[6]. Through the experiments, we apply $\gamma = 0.001$[6] as a regularization parameter in proposed method and the other conventional methods apply $\gamma = 0.1$. And initial solution is as follows :

$$\hat{f}_0 = H^T \cdot g \qquad (20)$$

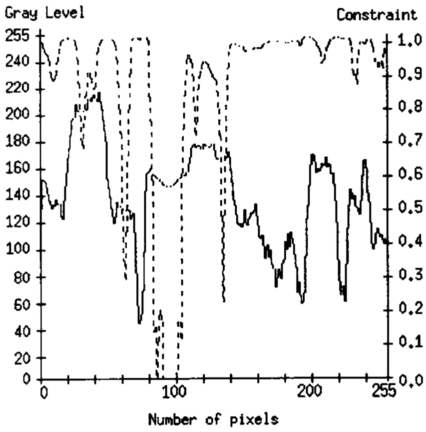


Fig.2 Variation of the constraint $P_k(...)$ according to the variation of the pixels.

Table 1, 2 show the MSE in each methods when "bridge" and "sbar" images are used as standard ones.

From above results, we verify that proposed method has fastest convergence ratio than the other methods.

Also, restoration results are in fig.3 and 4 when we process the degraded image 50 times iteration.

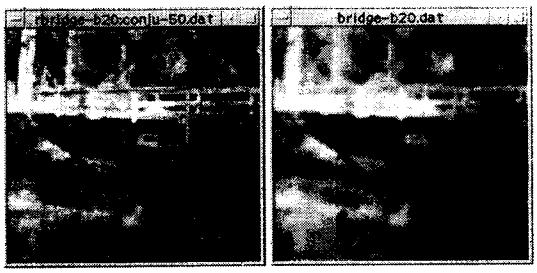
Table 1. Comparison of the MSE estimated from restored images by each restored methods when degraded "bridge" image having BSNR=20dB was used

No. of Iterations	Method 1	Method 2	Method 3
Degraded	493.79438		
10	292.21112	287.73708	298.37436
20	284.61986	273.46433	277.43064
30	282.64090	270.86158	270.54017
40	282.95538	270.63308	268.96037
50	284.02752	270.43864	269.08031

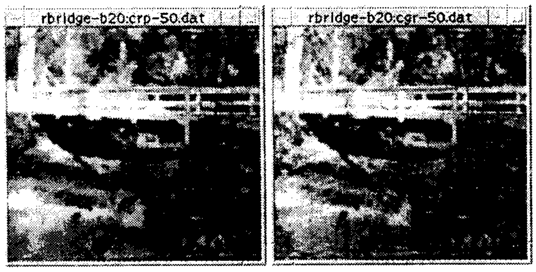
Table 2. Comparison of the MSE estimated from restored images by each restored methods when degraded "sbar" image having BSNR=20dB was used

No. of Iterations	Method 1	Method 2	Method 3
Degraded	476.69525		
10	367.92871	365.11677	361.49794
20	362.47955	361.19252	351.26363
30	362.29737	361.27285	349.04315
40	363.99854	361.23744	348.23990
50	366.41682	361.17657	347.66536

Method 1 : Conventional method of conjugate gradient.
Method 2 : Conventional regularized iterative method of conjugate gradient.
Method 3 : Proposed method.

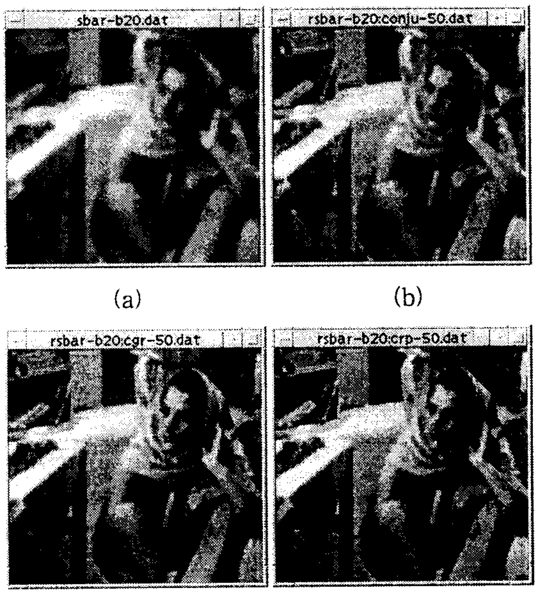


(a) (b)



(c) (d)

Fig. 3 Comparison of the restoration result,
(a) Degraded "bridge" image having $BSNR=20dB$
(b) 50 times iterative restored image by using conventional method of conjugate gradient.
(c) 50 times iterative restored image by using conventional regularized method of conjugate gradient.
(d) 50 times iterative restored image by using proposed method



(a) (b)

(c) (d)

Fig. 4. Comparison of the restoration results
(a) Degraded "sbar" image having $BSNR=20dB$
(b) 50 times iterative restored image by using conventional method of conjugate gradient.
(c) 50 times iterative restored image by using conventional regularized method of conjugate gradient.
(d) 50 times iterative restored image by using proposed method

From Fig.3, and 4, we find that proposed method has smaller magnificati-

on of the noise at the flat regions, and edge regions are more sharped than the other methods.

V. Conclusions

We propose the regularized iterative method of conjugate gradient applying constraint to restore the noisy blurred image.

The method of conjugate gradient has merits to apply a priori information into the restoration process and has enhanced convergence ratio compared with the other methods.

But, if observed images are degraded by noise and blur, it is nearly impossible to estimate accurate value of the noise variance from the degraded one. So it is difficult to obtain the optimal constraints. Also if direction vectors does not satisfy the conjugate property, restoration process brings decrement of the quality of restoration results by ringing effects and the magnification of noise.

Proposed method applies constraints to prevent the magnification of the noise at flat regions and to enhance restoration effects around the edge portions.

When "bridge" and "sbar" images degraded by 20 *BSNR* and out-of-focusing are restored by proposed method, we verify more enhanced restoration results having faster convergence ratio compared with the other methods.

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