⊠ 연구논문

Development and Analysis of Algorithms for Reliability Calculation of Coherent Structure

일관성 신뢰성 구조에 대한 알고리즘의 개발 및 분석

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요 지

본 논문은 2-상태 시스템의 신뢰도 계산을 위하여 Heidtmann[3]이 제시한 가역 알고리즘을 부울 대수를 사용하여 경로집합으로부터 절단집합을 유도하거나, 역으로 절단집합으로부터 경로집합을 유도함으로서 효율성을 입증하였으며, 포함-불포함 공식(inclusion-exclusion formula)과 추축 분해법칙(pivotal decomposition rule)을 이용하여 직·병렬구조 및 복합구조의 신뢰도를 계산할 수 있는 알고리즘을 개발, 분석하였다.

1. Introduction

The problem of constructing reliable basic components by appropriate redundancy relatively used by Von Neumann in 1956. He showed how to combine a number of unreliable "Sheffer Stroke" organs to obtain an element which acts like a Sheffer stroke organ of higher reliability. Moore and Shannon, inspired by the Von Neumann's paper, carry out an elegant analysis for relay circuits in which they show that by the proper incorporation of redundancy, arbitrarily reliable circuits can be constructed from arbitrarily unreliable relays. Birbaum, Esary and Saunders generalize the concepts and extend some of the results of Moore and Shannon to the large natural class of structures having the property that replacing failed components by working components cannot cause a working structure to fail called coherent structures. The reliability literature of the past 10 years contains many papers with reliability calculation of coherent structure.

This paper extends [9] and discussed the some algorithms for reliability calculation. Problems related to the coherent structure of the system are based on [6]. The path set and cut set method for determining system reliability [1] is used. In section 2, we faintly survey a inversion algorithm by Heidtmann to the case of inverting paths and cuts of 2-state systems. In section 3 and 4, we developed inclusion-exclusion algorithm and pivotal decomposition algorithm by use of inclusion-exclusion formula and decomposition rule.

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Notation

N	set of integers from 1 to n
I	subset of N
2^N	power set of N
$N\{A\}$	number of element of A
A'	inverse of A
A^*	star function of A
x_i	boolean variable of i-th component
$\bar{x}_i$	complement boolean variable of i-th component

2. Inversion Algorithm

K. D. Heidtmann suggested an inversion algorithm in his paper [3]. Now we will try to describe his algorithm in simpler terms. Each component of a system of n component is uniquely represented by its index $i \in N$, and any assembly of components by a subset I of N. So any path or cut is a subset of N, and set of all paths, or the set of all cuts, is a subset of the power set 2^N. The concept of inverse combines two complement to invert paths and cuts as subsets A and A' of 2^N; that is, if A is set of paths, then A' is set of cuts; and vice versa.

Let $A \subset 2^N$, $I \subset N$. From the complement $\bar{I}$ of every I for $I \in A$. Eliminate from 2^N all such $\bar{I}$. The result is A', the inverse of A. More formally, let $A \subset 2^N$, $A' \subset 2^N$, $I \subset N$. Then A′ is the inverse of A if and only if for every possible I, either $I \in A$ or $\bar{I} \in A'$. The inverse property is reciprocal: $(A')' = A$. The number of elements of A, plus the number of elements in A' is 2^n ; $n\{A\} + n\{A'\} = 2^n$. There exists one and only one inverse of A.

Let $A \subset 2^N$, $I \subset N$. From the complement $\bar{I}$ of every for $I \in A$. A^* is the set of all those $\bar{I}$. A and $(A')^*$ are a partition of 2^N. $A' = (A^{c*}) = (A^c)^*$. These inverse and star relationships were derived and proved in Lee(1996)[8].

In applying the inverse concept by hand calculation, many sets must be looked at because 2^N contains 2^n elements. Thus it is helpful to verify the correctness of a computed A' by the following test which is based on the facts that A and $(A')^*$ form a partition of 2^N, and $n\{2^N\} = 2^n$.

Test : Let A' be the inverse of A, $A \subset 2^N$. Then A and A' satisfy

$$n\{A\} + n\{A'\} = 2^n.$$

For automated calculation, the inversion algorithm can be used. After execution, A' contains all cuts if A contains all paths, and vice versa.

Inversion Algorithm

Input : A, N

step 1. Compute A^*. The I are the elements of 2^N.

step 2. Set $I \leftarrow \emptyset$ and $A' \leftarrow \emptyset$.

step 3. If $I \not\in A^*$ *then* $A' \leftarrow A' \cup \{ I \}$.

step 4. If $I \neq N$ *then* replace I by its successor and go to 3.

step 5. Stop(A' is inverse of A).

The test yields n{ A}+ n{ A'} = 3+ 5 = 2^3.

Example 1. The bridge structure is shown in the following diagram

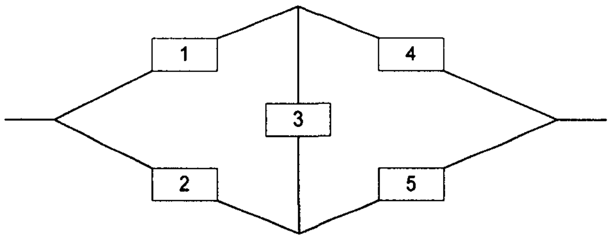

Fig.1 5-component bridge system

There are five components; $n=5$, $N=\{1, 2, 3, 4, 5\}$

$2^N=\{\emptyset,\{1\},\cdots,\{5\},\{1,2\},\cdots,\{4,5\},\{1,2,3\},\cdots,\{3,4,5\},\{1,2,3,4\},\cdots,$
$\{2,3,4,5\},\{1,2,3,4,5\}\}$

$n\{2^N\}=2^5$.

This system has 16 paths which conclude 4 minimal path sets.

$$A=\{I_1,I_2,\cdots,I_{16}\}=\{\{1,4\},\{2,5\},\cdots,\{1,2,3,4,5\}\}.$$

$$\overline{I_1}=\{\overline{1,4}\}=\{2,3,5\},\overline{I_2}=\{\overline{2,5}\}=\{1,3,4\},\cdots,\overline{I_{16}}=\{\overline{1,2,3,4,5}\}=\emptyset.$$

Thus $A^*=\{\{2,3,5\},\{1,3,4\},\cdots,\emptyset\}$.

By remove the elements of A^* from 2^N.

$$A'=\{\{1,2,3\},\{4,5\},\cdots,\{1,2,3,4,5\}\}.$$

The test yields : $n\{A\}+n\{A'\}=16+16=2^5$.

According to above example, inversion algorithm is useful to find inverting paths and cuts of 2-states systems, but it depend upon set theory. Therefore, we need another algorithm in order to calculate system reliability easily.

Now, we present a algebraic technique computing system reliability for above system. The minimal paths are {1,4}, {1,3,5}, {2,5}, {2,3,4}.

Let x_i is Boolean variable indicating whether component i is good ($x_i=1$) or failed ($x_i=0$), then the path polynomial is

$$x_1x_4+x_1x_3x_5+x_2x_5+x_2x_3x_4. \qquad (1)$$

With regard to the usual Boolean operations of addition and multiplication, any assignment of 0-1 values to the x_i's that makes the path polynomial equal to 1 corresponds to a good state of the system. An inverse polynomial to the path can be obtained by complimenting the given polynomial and using DeMorgan's laws. Complimentation of (1) results in:

$$(\overline{x_1}+\overline{x_4})(\overline{x_1}+\overline{x_3}+\overline{x_5})(\overline{x_2}+\overline{x_5})(\overline{x_2}+\overline{x_3}+\overline{x_4}) \qquad (2)$$

If we expanded above formula (2) using the distributive law and deleted non-minimal elements using absorption law, the result is the cut set polynomial which provides an enumeration of all minimal cut sets:

$$\overline{x_1}\,\overline{x_2}+\overline{x_1}\,\overline{x_3}\,\overline{x_5}+\overline{x_2}\,\overline{x_3}\,\overline{x_4}+\overline{x_4}\,\overline{x_5}. \qquad (3)$$

Any assignment of 0-1 values to the x_i's that makes the cut set polynomial equal to 1 corresponds to system failure; ie, no path of good arcs exists in bridge system.
This algorithm is very simple; is equally·efficient for simple and complex and system.

3. Inclusion-Exclusion Algorithm

The inclusion-exclusion rule came from additive law of probability. So the inclusion-exclusion method provides successive upper and lower bounds on system reliability which converge to the exact system reliability. In system reliability calculations by the inclusion-exclusion method, large numbers of pairs of identical terms with opposite signs cancel. For any system with n minimal path sets the number of terms generated in step i of the method is $\binom{n}{i}$, so that the Poincaré formula consist of $\sum_{i=1}^{n}\binom{n}{i}=2^n-1$ terms.

Two of the terms cancel if a union of i minimal path sets contains exactly the same components as a union of j minimal path sets ($1\le i,\ j\le n,\ |i-j|=1$) Therefore the reliability analysis of all systems having pairwise disjoint minimal path sets, i.e. which have redundant component, is affected by this cancelling terms. For nearly all large complex systems the number of cancelling terms is enormous, so that avoiding these terms affords an important computational advantage.

Reliability analysis by the original method of inclusion-exclusion assumes the knowledge of all minimal path or cut sets[1].

Let $E_i(\overline{E}_i)$ be the event that i-th component x_i is functioning (failed) with probability $p_i(1-p_i)$.

Let $A_r(\overline{A_r})$ be the event that all components in r-th minimal path set P_r is functioning (failed).

i.e.

$$A_r \equiv \bigcap_{x_i\subset P_r} E_i, \qquad \overline{A_r} \equiv \bigcap_{x_i\subset K_r} \overline{E}_i$$

where K_r is r-th minimal cut set.

Then

$$P(A_r) = P(\bigcap_{x_i \subset P_r} E_i), \quad P(\overline{A_r}) = P(\bigcap_{x_i \subset K_r} E_i) \tag{4}$$

System success corresponds to event $U_{r=1}^{n} A_r$ if the system has n minimal path sets.

The the system reliability function

$$h(p) = P[\bigcup_{r=1}^{n} A_r] \tag{5}$$

Let

$$S_k = \sum_{1 \le i_1 < i_2 \cdots i_k \le n} P[Ai_1 \cap Ai_2 \cap \cdots \cap Ai_k]$$

by the inclusion-exclusion principal[2]

$$h(p) = \sum_{k=1}^{n} (-1)^{k-1} S_k \tag{6}$$

and

$$\begin{aligned} h(p) &\le S_1 \\ h(p) &\ge S_1 - S_2 \\ h(p) &\le S_1 - S_2 + S_3 \end{aligned} \tag{7}$$

and so on.

Now we define the approximation to system reliability function $h(p)$ of step m by

$$h^{(m)}(p) \equiv \sum_{k=1}^{m} (-1)^{k-1} S_k \tag{8}$$

For $m > 1$

$$h^{(m)}(p) = h^{(m-1)}(p) + (-1)^{m-1} S_m \tag{9}$$

$$h^{(n)}(p) = h(p) \tag{10}$$

Although it is not true in general that the upper bounds decrease and the lower bounds increase, in practice it may be necessary to calculate only a few $S_k's$ to obtain a close approximation.

Of course similar formulas for computing system unreliability $\overline{h}(p)$ in terms of minimal cut sets and component unreliabilities $1 - p_i$ can be given. Now we state the algorithm in detail.

Inclusion- Exclusion Algorithm

Input : n, p_1, p_2, $\cdots$, p_n, N_0 or ε

Output : $h(p)$

step 1. set $i = 1$

step 2. while $i \le N_0$ do step 3~5

step 3. set $S_i = \sum_{1 \le r_1 < \cdots < r_i \le n} P_{r_1} \cdots P_{r_i}$ (*compute* S_i)

$$h(p) = \sum_{i=1}^{n} (-1)^{i-1} S_i$$

step 4. if $|S_i - S_{i-1}| < \varepsilon$
output $h(p)$
stop
step 5. set $i = i+1$
$S_i = S_{i+1}$
step 6. output (method failed after N_0 iterations, N_0 or $h(p)$)

4. Pivotal decomposition algorithm

This section, gives the algorithm for pivotal decomposition rule. The following identity holds for any structure function ϕ of order n:

$$\phi(x) = x_i \cdot \phi(1_i, x) + (1-x_i) \cdot \phi(0_i, x) \tag{11}$$

We immediately obtain the corresponding pivotal decomposition of the reliability function.

$$\begin{aligned} h(p) &= E[\phi(X)] \\ &= p_i \cdot h(1_i, p) + (1-p_i) \cdot h(0_i, p) \quad i = 1, \cdots, n \end{aligned} \tag{12}$$

Now we proposed following algorithm.

Algorithm for series system
Input : $n,\ p_1,\ p_2,\ \cdots,\ p_n$
Output : $h(p)$
step 1. $i = 1$
step 2. while $i \leq n$ do step 3~4
step 3. set $h(0_i, p) = 0$
$h(1_i, p) = p_{i+1} \cdot h(1_{i+1}, p) + (1-p_{i+1}) \cdot h(0_{i+1}, p)$
$h(p) = p_i \cdot h(1_i, p) + (1-p_i) \cdot h(0_i, p)$ (*compute* $h(p)$)
step 4. $i = i + 1$
step 5. output $h(p)$

Algorithm for parallel system
Input : $n,\ p_1,\ p_2,\ \cdots,\ p_n$
Output : $h(p)$
step 1. $i = 1$
step 2. while $i \leq n$ do step 3~4
step 3. set $h(1_i, p) = 1$
$h(0_i, p) = p_{i+1} h(1_{i+1}, p) + (1-p_{i+1}) h(0_{i+1}, p)$
$h(p) = p_i \cdot h(1_i, p) + (1-p_i) \cdot h(0_i, p)$ (*compute* $h(p)$)
step 4. $i = i + 1$
step 5. output $h(p)$

5. Numerical Examples

Now we present several examples. we applied the inclusion-exclusion method in actual practice which concerned airplane operation system (Lee 1991, 1993)[5],[6]. Suppose that an airplane engine will operate, when in flight, with probability p_i independently from engine to engine; Suppose that the airplane will make a successful flight if at least 50% of its engines remain operative.

Example 2. (1-out-of-2:G system) We consider 2-engine plane. From (Lee 1993-1)[6], it has two minimal path sets

$$P_1 = \{1\}, \quad P_2 = \{2\}$$

Thus the first bound on the reliability $h^{(1)}(p)$ is

$$h^{(1)}(p) = S_1 = \sum_{i=1}^{2} P(Ai)$$

$$= p_1 + p_2$$

And the second bound is

$$h^{(2)}(p) = h^{(1)}(p) - S_2$$

$$= h^{(1)}(p) - P(A_1 \cap A_2)$$

$$= p_1 + p_2 - p_1 p_2$$

Hence by (10), system reliability

$$h(p) = h^{(2)}(p)$$

The 1-out-of-2 : G system is also a 2-out of-2 : F system with only one minimal cut set

$$K_1 = \{1, 2\}$$

Hence unreliability

$$\overline{h}(p) = P(\overline{A_1})$$

$$= (1-p_1)(1-p_2)$$

Next we consider 4-engine plane.

Example 3.(2-out-of-4: G system) From[5] it has six minimal path sets;

$$P_1 = \{1, 2\}, \quad P_2 = \{1, 3\}, \quad P_3 = \{1, 4\}$$

$$P_4 = \{2, 3\}, \quad P_5 = \{2, 4\}, \quad P_6 = \{3, 4\}$$

Thus the first bound on the reliability is

$$h^{(1)}(p) = S_1 = \sum_{i=1}^{6} P(Ai)$$

$$= p_1 p_2 + p_1 p_3 + p_1 p_4 + p_2 p_3 + p_2 p_2 + p_3 p_4$$

Since

$$P(A_1 \cap A_2) = p_1 p_2 p_3, \quad P(A_1 \cap A_6) = p_1 p_2 p_3 p_4$$

Hence the second bound is

$$h^{(2)}(p) = h^{(1)}(p) - S_2$$

$$= h^{(1)}(p) - \sum_{1 \le i_1 < i_2 \le 6} P(Ai_1 \cap Ai_2)$$

$$= h^{(1)}(p) - 3(p_1 p_2 p_3 + p_1 p_2 p_4 + p_1 p_3 p_4 + p_2 p_3 p_4 + p_1 p_2 p_3 p_4)$$

Similarly

$$
\begin{aligned}
h^{(3)}(p) &= h^{(2)}(p) + S_3 \\
&= h^{(2)}(p) - \sum_{1\le i_1\le i_2\le i_3\le 6} P(Ai_1 \cap Ai_2 \cap Ai_3) \\
&= h^{(2)}(p) + p_1 p_2 p_3 + p_1 p_2 p_4 + p_1 p_3 p_4 + p_2 p_3 p_4 + 16 p_1 p_2 p_3 p_4
\end{aligned}
$$

The 4 terms of $h^{(3)}(p)$ being products of 3 factors cancel 4 terms of $h^{(2)}(p)$, and the 3 terms $p_1p_2p_3p_4$ of $h^{(2)}(p)$ cancel 3 of the 16 terms of $h^{(3)}(p)$.

This results in

$$h^{(4)}(p) = h^{(3)}(p) - 15 p_1 p_2 p_3 p_4$$

$$h^{(5)}(p) = h^{(4)}(p) + 6 p_1 p_2 p_3 p_4$$

$$h(p) = h^{(6)}(p) = h^{(5)}(p) - p_1 p_2 p_3 p_4$$

with 46 cancelling terms.

The 2-out-of-4 : G system is also a 3-out-of-4 : F system with four minimal cut sets

$K_1 = \{1, 2, 3\}$ $K_2 = \{1, 2, 4\}$ $K_3 = \{1, 3, 4\}$ $K_4 = \{2, 3, 4\}$

$$\overline{h^{(1)}}(p) = \sum_{i=1}^{4} P(\overline{A}i)$$

$$= (1-p_1)(1-p_2)(1-p_3) + (1-p_1)(1-p_2)(1-p_4)$$

$$+ (1-p_1)(1-p_3)(1-p_4) + (1-p_2)(1-p_3)(1-p_4)$$

$$\overline{h^{(2)}}(p) = \overline{h^{(1)}}(p) - 6(1-p_1)(1-p_2)(1-p_3)(1-p_4)$$

$$\overline{h^{(3)}}(p) = \overline{h^{(2)}}(p) + 4(1-p_1)(1-p_2)(1-p_3)(1-p_4)$$

$$\overline{h}(p) = \overline{h^{(4)}}(p) = \overline{h^{(3)}}(p) - (1-p_1)(1-p_2)(1-p_3)(1-p_4)$$

There are 8 cancelling terms instead of 46, and much less computation because $2^6 - 2^4 = 48$ fewer terms.

Example 4. We consider 3-component series system and 3-component parallel system with success probabilities $p_1 = 0.6$, $p_2 = 0.7$, $p_3 = 0.8$ in Fig.2 and Fig.3. By pivotal decomposition algorithm, series system reliability is

$$
\begin{aligned}
h(p) &= p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p) \\
&= p_1\{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\} + (1-p_1) h(0_1, p) \\
&= p_1\{p_2 p_3 + (1-p_2) \cdot 0\} + (1-p_1) \cdot 0 \\
&= p_1 p_2 p_3 \\
&= 0.336
\end{aligned}
$$

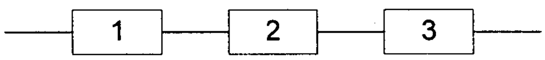

Fig.2 3-component series system

parallel system reliability is

$$h(p) = p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p)$$

$$= p_1 \cdot 1 + (1-p_1)\{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\}$$

$= p_1 + (1-p_1)\{p_2 + (1-p_2)p_3\}$
$= 0.6 + 0.4(0.7 + 0.3 \times 0.8)$
$= 0.976$

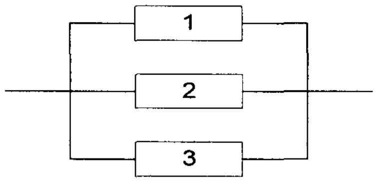

Fig.3 3-component parallel system

Example 5.(Generalized Example 4) By pivotal decomposition algorithm, we obtain following reliability function.

2-component series system ;

$$h(p) = p_1 h(1_1, p) + (1-p_1) h(0_1, p)$$
$$= p_1 p_2 + (1-p_1) \cdot 0$$
$$= p_1 p_2$$

3-component series system ;

$$h(p) = p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p)$$
$$= p_1 \{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\} + (1-p_1) h(0_1, p)$$
$$= p_1 \{p_2 p_3 + (1-p_2) \cdot 0\} + (1-p_1) \cdot 0$$
$$= p_1 p_2 p_3$$

4-component series system;

$$h(p) = p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p)$$
$$= p_1 \{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\} + (1-p_1) h(0_1, p)$$
$$= p_1 [p_2 \{p_3 h(1_3, p) + (1-p_3) h(0_3, p)\} + (1-p_2) h(0_2, p)] + (1-p_1) h(0_1, p)$$
$$= p_1 [p_2 \{p_3 p_4 + (1-p_3) \cdot 0\} + (1-p_2) \cdot 0] + (1-p_1) \cdot 0$$
$$= p_1 p_2 p_3 p_4$$

2-component parallel system ;

$$h(p) = p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p)$$
$$= p_1 \cdot 1 + (1-p_1)\{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\}$$
$$= p_1 + (1-p_1)\{p_2 + (1-p_2) \cdot 0\}$$
$$= p_1 + (1-p_1) p_2$$

3-component parallel system ;

$$h(p) = p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p)$$
$$= p_1 \cdot 1 + (1-p_1)\{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\}$$

$$= p_1 + (1-p_1)[p_2 + (1-p_2)\{p_3 h(1_3, p) + (1-p_3)h(0_3, p)\}]$$
$$= p_1 + (1-p_1)[p_2 + (1-p_2)\{p_3 + (1-p_3)\cdot 0\}]$$
$$= p_1 + (1-p_1)\{p_2 + (1-p_2)\cdot p_3\}$$

4-component parallel system ;

$$h(p) = p_1 \cdot h(1_1, p) + (1-p_1)\cdot h(0_1, p)$$
$$= p_1 \cdot 1 + (1-p_1)\{p_2 h(1_2, p) + (1-p_2)h(0_2, p)\}$$
$$= p_1 + (1-p_1)[p_2 + (1-p_2)\{p_3 h(1_3, p) + (1-p_3)h(0_3, p)\}]$$
$$= p_1 + (1-p_1)[p_2 + (1-p_2)\{p_3 + (1-p_3)\cdot\{p_4 h(1_4, p) + (1-p_4)h(0_4, p)\}\}]$$

For general non series-parallel system (have unequal probabilities of success), the only known practical method of exact analysis are the path and cut-set method.

Example 6. (8-component complex system) Consider complex system in fig.4.

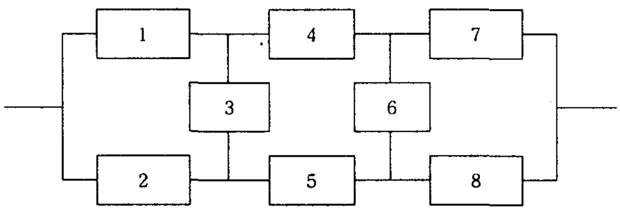

Fig.4 8-component complex system

There are 8 path set;

{1,4,7}, {2,5,8}, {1,3,5,8}, {1,4,6,8}, {2,3,4,7}, {2,5,6,7}, {1,3,5,6,7}, {2,3,4,6,8}

System reliability is

$$h(p) = p_1p_4p_7 + (1-p_1p_4p_7)\,[p_2p_5p_8 + (1-p_2p_5p_8)\{p_1p_3p_5p_8 + (1-p_1p_3p_5p_8)\{p_1p_4p_6p_8$$
$$+ (1-p_1p_4p_6p_8)\{p_2p_3p_4p_7) + (1-p_2p_3p_4p_7)\cdot\{p_2p_5p_6p_7 + (1-p_2p_5p_6p_7)$$
$$\{p_1p_3p_5p_6p_7 + (1-p_1p_3p_5p_6p_7)\cdot(p_2p_3p_4p_6p_8)\}\}\}\}\}]$$

Now, we consider following example in order to compare two algorithms in computational complexity.

Example 7. In example 1, there are 4 minimal path sets {1,4}, {1,3,5}, {2,5}, {2,3,4}.

By inclusion-exclusion algorithm, reliability function is

$$h(p) = p_1p_4 + p_1p_3p_5 + p_2p_5 + p_2p_3p_4 - (p_1p_3p_4p_5 + p_1p_2p_4p_5 + p_1p_2p_3p_4 + p_1p_2p_3p_5 + p_1p_2p_3p_4p_5 + p_2p_3p_4p_5)$$
$$+ (p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5) - p_1p_2p_3p_4p_5$$
$$= p_1p_4 + p_1p_3p_5 + p_2p_5 + p_2p_3p_4 - (p_1p_3p_4p_5 + p_1p_2p_4p_5 + p_1p_2p_3p_4 + p_2p_3p_4p_5) + 4p_1p_2p_3p_4p_5$$

By pivotal decomposition algorithm, reliability function is

$$h(p) = p_1p_4 + (1-p_1p_4)[p_1p_3p_5 + (1-p_1p_3p_5)\{p_2p_5 + (1-p_2p_5)p_2p_3p_4\}].$$

Thus pivotal decomposition algorithm is more useful for reliability calculation in 5-bridge structure.

4. Conclusion

In this paper, We suggest some algorithms for system reliability calculation of coherent structure. Inversion algorithm is most useful to find inverting paths and cuts of 2-states systems, but it is not more useful than inclusion-exclusion or decomposition algorithm in reliability calculation of coherent structure.

Following tables represent reliability calculation of n-component structure used two algorithms.

table 1. series structure

	inclusion - exclusion	pivotal decomposition
n=2	p_1p_2	$p_1p_2+(1-p_1)0$
n=3	$p_1p_2p_3$	$p_1\{p_2p_3+(1-p_2)0\}+(1-p_1)0$
n=4	$p_1p_2p_3p_4$	$p_1[p_2\{p_3p_4+(1-p_3)0+(1-p_2)0\}]+(1-p_1)0$

table 2. parallel structure

	inclusion - exclusion	pivotal decomposition
n=2	$p_1+p_2-p_1p_2$	$p_1+(1-p_1)p_2$
n=3	$p_1+p_2+p_3-(p_1p_2+p_1p_3+p_2p_3)+p_1p_2p_3$	$p_1+(1-p_1)\{p_2+(1-p_2)p_3\}$
n=4	$p_1+p_2+p_3+p_4-(p_1p_2+p_1p_3+p_1p_4+p_2p_3$ $+p_2p_4+p_3p_4)+(p_1p_2p_3+p_1p_2p_4+p_1p_3p_4$ $+p_2p_3p_4)-p_1p_2p_3p_4$	$p_1+(1-p_1)[p_2+(1-p_2)\{p_3+(1-p_3)p_4\}]$

By above tables and example 7, we show that in case of series structure, the inclusion-exclusion algorithm is proper in computational complexity reduction, but pivotal decomposition algorithm is proper in computational complexity reduction in case of parallel structure. We expect that our method, applied in this paper, is further extended to the case when components of the system are given multi-states. Further research, which is outside the scope of this paper, must be undertaken to compare the computation speeds between the two algorithms.

References

[1] R. E. Barlow, F. Proschan, *Statistical Theory of Reliability and Life testing*, Holt, Reinhardt & Winston Inc., 1975.

[2] W. Feller, *An Introduction to Probability Theory and Its Applications* I. II. John Wiley & Sons Inc., New York, 1968.

[3] K. D. Heidtmann, "Inverting Paths and Cuts of 2-State Systems", *IEEE Transactions on Reliability*, Vol. R-32 No. 5, pp.469-471, 1983.

[4] R. L. Kenyon, R. J. Newell, "Steady-State Availability of k-out of-n: G system with single Repair", *IEEE Transactions on Reliability*, Vol. R-32 No. 2, 1983.

[5] S. C. Lee, "System Diagrams and Event Trees for Coherent Structure", *Bulletin of the Institute for basic science,* Vol. 12, Inha Univ., 1991.

[6] S. C. Lee, "Probability of System Failure of Coherent Structure", *Bulletin of the Institute for Applied science,* Korea Aviation Univ., 1993.

[7] S. C. Lee, "Determination of Probability of Component or Subsystem Failure", *Journal of Korean Society for Quality Control,* Vol. 2, No. 2, pp.121-130, 1993.

[8] S. C. Lee, "Reliability Expression for Complex System", *Journal of Korean Communications in Statistics,* Vol. 3, No. 3, pp.125-133, 1996.

[9] S. C. Lee, "Algorithm for System Reliability Calculation", *Journal of Namseoul Univ.,* Vol. 3, pp.75-87, 1997.

[10] 三根久, 河合一, *信頼性 保全性の基礎數理*, 日科技連, 1984.