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Jackknife Estimation for Mean in Exponential Model with Grouped and Censored Data[1]

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Abstract

In this paper, we propose some jackknife estimators for mean in the exponential model with grouped and censored data. Also, we compare the proposed jackknife estimators to other approximate estimators in terms of the mean square error and bias.

1. Introduction

The model used in this paper is based on exponential life-time distribution, where data are grouped and censored at a specified time. In this case we have very poor information for lifetime when sample size N and number of inspection k are small. Here we consider three computationally simpler estimation methods. The first method assume that failures occur at interval center, the second method is based on a Taylor series expansion of the probability of failure in each interval, and the last method is based on the first step iteration of the Newton-Raphson procedure to solve the likelihood equation.

Kulldorff(1961) and Ehrenfeld(1962) considered the large sample properties of the maximum likelihood estimator(MLE) for the grouped and censored exponential data. Kulldorff(1961) showed that mid-point estimator is biased and not consistent. Nelson(1982) showed that mid-point estimator could be useful for practical purpose if interval widths were small relative to mean(θ). Meeker(1986) considered three inspection schemes until censoring; equally spaced in time, equally spaced in log time and equal probability. Seo and Yum(1993) reported some Monte Carlo simulation results for three approximate estimators in exponential grouped and censored data. They compared these approximate estimators to exact MLE in terms of mean squared error(MSE) and bias.

The jackknife method is resampling method which was at first introduced by Quenouille(1956) for the purpose of reducing bias. A review concerning this resampling plan is

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given in Miller(1974).

In this paper, we compare three approximate estimators for mean in the exponential model to their jackknife estimators in terms of MSE and bias. In Section 2, we explain notations, assumption of this model and three typed inspection scheme. And we introduce mid-point estimator for grouped and censored exponential data and derive the jackknife estimator using this estimator. In Section 3, we introduce the approximate estimators based on Taylor series expansion and the Newton-Raphson method, respectively, and evaluate jackknife estimators using these two approximate estimators. In Section 4, through the Monte Carlo simulation study we compare three approximate estimators to the jackknife estimators using these estimators in terms of MSE and bias.

2. Mid-Point Estimator

2.1 Notations and model

Let $T_1, T_2, \ldots, T_N$ be identically independent lifetimes with distribution fuction F, and let τ_i, $i = 1, 2, \cdots, k$, be the ith inspection time, where k is the number of inspections;

$$\tau_0 = 0, \quad \tau_{k+1} = \infty \quad \text{and } \tau_k = \text{censoring time.}$$

Then we define the probability p_i that a test unit fails in $(\tau_{i-1}, \tau_i]$, $i = 1, 2, \cdots, k$, as

$$p_i = \Pr[\tau_{i-1} < T \le \tau_i],$$

and the probability p_c that a test unit is censored, as

$$p_c = \Pr[T > \tau_k].$$

Let x_i's be the numbers of failures in each inspection interval $(\tau_{i-1}, \tau_i]$, $i=1, 2, \cdots, k$;

$$x_{k+1} = \text{number of test units censored,}$$

$$r = \sum_{i=1}^{k} x_i.$$

We have assumptions as follow;

(1) N units are available for life test at time 0.

(2) The lifetimes of test units are mutually independent and follow an exponential distribution with p.d.f.

$$f(t;\theta) = \frac{1}{\theta}\exp\left(\frac{-t}{\theta}\right), \quad t>0, \quad \theta>0.$$

(3) The life test is terminated at a specified time.

(4) ith inspection is conducted at τ_i, $i=1, 2, \cdots, k$; the last inspection occurs at the

censoring time τ_k.

For inspection times, we consider three schemes from Meeker(1986); in equally spaced(ES) inspection scheme, τ_i are defined as

$$\tau_i = i\frac{\tau_k}{k}, \quad i=1,\ 2,\ \cdots,\ k-1,$$

in equal probability(EP) inspection scheme, these are defined as

$$\tau_i = -\theta \ln\left(1-\frac{(1-p_c)i}{k}\right), \quad i=1,\ 2,\ \cdots,\ k-1,$$

and in equally spaced inspection scheme of log time(ESL), these are defined as

$$\tau_i = \tau_1\left(\frac{\tau_k}{\tau_1}\right)^{\frac{i-1}{k-1}}, \quad i=2,\ 3,\ \cdots,\ k-1,$$

where τ_1 is the same as in EP.

2.2 Mid-point estimation

When we know exact failure time, the MLE of θ is

$$\hat{\theta}_c = \frac{\left(\sum_{j=1}^{r} t_j + \tau_k x_{k+1}\right)}{r}, \quad r > 0,$$

where t_j's are the exactly jth failure time until τ_k, $j = 1, 2, \cdots, r$. However in grouped data scheme, we can not obtain closed form of MLE. Hence we assume that x_i failures have occured at the mid-point of the interval m_i, $m_i = \frac{(\tau_i + \tau_{i-1})}{2}$, $i = 1, 2, \cdots, k$. Then we can obtain a mid-point estimator of θ by using ML procedure for continuous inspection as follows;

$$\hat{\theta}_0 = \frac{\left(\sum_{i=1}^{k} m_i x_i + \tau_k x_{k+1}\right)}{r}, \quad r>0,$$

$\hat{\theta}_0$ can be useful for practical purposes if interval widths are small relative to θ. However $\hat{\theta}_0$ is biased and not consistent.

To obtain jackknife estimator using mid-point estimator we use the method proposed by Quenouille(1956). Estimator $\widehat{\theta_0^i}$ can be obtained by deleting one obsevation from x_i observations in interval $(\tau_{i-1}, \tau_i]$, $i = 1, 2, \cdots, k+1$, and depends on whether the deleted observation is censored or not censored. Hence the estimator $\widehat{\theta_0^i}$ is given as

$$\widehat{\theta}_0^i = \begin{cases} \frac{1}{r-1}\left\{ \sum_{j=1 \neq i}^{k} m_j x_j + m_i(x_i - 1) + \tau_k x_{k+1} \right\} & i = 1,\ 2,\ \cdots,\ k, \\ \frac{1}{r}\left\{ \sum_{j=1}^{k} m_j x_j + \tau_k(x_{k+1} - 1) \right\} & i = k+1. \end{cases} \tag{2.1}$$

Then Quenouille's method yields the pseudo jackknife

$$\widehat{J}_0^i = N\widehat{\theta}_0 - (N-1)\widehat{\theta}_0^i, \quad i = 1,\ 2,\ \cdots,\ k+1.$$

Each pseudo jackknife exists exactly $x_1, x_2, \cdots, x_{k+1}$ times, respectively. Therefore the jackknife estimator $\widehat{J}_0$ using mid-point estmator is the arithmetic mean of the N pseudo jackknifes given by

$$\widehat{J}_0 = \frac{1}{N}\sum_{i=1}^{k+1} x_i \widehat{J}_0^i.$$

It is obvious that if the number of inspection k is small then only a few pseudo jackknife exist. Note that both the mid-point estimator and the jackknife estimator using this estimator are very sensitive to the probability of censoring when number of inspection is small.

3. Approximate Maximum Likelihood Estimators

For the grouped data, the corresponding likelihood is given by

$$L(\theta) = N! \cdot \left(\prod_{i=1}^{k+1} x_i!\right)^{-1} \cdot \prod_{i=1}^{k+1} p_i^{x_i}$$

where

$$\begin{aligned} p_i &= \Pr(\tau_{i-1} < T \le \tau_i) \\ &= \exp\left(-\frac{\tau_{i-1}}{\theta}\right) - \exp\left(-\frac{\tau_i}{\theta}\right), \quad i = 1,\ 2,\ \cdots,\ k+1, \end{aligned}$$

and log likelihood is

$$\ln L(\theta) = C + \sum_{i=1}^{k+1} x_i \cdot \ln(p_i)$$

where C is constant with respect to θ. Then MLE of θ is obtained by solving the equation

$$\frac{d\ln L(\theta)}{d\theta} = \sum_{i=1}^{k+1} x_i \cdot \frac{d\ln(p_i)}{d\theta} = 0. \tag{3.1}$$

Equation (3.1) yields no closely formed solution for θ except for an equally spaced inspection scheme or a single inspection. Therefore we use two approximate MLE's to obtain closed form of solution.

3.1 Approximate estimation based on Taylor series expansion

Tallis(1967) considered an approximate MLE of θ based on Taylor series expansion of the probability of failure p_i, $i=1, 2, \cdots, k$. He suggested the following approximation

$$\frac{d\ln(p_i)}{d\theta} \approx \frac{d}{d\theta}(f(m_i;\theta)) + \frac{d_i^2}{24}\cdot\frac{d}{d\theta}\left(\frac{f'(m_i;\theta)}{f(m_i;\theta)}\right),\quad i=1, 2, \cdots, k, \tag{3.2}$$

where f is the p.d.f. of lifetime T, f' is the second derivative of f, and $d_i=\tau_i-\tau_{i-1}$. By inserting equation (3.2) in equation (3.1) for $i=1, 2, \cdots, k$, we have

$$\frac{d\ln L}{d\theta} \approx \sum_{i=1}^{k} x_i\left\{\frac{d\ln f}{d\theta} + \frac{d_i^2}{24}\frac{d}{d\theta}\left(\frac{f'}{f}\right)\right\} + x_{k+1}\frac{d\ln(p_{k+1})}{d\theta}. \tag{3.3}$$

Since f is an exponential p.d.f with mean θ , (3.3) is simplified as

$$\frac{d\ln L}{d\theta} \approx \sum_{i=1}^{k}\left(-\theta^{-1} + m_i\theta^{-2} - \frac{d_i^2}{12}\theta^{-3}\right) + x_{k+1}\tau_k\theta^{-2}. \tag{3.4}$$

An approximate MLE $\hat{\theta}_a$ is obtained as

$$\hat{\theta}_a = \frac{\hat{\theta}_0}{2} \pm \frac{1}{2}\left(\hat{\theta}_0^{\,2} - \frac{\sum_{i=1}^{k} x_i d_i^2}{3r}\right)^{1/2},\quad r>0, \tag{3.5}$$

where $\hat{\theta}_0$ is the mid-point estimator. We take "+" in optimal sign in equation (3.5) since $\hat{\theta}_a$ must approach $\hat{\theta}_0$ as $k\to\infty$ (*that is,* $d_i\to 0$). Kendell and Anderson(1971) obtained the same estimator based on a different approximation. Now to obtain the jackknife estimator using the mid point estimator, we evaluate estimators $\widehat{\theta_a^i}$ which can be obtained by deleting one sample in $(\tau_{i-1}, \tau_i]$, $i=1, 2, \cdots, k$. The estimators $\widehat{\theta_a^i}$, $i=1, 2, \cdots, k+1$, are given by

$$\widehat{\theta_a^i} = \begin{cases} \dfrac{\widehat{\theta_0^i}}{2} + \dfrac{1}{2}\left(\widehat{\theta_0^i}^{\,2} - \dfrac{\sum_{j=1\neq i}^{k} x_j d_j^2 + (x_i-1)d_i^2}{3(r-1)}\right)^{1/2} & i=1, 2, \cdots, k, \\[2ex] \dfrac{\widehat{\theta_0^i}}{2} + \dfrac{1}{2}\left(\widehat{\theta_0^i}^{\,2} - \dfrac{\sum_{j=1}^{k} x_j d_j^2}{3r}\right)^{1/2} & i=k+1, \end{cases}$$

where $\widehat{\theta_0^i}$ is defined in equation (2.1). By the Quenouille's method the pseudo jackknife $\widehat{J_a^i}$ can be obtained as follows

$$\widehat{J_a^i} = N\hat{\theta}_a - (N-1)\widehat{\theta_a^i},\quad i=1, 2, \cdots, k+1.$$

Therefore the jackknife estimator $\hat{J}_a$ using the approximate estimator based on Taylor series expansion is

$$\hat{J}_a = \frac{1}{N}\sum_{i=1}^{k+1} x_i \widehat{J^i_a}.$$

Note that if all observation are censored (that is, r=0), the acceptable $\hat{\theta}_a$ and $\hat{J}_a$ can not be obtained. The case of $r = 0$ tends to occur when p_c is large and N is small. If the number of inspection and the probability of censoring are very small, then the quantity in the brace of equation (3.4) may be negative and hence acceptable $\hat{\theta}_a$ and $\hat{J}_a$ can not be obtained.

3.2 Approximate estimation using Newton-Raphson method

We can obtain another approximate estimator by using first step iteration of the Newton-Raphson method. Let $\hat{\theta}_0$ be chosen as an initial value in the Newton-Raphson procedure. Then the aprroximate estimator $\hat{\theta}_{a'}$ is

$$\hat{\theta}_{a'} = \frac{\hat{\theta}_0 + \frac{dL}{d\theta}\mid_{\theta=\widehat{\theta_0}}}{\hat{I}_0}$$

where

$$\hat{I}_0 \equiv -\frac{d^2L}{d\theta^2}\mid_{\theta=\widehat{\theta_0}}.$$

If we replace $\hat{I}_0$ with the following simpler observed information for a continuous inspection

$$\widehat{I_0} = \frac{r}{\hat{\theta}_0^{\,2}},$$

$$\hat{\theta}_{a'} = \hat{\theta}_0\left(1 - \frac{\sum_{i=1}^{k} x_i\left(\frac{di}{\hat{\theta}_0}\right)^2}{12r}\right), \quad r > 0.$$

The estimators $\widehat{\theta^i_{a'}}$ which can be obtained by deleting one sample in $(\tau_{i-1}, \tau_i]$, $i=1, 2, \cdots, k+1$, are given by

$$\widehat{\theta^i_{a'}} = \begin{cases} \widehat{\theta^i_0}\left(1 - \frac{\sum_{j=1\neq i}^{k} x_j d_j^2 + (x_i - 1)\, d_i^2}{12(r-1)} \cdot \widehat{\theta_0^{i}}^{-2}\right) & i=1, 2, \cdots, k, \\ \widehat{\theta^i_0}\left(1 - \frac{\sum_{j=1}^{k} x_j d_i^2}{12r} \cdot \widehat{\theta_0^{i}}^{-2}\right) & i=k+1, \end{cases}$$

Also the pseudo jackknife $\widehat{J^i_{a'}}$ is calculated as follows;

$$\widehat{J^i_{a'}} = N\hat{\theta}_{a'} - (N-1)\widehat{\theta^i_{a'}}, \quad i = 1, 2, \cdots, k+1.$$

Hence the jackknife estimator $\hat{J}_{a'}$ using approximate estimator based on Newton-Raphson method is

$$\hat{J}_{a'} = \frac{1}{N}\sum_{i=1}^{k+1} x_i \widehat{J_{a'}}.$$

4. Simulation Study

Design parameters in the simulation experiments include sample size N, probability of censoring p_c, number of inspections k and inspection schemes. We consider the follow combinations;

$$N = 20,\ 30,\ 50,\ 100$$
$$k = 2,\ 3,\ 5,\ 7,\ 10$$
$$p_c = 0.05,\ 0.1,\ 0.3,\ 0.5$$
$$\tau_k = -\theta \ln(p_c) \ : \text{censoring time.}$$

For each combination of N, k, p_c and inspection schemes, exponential random variables are generated by using IMSL, and $\hat{\theta}_0$, $\hat{\theta}_a$, $\hat{\theta}_{a'}$ and the jackknife estimators using these estimators are calculated. For each combination of parameters, we obtain biases and MSE's of estimators through 3000 simulations, respectively. There are some parts of the simulation results in Table 1 and Table 2 and the rest are available on request.

In these tables, the mid-point estimator, the approximate estimator based on Taylor series expansion and the approximate estimator using Newton-Raphson method are denoted by MID, TS and NR, respectively, and the maximum likelihood estimator and the jackknife estimator are denoted by MLE and JACK, respectively.

From Table 1 and Table 2, we observe the facts as follow;

In the mid-point estimator,

(1) Jackknife estimator has generally smaller bias and MSE than approximate estimators, especially in the cases of small sample size, small number of inspection and large probability of censoring.

(2) Mid-ponit estimator and jackknife estimator are generally overestimated.

(3) Jackknife estimator is biased and also not consistent.

(4) Estimator in equally spaced scheme is slightly efficient when p_c is small.

(5) In small number of inspection, bias of estimator is relative to probability of censoring.

In the approximate estimator based on Taylor series expansion,

(1) Jackknife estimator has generally smaller bias and MSE than approximate estimators except for very small p_c, especially in the cases of small sample size, small number of inspection and large probability of censoring.

(2) For small probability of censoring and small numer of inspection, approximate estimator and jackknife estimator may not exist.

(3) Approximate estimator is overestimated, but jackknife estimator is generally underestimated.

(4) Estimator in equally spaced scheme is slightly efficient when p_c is small.

(5) In small sample size and large probability of censoring, bias and MSE are invariable for number of inspection k.

In the approximate estimator using Newton-Raphson method,

(1) Jackknife estimator has generally smaller bias and MSE than approximate estimators except for very small p_c, especially in the cases of small sample size, small number of inspection and large probability of censoring.

(2) Approximate estimator is overestimated but jackknife estimator is generally underestimated.

(3) Estimator in equally spaced scheme is slightly efficient when p_c is small.

(4) In small sample size and large probability of censoring, bias and MSE are invariable for number of inspection k.

Table 1
Biases for three approximate estimators and their jackknife estimators

$N=20$

k	insp	Pc	0.05			0.1			0.3			0.5		
		M	MID	TS	NR	MID	TS	NR	MID	TS	NR	MID	TS	NR
2	ES	MLE	.9242	.2077	.1113	.5807	.0543	.0653	.3264	.1684	.1747	.4313	.3794	.3801
		JACK	.8788	.2585	.0855	.5017	.0193	.0009	.1004	-.0506	-.0465	-.0742	-.1242	-.1238
	EP	MLE	1.1646	.1565	.1527	.6999	.0030	.0907	.3354	.1666	.1735	.4355	.3825	.3832
		JACK	1.1192	.1854	.1086	.6209	-.0492	.0162	.1094	-.0542	-.0492	-.0700	-.1215	-.1210
	ESL	MLE	1.1646	.1565	.1527	.6999	.0030	.0907	.3354	.1666	.1735	.4355	.3825	.3832
		JACK	1.1192	.1854	.1086	.6209	-.0492	.0162	.1094	-.0542	-.0492	-.0700	-.1215	-.1210
10	ES	MLE	.0527	.0133	.0137	.0648	.0416	.0418	.1816	.1754	.1754	.3827	.3806	.3806
		JACK	.0073	-.0300	-.0297	-.0140	-.0361	-.0360	-.0443	-.0503	-.0503	-.1228	-.1248	-.1248
	EP	MLE	.0843	.0153	.0164	.0759	.0427	.0430	.1809	.1739	.1739	.3831	.3810	.3810
		JACK	.0389	-.0314	-.0304	-.0029	-.0362	-.0360	-.0450	-.0518	-.0518	-.1223	-.1244	-.1244
	ESL	MLE	.0880	.0154	.0165	.0894	.0415	.0420	.1880	.1736	.1736	.3856	.3808	.3808
		JACK	.0427	-.0305	-.0296	.0105	-.0373	-.0368	-.0379	-.0521	-.0521	-.1199	-.1246	-.1246

$N = 50$

k	insp	Pc	0.05			0.1			0.3			0.5		
		M	MID	TS	NR	MID	TS	NR	MID	TS	NR	MID	TS	NR
2	ES	MLE	.9139	-.0462	.1136	.5604	-.0075	.0557	.2055	.0514	.0565	.1765	.1256	.1262
		JACK	.8964	-.0246	.1039	.5305	-.0230	.0314	.1266	-.0249	-.0205	.0197	-.0304	-.0299
	EP	MLE	1.1575	-.1142	.1506	.6779	-.0152	.0750	.2135	.0477	.0535	.1792	.1269	.1276
		JACK	1.1400	-.0958	.1340	.6480	-.0353	.0470	.1346	-.0292	-.0240	.0225	-.0291	-.0286
	ESL	MLE	1.1575	-.1142	.1506	.6827	-.0152	.0750	.2666	.0477	.0535	.2841	.1269	.1276
		JACK	1.1400	-.0958	.1340	.6480	-.0353	.0470	.1346	-.0292	-.0240	.0225	-.0291	-.0286
10	ES	MLE	.0470	.0088	.0091	.0387	.0162	.0163	.0615	.0554	.0554	.1305	.1284	.1284
		JACK	.0295	-.0079	-.0076	.0088	-.0132	-.0131	-.0172	-.0233	-.0233	-.0262	-.0282	-.0282
	EP	MLE	.0776	.0083	.0093	.0517	.0188	.0191	.0614	.0546	.0546	.1292	.1271	.1271
		JACK	.0601	-.0096	-.0087	.0218	-.0110	-.0108	-.0173	-.0242	-.0241	-.0275	-.0296	-.0296
	ESL	MLE	.0837	.0110	.0121	.0648	.0174	.0179	.0680	.0538	.0538	.1320	.1273	.1273
		JACK	.0662	-.0066	-.0055	.0348	-.0124	-.0120	-.0107	-.0249	-.0249	-.0246	-.0293	-.0293

Table 2

MSE's for three approximate estimators and their jackknife estimators

$N = 20$

k	insp	Pc	0.05			0.1			0.3			0.5		
		M	MID	TS	NR	MID	TS	NR	MID	TS	NR	MID	TS	NR
2	ES	MLE	1.9420	1.3032	1.4019	1.6694	1.5732	1.5761	2.3312	2.3737	2.3632	4.0393	4.0548	4.0533
		JACK	1.7847	1.1359	1.3129	1.4552	1.3732	1.4288	1.7476	1.8413	1.8335	2.2032	2.2566	2.2554
	EP	MLE	2.7531	1.5035	1.5411	1.9589	1.7242	1.6290	2.3926	2.4191	2.4088	4.0255	4.0380	4.0366
		JACK	2.5721	1.3676	1.4532	1.7237	1.5552	1.4828	1.7999	1.8849	1.8759	2.1916	2.2334	2.2422
	ESL	MLE	2.7531	1.5035	1.5411	1.9589	1.7242	1.6290	2.3926	2.4191	2.4088	4.0255	4.0380	4.0366
		JACK	2.5721	1.3676	1.4532	1.7237	1.5552	1.4828	1.7999	1.8849	1.8759	2.1916	2.2334	2.2422
10	ES	MLE	1.3298	1.3480	1.3474	1.4837	1.4943	1.4941	2.3194	2.3218	2.3218	3.9946	3.9953	3.9953
		JACK	1.2497	1.2694	1.2690	1.3492	1.3620	1.3619	1.7997	1.8043	1.8043	2.2180	2.2202	2.2202
	EP	MLE	1.3696	1.3545	1.3543	1.4977	1.4980	1.4979	2.3178	2.3195	2.3195	4.0008	4.0014	4.0014
		JACK	1.2863	1.2781	1.2778	1.3609	1.3662	1.3660	1.9788	1.8030	1.8030	2.2222	2.2244	2.2244
	ESL	MLE	1.3724	1.3647	1.3642	1.4974	1.4985	1.4982	2.3199	2.3222	2.3221	4.0021	4.0029	4.0029
		JACK	1.2886	1.2879	1.2873	1.3590	1.3673	1.3669	1.7972	1.8049	1.8048	2.2204	2.2248	2.2248

N = **50**

k	insp	Pc	0.05			0.1			0.3			0.5		
		M	MID	TS	NR	MID	TS	NR	MID	TS	NR	MID	TS	NR
2	ES	MLE	1.2804	.6697	.5834	.8276	.6537	.6117	.8070	.8135	.8097	1.2109	1.2170	1.2164
		JACK	1.2367	.6183	.5677	.7759	.6246	.5883	.7251	.7532	.7493	1.0227	1.0430	1.0424
	EP	MLE	1.9131	.7540	.6477	1.0425	.6946	.6509	.8246	.8235	.8196	1.2102	1.2149	1.2144
		JACK	1.8607	.7032	.6308	.9833	.6700	.6268	.7410	.7637	.7595	1.0213	1.0407	1.0401
	ESL	MLE	1.9131	.7540	.6477	1.0425	.6946	.6509	.8246	.8235	.8196	1.2102	1.2149	1.2144
		JACK	1.8607	.7032	.6308	.9833	.6700	.6268	.7410	.7637	.7595	1.0213	1.0407	1.0401
10	ES	MLE	.5445	.5507	.5505	.5870	.5910	.5909	.7946	.7957	.7957	1.2076	1.2079	1.2079
		JACK	.5310	.5381	.5380	.5662	.5713	.5712	.7352	.7371	.7371	1.0338	1.0347	1.0347
	EP	MLE	.5670	.5577	.5576	.5935	.5929	.5929	.7942	.7949	.7949	1.2060	1.2063	1.2063
		JACK	.5523	.5456	.5454	.5717	.5731	.5731	.7347	.7365	.7365	1.0327	1.0336	1.0336
	ESL	MLE	.5661	.5593	.5591	.5956	.5949	.5948	.7956	.7967	.7966	1.2050	1.2054	1.2054
		JACK	.5512	.5470	.5468	.5731	.5752	.5750	.7351	.7383	.7382	1.0309	1.0326	1.0326

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