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A Doubly Winsorized Poisson Auto-model

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Abstract

This paper introduces doubly Winsorized Poisson auto-model by truncating the support of a Poisson random variable both from above and below, and shows that this model has a same form of negpotential function as regular Poisson auto-model and one-way Winsorized Poisson auto-model. Strategies for maximum likelihood estimation of parameters are discussed. In addition to exact maximum likelihood estimation, Monte Carlo maximum likelihood estimation may be applied to this model.

1. Introduction

Poisson spatial auto-models proposed by Besag (1974) have been popular for the analysis of count data that exhibit dependence. Such models are particularly useful for spatial processes defined on lattice-indexed random fields. Patterns of mortality or morbidity in epidemiological studies have been described using these models (e.g., Clayton and Kaldor 1987). However, Besag (1974) showed that Poisson auto-models may be used to characterize only negative spatial dependence due to the 'summability' condition, a requirement that the joint probability distribution for a set of spatial random variables have a finite normalizing constant. Based on the concept that converting the infinite supports of Poisson random variables to finite supports can ensure that the summability condition is met, Kaiser and Cressie (1997) proposed a Winsorized Poisson auto-model using truncated random variables. This Winsorized Poisson auto-model can be used to incorporate either positive or negative spatial dependencies among a set of variables. Lee and Kaiser (1997) used this model in describing spatial dependence in occurrences of Sweet Birch trees in the northeastern United States. In this example, zero counts were too abundant in the observed data for a Poisson model to provide an adequate fit, and zero values were deleted from the data proir to analysis. It is not uncommon in sets of data composed of small counts that the zero class is overly-represented for description through use of a Poisson model (e.g., Cohen 1960). In these situations, one approach is to truncate a Poisson distribution from below.

In this paper, we adapt the Winsorized Poisson auto-model to include truncation from both

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above and below. The primary difficulty in defining a valid auto-model for doubly Winsorized Poisson random variables is exclusion of zero from the support of a Poisson random variable. In Besag's development of auto-models (1974), a basic assumpton was that zero be a possible value in the support of each random variable. Auto-models may be developed without the use of zero values, providing greater flexibility in the formulation of models for a number of situations (Kaiser and Cressie 1996).

Winsorization makes it possible to evaluate the joint distribution normalizing constant in Poisson auto-models because the support of each random variable is then finite. Therefore, exact maximum likelihood estimation can be accomplished theoretically, but in practice would require the use of a computer having no cpu time limit. In actual use of Winsorized Poisson auto-models, Monte Carlo maximum likelihood estimation method is more efficient since the unnormalized joint distribution is not evaluated for all possible values but rather is approximated as an average over samples from some density, called an importance distribution (e.g., Penttinen 1984; Geyer and Thompson 1992; Lee and Kaiser 1997).

The remainder of the paper is organized as follows. In Section 2, the formulation of auto-models will be briefly reviewed. In Section 3, a Poisson distribution Winsorized both from above and below will be used to construct a class of doubly-Winsorized Poisson auto-models. Section 4 contains possible maximum likelihood estimation methods for the models developed in Section 3, and section 5 presents a small example that is used to illustrate the development of previous sections.

2. The Formulation of Poisson Auto-Models

In this section, some notation is introduced and general results that will be needed later in this paper are briefly reviewed. These results allow a formulation that is a slight variation of the framework based on Markov random fields developed originally by Besag (1974). Let s_i denote a physical location in a geographic region of interest, and let $D \equiv \{ s_i : i = 1, \cdots, n\}$ be a finite lattice (regular or irregular) defined by these sites. The random process associated with these geographic locations will be denoted as $Z \equiv \{Z(s_i) : s_i \in D\}$. Auto-models are formulated on the basis of a Markov random field defined by the specification of a neighborhood for each component of Z. A site s_j is a neighbor of a site s_i if the conditional distribution of $Z(s_i)$ given $\{z(s_k) : k \neq i\}$ depends functionally on the value of $z(s_j)$. Let $N_i \equiv \{ s_j : s_j \textit{ is a neighbor of } s_i\}$ be the neighborhood of site s_i and, for discrete random variables, let the probability mass function (pmf) of $Z(s_i)$, conditional on its neighbors, be given by

$$p(z(s_i)|z(N_i)) \equiv p(z(s_i)|\{z(s_j): s_j \in N_i\}).$$

A Poisson auto-model results from specifying that all components of Z have Poisson conditional pmfs. The quantity that connects such a set of conditional pmfs with the joint likelihood of Z is called the 'negpotential function' and may be defined here as $Q(z) \equiv \log\{\Pr(z)/\Pr(z_0)\}$, where $z_0 = (z_0, \cdots, z_0)'$ is in the support set, Ω, of the joint pmf of Z, and $Q(z)$is defined for all $z \in \Omega$. If one can calculate $Q(\cdot)$, the joint pmf of Z is available as

$$f(z) = \frac{\exp\{Q(z)\}}{\sum_{t \in \Omega} \exp\{Q(t)\}}. \tag{1}$$

Pairwise-only dependence is an assumption often made in spatial models and, with this assumption, the negpotential function may be written as (Besag, 1974)

$$Q(z) = \sum_{1 \le i \le n} z(s_i) G_i(z(s_i)) + \sum\sum_{1 \le i < j \le n} z(s_i) z(s_j) G_{ij}(z(s_i), z(s_j)), \tag{2}$$

where $G_{ij}(\cdot, \cdot)$ is zero if $z(s_j)$ is not an element of the set N_i. This restriction on the limits of summation in the interaction term arises from a theorem of fundamental importance due to Hammersley and Clifford (Hammersley and Clifford 1971, Besag 1974). Cressie (1993, p. 416) and Kaiser and Cressie (1996) demonstrate that the terms of equation (2) may be written as functions of conditional pmfs,

$$z(s_i) G_i(z(s_i)) = \log\left\{\frac{p(z(s_i)|\, z_0(N_i))}{p(z_0(s_i)|\, z_0(N_i))}\right\}, \tag{3}$$

and

$$z(s_i) z(s_j) G_{ij}(z(s_i), z(s_j)) = \log\left\{\frac{p(z(s_i)|z(s_j),\, z_0(N_i^{-j}))}{p(z_0(s_i)|z(s_j),\, z_0(N_i^{-j}))} \frac{p(z_0(s_i)|\, z_0(N_i))}{p(z(s_i)|\, z_0(N_i))}\right\}, \tag{4}$$

where $N_i^{-j} = \{z(s_k): s_k \in N_i, k \neq j\}$, $z_0(s_i)$ denotes that $z(s_i) = z_0$, and $z_0(N_i)$ denotes that $\{z(s_j) = z_0: s_j \in N_i, j \neq i\}$. This result indicates the way in which the negpotential function can be constructed directly from a set of conditional pmf specificatons. The last general result that will be needed here is proved in a theorem by Kaiser and Cressie (1996). Within the context of pairwise-only dependence, the result says that any specification of conditional pmfs $\{p(z(s_i)|z(N_i)): i=1,\cdots,n\}$ such that the resulting terms for $z(s_i)z(s_j)G_{ij}(z(s_i),z(s_j))$ are symmetric in i and j, leads to a well-defined joint probability model for Z as long as the summability condition is met, that is, as long as

$$(5) \qquad \sum_{t\in\Omega} \exp(Q(t)) < \infty.$$

If this is the case, the joint pmf and likelihood are available through equation (1) and (2). The existing method to construct a Poisson auto-model depends on the following result of Besag (1974). For models in which the conditional pmfs are specified as belonging to an exponential family,

$$(6) \qquad p(z(s_i)|z(N_i)) = \exp\{A_i(z(N_i))B_i(z(s_i)) - D_i(z(N_i)) + C_i(z(s_i))\},$$

the functions $A_i(z(N_i));\ i=1,\cdots,n$ must satisfy

$$(7) \qquad A_i(z(N_i)) = \alpha_i + \sum_{j=1}^{n} \eta_{ij} B_j(z(s_j)),$$

where $\eta_{ij} = \eta_{ji}$ for all i and j, and $\eta_{ij}=0$ if s_j is not in N_i, the neighborhood of s_i. A standard Poisson specification for the conditional pmfs (6) results from taking

$B_i(z(s_i)) = z(s_i), D_i(z(N_i)) = \exp\{A_i(z(N_i))\}$, and $C_i(z(s_i)) = -\log(z(s_i)!)$.

Using the equations (2), (3), and (5), the negpotential function for this regular Poisson auto-model becomes

$$(8) \qquad Q(z) = \sum_{1\le i\le n} [\alpha_i z(s_i) - \log(z(s_i)!)] + \sum_{1\le i<j\le n} \eta_{ij} z(s_i)z(s_j),$$

where $\eta_{ij}=0$ if $s_j \notin N_i$. The joint support Ω of a regular Poisson auto-model is the n-fold Cartesian product of the set of nonnegative integers. Now, the summability condition (5) does not hold for $Q(\cdot)$ given by equation (8) should any one of the $\{\eta_{ij}\}$ be positive (Besag, 1974). Thus, for a well defined Poisson auto-model, we must have $\eta_{ij} \le 0$ for all i and j, which implies that the model must contain only negative dependence relations among the elements of Z. Kaiser and Cressie (1997) defined Winsorized Poisson conditional pmfs, for $0 \le z(s_i) \le R$ for all $i=1,\cdots,n$, as

(9) $$p(z(s_i)|z(N_i)) = \exp\{A_i(z(N_i))z(s_i) - D_i(z(N_i)) + \log(z(s_i)!)\},$$

where

$$D_i(z(N_i)) = \begin{cases} \exp\{A_i(z(N_i))\}, & \text{if } z(s_i) \le R-1 \\ \exp\{A_i(z(N_i))\} - \psi, & \text{if } z(s_i) = R, \end{cases}$$

for some $0 < \psi_i < \exp\{A_i(z(N_i))\}$. In these expression, R is a large positive integer that must be specified in model formulation. They showed that if $A_i(z(N_i)) = \alpha_i + \sum_{j=1}^{n} \eta_{ij} z(s_j)$, the negpotential function from the definition of (9) is identical with (8) but that the η_{ij}'s can have not only negative but also positive values. This implies that the model can be used to characterize either negative or positive dependence among the elements of Z.

3. Models for Doubly-Winsorized Poisson Conditionals

In this section, an auto-model for Poisson random variables Winsorized from below as well as from above is developed. Double Winsorization of a regular Poisson random variable is considered first, and then these distributions are used in construction of an auto-models.

3.1 Poisson Winsorization

Let X be a Poisson random variable with pmf

$$f(x|\lambda) = \frac{\lambda^x}{x!}\exp(-\lambda), \quad x \in \{0, 1, \cdots, \},$$

for $\lambda > 0$. With $I(a)$ denoting the indicating function for an event a, define the doubly-winsorized random variable
$Z \equiv R_L I(X \le R_L) + X I(R_L + 1 \le X \le R_U - 1) + R_U I(X \ge R_U)$, for finite, non-negative integer values R_L and R_U. Then the pmf of Z may be written as

$$\begin{aligned} p(z; \lambda, R_L, R_U) &= \left\{ \sum_{t=0}^{R_L} \frac{\lambda^t}{t!}\exp(-\lambda) \right\} I(z = R_L) + \left\{ \frac{\lambda^z}{z!}\exp(-\lambda) \right\} I(R_L + 1 \le z \le R_U - 1) \\ &+ \left\{ 1 - \sum_{t=0}^{R_U - 1} \frac{\lambda^t}{t!}\exp(-\lambda) \right\} I(z = R_U). \end{aligned}$$

From Taylor's formula for $\exp(-\lambda)$, we have that

$$1-\sum_{t=0}^{R_U-1}\frac{\lambda^t}{t!}\exp(-\lambda)=\frac{\lambda^{R_U}}{R_U!}\exp(\psi-\lambda),\quad \text{for some } 0<\psi<\lambda,$$

and

$$\sum_{t=0}^{R_L}\frac{\lambda^t}{t!}\exp(-\lambda)=\frac{\lambda^{R_L}}{R_L!}\exp(-\lambda)\left\{1+\sum_{t=0}^{R_L-1}\frac{\lambda^{t-R_L}}{t!/R_L!}\right\}\equiv\frac{\lambda^{R_L}}{R_L!}\exp(-\lambda)g(\lambda).$$

Hence,

(10)

$$\begin{aligned} p(z|\lambda,R_L,R_U) &=\left\{\frac{\lambda^{R_L}}{R_L!}\exp[\log(g(\lambda))-\lambda]\right\}I(z=R_L)+\left\{\frac{\lambda^z}{z!}\exp(-\lambda)\right\}I(R_L+1\le z\le R_U-1)\\ &+\left\{\frac{\lambda^{R_U}}{R_U!}\exp(\psi-\lambda)\right\}I(z=R_U);\ z\in\{R_L,R_L+1,\cdots,R_U\}. \end{aligned}$$

The doubly-winsorized Poisson pmf (10) may be written in canonical exponential family form as

$$p(z|\lambda,R_L,R_U)=\exp\{\theta z-D(\theta)-\log(z!)\},$$

where $\theta=\log(\lambda)$ and, for $0<\psi<\exp(\theta)$ and $g(\cdot)$ as defined above,

$$D(\theta)=\begin{cases}\exp(\theta)+\log(g(\theta)), & \text{if } z=R_L\\ \exp(\theta), & \text{if } R_L+1\le z\le R_U-1\\ \exp(\theta)-\psi, & \text{if } z=R_U.\end{cases}$$

3.2 Spatial Formulation

We shall now formulate a spatial model for the random process $\boldsymbol{Z}$, where each component, conditional on its neighbors, follows the distribution of a doubly-Winsorized Poisson variable. Here, Ω in (5) is the n-fold Cartesian product of the set $\{R_L,R_L+1,\cdots,R_U-1,R_U\}$.

Writing the conditional pmf of each component of $\boldsymbol{Z}$ in canonical exponential family form gives, for $i=1,\cdots,n$,

(11) $$p(z(s_i)|z(N_i)) = \exp\{A_i(z(N_i))z(s_i) - D_i(z(N_i)) + \log(z(s_i)!)\},$$

where

$$D_i(z(N_i)) = \begin{cases} \exp\{A_i(z(N_i))\} + \log[g(A_i(z(N_i)))], & \text{if } z(s_i) = R_L \\ \exp\{A_i(z(N_i))\}, & \text{if } R_L + 1 \le z(s_i) \le R_U - 1 \\ \exp\{A_i(z(N_i))\} - \psi_i, & \text{if } z(s_i) = R_U. \end{cases}$$

We have the following results.

Lemma:

If $\eta_{ij} = \eta_{ji}$ and $\eta_{ii} = 0$ for all $i, j = 1, \cdots, n$, then

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \eta_{ij} z(s_j) = \sum_{1\le i<j\le n} \eta_{ij} z(s_i) + \sum_{1\le i<j\le n} \eta_{ij} z(s_j).$$

Proof:

$$\begin{aligned} \sum_{i=1}^{n}\sum_{j=1}^{n} \eta_{ij} z(s_j) &= z(s_1)\sum_{j=1}^{n}\eta_{1j} + z(s_2)\sum_{j=1}^{n}\eta_{2j} + \cdots + z(s_{n-1})\sum_{j=1}^{n}\eta_{n-1,j} + z(s_n)\sum_{j=1}^{n}\eta_{nj} \\ &= z(s_1)\sum_{j=2}^{n}\eta_{1j} + z(s_2)\sum_{j=3}^{n}\eta_{2j} + \cdots + z(s_{n-1})\sum_{j=n}^{n}\eta_{n-1,j} \\ &\quad + z(s_2)\sum_{j=1}^{1}\eta_{2j} + \cdots + z(s_{n-1})\sum_{j=1}^{n-2}\eta_{n-1,j} + z(s_n)\sum_{j=1}^{n-1}\eta_{nj} \\ &\quad + z(s_1)\eta_{11} + z(s_2)\eta_{22} + \cdots + z(s_n)\eta_{nn}. \end{aligned}$$

Using the conditions given on $\{\eta_{ij}; i, j = 1, \cdots, n\}$, it is easy to verify that right hand side becomes

$$\sum_{1\le i<j\le n} \eta_{ij} z(s_i) + \sum_{1\le i<j\le n} \eta_{ij} z(s_j).$$ ‡

Proposition:

Given a set of doubly-Winsorized Poisson pmfs of the form (11), a valid joint distribution of the form (1) exists, and may be identified using a negpotential function (2) constructed from equations (3) and (4) if

(12) $$A_i(z(N_i)) = \alpha_i + \sum_{j=1}^{n} \eta_{ij} z(s_j),$$

where $\eta_{ij}=\eta_{ji}$, and $\eta_{ij}=0$ if $s_j \notin N_i$.

Proof:
Validity of the model is verified by substitution of (11) with (12) into equation (4) with $z_0=R_L$ which yields

$$z(s_i)z(s_j)G_{ij}(z(s_i),z(s_j))=\eta_{ij}\{z(s_i)-R_L\}\{z(s_j)-R_L\}. \tag{13}$$

If $\eta_{ij}=\eta_{ji}$ for all $i,j=1,\cdots,n$, then (13) is symmetric in i and j. By theorem 3 of Kaiser and Cressie (1996), a valid joint distribution having the specified conditionals pmfs defined as in (11) exists, and may be identified up to a normalizing constant if the summability condition (5) is met. To verify that (5) holds, we need an expression for the negpotential function. Substitution of (11) and (12) into equation (3) gives

$$z(s_i)G_i(z(s_i))=(\alpha_i+R_L\sum_{j=1}^{n}\eta_{ij})(z(s_i)-R_L)-\log(z(s_i)!)+\log(R_L!). \tag{14}$$

Substitution of (13) and (14) into equation (2) and use of the Lemma given previously then yields the negpotential function, modulo an additive constant as,

$$\begin{aligned}
Q(z)&=\sum_{1\le i\le n}z(s_i)G_i(z(s_i))+\sum\sum_{1\le i<j\le n}z(s_i)z(s_j)G_{ij}(z(s_i),z(s_j))\\
&=\sum_{i=1}^{n}\{(\alpha_i+R_L\sum_{j=1}^{n}\eta_{ij})(z(s_i)-R_L)\}-\log(z(s_i)!)\\
&+\sum_{1\le i<j\le n}\eta_{ij}\{z(s_i)-R_L\}\{z(s_j)-R_L\}\\
&=\sum_{1\le i\le n}[\alpha_i z(s_i)-\log(z(s_i)!)]+\sum_{1\le i<j\le n}\eta_{ij}z(s_i)z(s_j).
\end{aligned}$$

The summability condition (5) is easily verified for any real α_i and η_{ij} since the joint support set, Ω, is finite with $(R_U-R_L+1)^n$ elements.

‡

As seen from the proof of this proposition, the negpotential function, modulo an additive constant, of a doubly-Winsorized Poisson auto-model is identical in form to that of a singly-Winsorized Poisson auto-model (Kaiser and Cressie 1997) and also that of a regular Poisson auto-model (Besag 1974). However, the conditional pmfs (11) used to formulate the Doubly-Winsorized Poisson auto-model differ from those used in the other models, as does the joint support set Ω and, in the case of the regular Poisson auto-model, necessary restrictions on the dependence parameters η_{ij}.

The regular Poisson auto-model of Besag (1974) is useful for modeling spatially dependent count data, but requires that such dependence be negative. The Winsorized Poisson auto-model of Kaiser and Cressie (1997) allows both negative and positive spatial dependence, but requires that the zero vector remain a possible value of the joint support. The doubly-Winsorized Poisson auto-model intoduced here allows exclusion of this class, while maintaining the flexibility to model both negative and positive spatial dependence.

4. Maximum Likelihood Estimation

To render full maximum likelihood estimation feasible, it is necessary to reduce the number of parameters allowed by a set of conditional pmfs given by (11). One way to do this is to restrict the parameters of (12) such that $\alpha_i = \alpha,\ i = 1, \cdots, n$ and $\eta_{ij} = \eta,\ i, j = 1, \cdots, n$. Two estimation methods will be introduced for a model given by conditional pmfs in (11) and parameterized as in (12) with only parameters α and η. Let $\theta \equiv (\alpha, \eta)'$, then the log likelihood formed from the joint pmf of $\boldsymbol{Z}$ may be written, for $\boldsymbol{z} \in \Omega$, as

$$L(\theta) = Q(\boldsymbol{z}|\theta) - \log\{k(\theta)\}, \tag{15}$$

where

$$Q(\boldsymbol{z}|\theta) = \sum_{1 \le i \le n} [\alpha z(s_i) - \log(z(s_i)!)] + \eta \sum_{1 \le i < j \le n} z(s_i) z(s_j),$$

and

$$k(\theta) = \sum_{\boldsymbol{t} \in \Omega} \exp\{Q(\boldsymbol{t}|\theta)\}. \tag{16}$$

4.1 Exact MLEs

MLEs can be obtained by solving the likelihood equations resulting from (15). It is easy to show that, for $i = 1, 2$ and $j = 1, 2$,

$$\frac{\partial L(\theta)}{\partial \theta_i} = \frac{\partial Q(\boldsymbol{z}|\theta)}{\partial \theta_i} - E\left\{\frac{\partial Q(\boldsymbol{z}|\theta)}{\partial \theta_i}\right\}, \tag{17}$$

and

(18) $$\frac{\partial^2 L(\theta)}{\partial\theta_i\theta_j} = E\left\{\frac{\partial Q(z|\theta)}{\partial\theta_i}\right\}E\left\{\frac{\partial Q(z|\theta)}{\partial\theta_j}\right\} - E\left\{\frac{\partial Q(z|\theta)}{\partial\theta_i}\frac{\partial Q(z|\theta)}{\partial\theta_j}\right\}.$$

In these expression, for a real-valued function $h(\cdot)$,

$$E\{h(z;\theta)\} = \sum_{t\in\Omega}[h(t;\theta)\exp\{Q(t;\theta)\}/k(\theta)].$$

Since Ω is finite, derivatives in (17) and (18) may be evaluated exactly, and MLEs obtained from standard iterative techniques such as Newton-Raphson. If the sample size n is large, this strategy may be prohibitively time consuming, and more efficient methods will be required for practical applications.

4.2 Monte Carlo MLEs

An alternative approach toward maximum likelihood estimation is to form a Monte Carlo approximation to (16) using an importance density. That is,

$$\begin{aligned} k(\theta) &= \sum_{t\in\Omega}\exp\{Q(t|\theta)\} \\ &= \sum_{t\in\Omega}\frac{\exp\{Q(t|\theta)\}}{m(t)}m(t), \end{aligned}$$

which can be approximated by

(19) $$k_M(\theta) \equiv \frac{1}{M}\sum_{r=1}^{M}\frac{\exp\{Q(y_r|\theta)\}}{m(y_r)},$$

where $y_1, \cdots, y_M$ is a sample from $m(y)$, which is called an importance distribution. A Monte Carlo approximation to log likelihood (15) is then defined as

(20) $$L_M(\theta) \equiv Q(z|\theta) - \log\{k_M(\theta)\}.$$

Monte Carlo MLEs can then be obtained by maximizing (20) with respect to θ. For various approaches to selection of an importance distribution, see Penttinen (1984), Geyer and Thompson (1992), and Lee and Kaiser (1997).

It may be shown that,

(21)
$$\frac{\partial L_M(\theta)}{\partial \theta_i} = \frac{\partial Q(z|\theta)}{\partial \theta_i} - E_M\left\{\frac{\partial Q(z|\theta)}{\partial \theta_i}\right\},$$

and

(22)
$$\frac{\partial^2 L_M(\theta)}{\partial \theta_i \theta_j} = E_M\left\{\frac{\partial Q(z|\theta)}{\partial \theta_i}\right\}E_M\left\{\frac{\partial Q(z|\theta)}{\partial \theta_j}\right\} - E_M\left\{\frac{\partial Q(z|\theta)}{\partial \theta_i}\frac{\partial Q(z|\theta)}{\partial \theta_j}\right\},$$

where, for a real-valued function $h(\cdot)$,

$$E_M\{h(z;\theta)\} = \frac{1}{Mk_M(\theta)}\sum_{r=1}^{M}[h(y_r;\theta)\exp\{Q(y_r;\theta)\}/m(y_r)].$$

Thus, the maximizer of (20) can be obtained using a standard Newton-Raphson method, in which exact evaluations of derivatives are replaced with Monte Carlo approximations. For details on the use of Monte Carlo maximum likelihood for conditionally-specified models, see Lee and Kaiser (1997).

5. Example

In this section, a small simulated example is used to compare exact and Monte Carlo maximum likelihood estimation. Data were simulated along a regular 2x3 lattice, so that $s_i \equiv (u_i, v_i)$, $i = 1, \cdots, 6$, and $Z = \{Z(u_i, v_i): i = 1, \cdots, 6\}$. Neighborhoods were defined using a 'nearest neighbor' structure as

$$N_1 = \{s_2, s_4\},\ N_2 = \{s_1, s_3, s_5\}\ N_3 = \{s_2, s_6\},$$
$$N_4 = \{s_1, s_5\},\ N_5 = \{s_2, s_4, s_6\},\ N_6 = \{s_3, s_5\}.$$

Data values were generated using 4,000 iterations of a Gibbs sampling algorithm with conditional pmfs (11) having $A_i(\cdot)$ parametrized as in (15) with $\alpha_i = \alpha = 0.6; i = 1, \cdots, 6,$ and $\eta_{ij} = \eta = 0.03; i, j = 1, \cdots, 6.$ Truncation values were set as $R_L = 1$ and $R_U = 10$. Realized values from the simulation were $z(s_1) = 2$, $z(s_2) = 4$, $z(s_3) = 3$, $z(s_4) = 3$, $z(s_5) = 2$, and $z(s_6) = 2$. Exact MLEs were computed as described in Section 4.1 using a standard Newton-Raphson algorithm. With $R_L = 1, R_U = 10$, resultant estimates were $\hat{\alpha} = 0.861$ and $\hat{\eta} = 0.00457$. Monte Carlo MLEs were computed as described in section 4.2, resulting in $\hat{\alpha} = 0.966$ and $\hat{\eta} = 0.00227$. We can see somewhat significant difference between exact MLEs and Monte Carlo MLEs because of the Monte Carlo errors in

(20). These errors can be reduced by increasing the Monte Carlo sample size, M.

In this example, we illustrated that doubly-Winsorized Poisson auto-model introduced in section 3 can be estimated by standard iterative estimation methods, and, if needed, by more efficient method which approximates MLEs.

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정정 안내

$\alpha_{12}(t|s;Z)$

1 2

$\alpha_{23}(t|s;Z)$

$\alpha_{13}(t;Z)$ $\alpha_{24}(t|s;Z)$

3 4

Figure 1. An illness-death model with two transient and two absorbing states

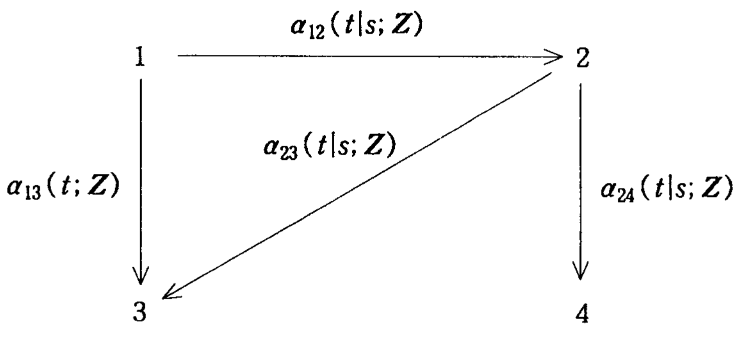

Figure 1. An illness-death model with two transient and two absorbing states