The Korean Communications
in Statistics Vol. 5, No. 2, 1998
pp. 417-429

Power Analysis of Distributions between Nonparametric Tests

Chan Keun Park[1)]

Abstract

This paper compares powers of the two nonparametric tests under a variety of population distributions through a simulation study. Both tests require that the two underlying populations have the same variance, but this assumption is relaxed in some of the comparisons.

1. Introduction

Suppose a researcher wishes to test the following set of hypotheses,

$$H_0 : \ \mathrm{M}x = \ \mathrm{M}y$$

$$H_a : \ \mathrm{M}x \neq \ \mathrm{M}y$$

where $\mathrm{M}x$ and $\mathrm{M}y$ represent the medians of their respective populations. It will be assumed the underlying distributions are unknown, but of the same type. The two most common nonparametric tests that test this set of hypotheses are the Mann-Whitney test and the median test (Conover, 1980).

The Mann-Whitney test is sometimes referred to as the Mann-Whitney-Wilcoxon test(Mann and Whitney, 1947) or the Wilcoxon test(Wilcoxon, 1945). The Mann-Whitney test assumes that the population distributions are the same, except possibly in location. To conduct the test, two random samples are taken from their respective populations, and these samples are denoted by $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$. There needs to be mutual independence between samples, and the measurement scale needs to be at least ordinal. The observations are tied, the average rank is assigned to each of these observations.

The test statistic is the sum of all the ranks assigned to the observations in the first sample. If the value of the test statistic is sufficiently small or sufficiently large, the null hypothesis is rejected. The lower critical values come from Conover (1980, pp448-452). These are denoted by $w_{\alpha/2}$. The upper critical values, $w_{1-\alpha/2}$, may be found by using the

1) Graduate School, Dept. of Statistics, North Dakota State Univ. Fargo ND 58102, USA.

relationship $w_{1-\alpha/2} = nm - w_{\alpha/2}$.

The median test can be one of the easiest and most useful procedures for testing the null hypothesis that independent random samples come from populations with equal medians (Mood, 1950). The median test assumes that the two populations have the same shape and that the observations within each sample are independent. The measurement scale needs to be at least ordinal. The median test also assumes that if the two populations have the same median, each population has the same probability p of an observation exceeding the grand median. Two independent random samples are taken, and these are denoted by $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$. The sample median of the combined samples is calculated. Let A denote the number of the observations in the first sample that are above the combined sample median, B denote the number of the observations in the second sample that are above the combined sample median, C denote the number of observations in the first sample that are less than or equal to the combined sample median, and D denote the number of observations in the second sample that are less than or equal to the combined sample median.

It has been shown that the sampling distribution of A and B follows a hypergeometric distribution. Because the critical values of this sampling distribution are generally time-consuming to find, a normal approximation is used. The following test statistic was used in this paper(Daniel, 1990):

$$T = \frac{(A/n_1) - (B/n_2)}{\sqrt{(\hat{p}(1-\hat{p})(1/n_1 + 1/n_2))}}$$

$$\text{where}\ ,\ \hat{p} = (A+B)/N$$

$$n_1 = A + C,$$

$$n_2 = B + D,$$

$$\text{and}\quad N = n_1 + n_2.$$

The critical values can be found in a standard normal table.

Since the Mann-Whitney test and the median test can be used to test for a difference between medians when the underlying distributions are unknown, we would like to know which test is the better one to use.

2. Historical Background of Literature

Some comparisons between the Mann-Whitney tests and the median test and between these two tests and other well-known tests have been made. The Mann-Whitney tests have been compared to the t test under specific conditions. These comparisons were made on the basis of each test's power. Gibbons and Chakraborti(1990) stated that the Student's t tests was more powerful than the Mann-Whitney test for any sample size if the population could be assumed normal with equal samples of size 10 and then for different sample sizes between 4 and 16 under normal distributions. Blair and Higgins(1980) checked the powers of the Mann-Whitney and the t test for samples of size 18 and 18, 9 and 27, 54 and 54, and 27 and 81, drawn from the mixed normal distribution, 95 % $N(0,1)$ and 5% $N(33,100)$. Rasmussen(1985) compared the powers of the Mann-Whitney test, the t test, and the t test that corrected outliers using the Grubbs-type outlier detection statistic. Zimmerman(1987) found the Mann-Whitney test was more powerful than t test when unequal samples are taken from normal distributions and the smaller sample is taken from the population with the smaller variance. The Kruskal-Wallis test is an extension of the Mann-Whitney test to k populations(Kruskal and Wallis, 1952). The median test may also be extended to k populations(Conover, 1980). Conover(1980) said that the Kruskal-Wallis test was usually more powerful than the median test because the Kruskal-Wallis test statistic was a function of the ranks of the observations in the combined sample, as was true with the Mann-Whitney test, while the median test statistic depended only on the knowledge of whether the observations were below or above the ground median. Conover, Wehmenen and Ramsey(1978) said that the Mann-Whitney test may be more powerful than the median test in the case of the double exponential distribution for small samples. Conover(1980) said that the median test may be applied in situations where the Mann-Whitney test was not valid. He said that the t test was the most powerful test if both populations had a normal distribution, but the t test is not always more powerful than any other test if the population does not have a normal distribution. We often use the notion of the asymptotic relative efficiency(ARE) when we discuss the power of statistical hypotheses tests. The limit of the ratio, Nb/Na, is called the ARE of the test A relative to test B, as Na approaches infinity, where Na and Nb are the sample sizes required for test A and Test B, respectively, to have the same power under the same level of significance. If the ARE of test A relative to test B is less than 1, test B is more efficient than test A. Hodges and Lehmann(1956) calculated the asymptotic relative efficiency of the Mann-Whitney test relative to the t test and found that it is never less than 0.864 if population distributions were symmetric and continuous. If the population is normal, they found the ARE is 0.950. They said that the ARE of the Mann-Whitney test relative to the t test was approximately 1.266 when population distributions were Gamma(3, 1). The ARE of the Mann-Whitney test relative to the median test is 1.5 for normal populations and 3.0 for uniform populations(Conover, Wehmanen, and Ramsey, 1978). Andrews(1954) calculated the ARE of the Mann-Whitney test relative to the t test and found it to be 0.955 for normal populations. He found the ARE of the median test relative to the Kruskal-Wallis test is 2/3 when populations are normal, and the ARE of the median test to the Kruskal-Wallis test is 1/3 when populations are uniform. Therefore, many statisticians consider the Mann-Whitney test as the best nonparametric two-sample test for location. Gibbons(1971) stated that the Mann-Whitney test generally has greater power than the median test as a test for location.

3. Design of Study

In comparing the powers of the Mann-Whitney test and the median test based on *independent random* samples from two populations, we consider the following types of distributions: Normal, Uniform, Exponential, and Cauchy.

To investigate the effect of location shifts in mixture distributions, we use the mixture of $N(0,1)$ and $N(3,36)$ and the mixture of $N(0,1)$ and $N(33,100)$. Each test is performed 5,000 times under the same conditions. The powers of the tests are estimated, based on the number of times the null hypothesis is rejected divided by 5,000. Equal sample sizes of 10 and 20 are used.

The relative powers of the two tests are first examined under the equal variance assumption and then when the equal variance assumption is relaxed. The first type of distribution to be considered here is the normal with the following cases considered:

$$N(0,1) \textit{ versus } N(\theta_1, 1).$$

The values θ_1, θ_2, θ_3, and θ_4 are found by computer simulation so that the power of the Mann-Whitney test in each of these cases is equal to approximately 0.95 for equal sample sizes of 20. The values θ_1, θ_2, θ_3, and θ_4 are then divided by 5, and a power comparison of the Mann-Whitney test and the median test are made for each of the following distributions:

a. $N(0,1)$ *versus* $N(\theta_1/5, 1)$,

b. $N(0,1)$ *versus* $N(2\theta_1/5, 1)$,

c. $N(0,1)$ *versus* $N(3\theta_1/5, 1)$,

d. $N(0,1)$ *versus* $N(4\theta_1/5, 1)$, and

e. $N(0,1)$ *versus* $N(\theta_1, 1)$.

We examine the relative power of the two tests in the same way for equal sample sizes of 10.

The powers of the two tests are also examined when the populations are a mixture of normal distributions. The mixed normal populations considere were 75% $N(0,1)$ and 25% $N(3,36)$, and 90% $N(0,1)$ and 10% $N(33,100)$. The following cases are considered:

$$\text{mixture of } N(0,1) \text{ and } N(3,36) \textit{ versus}$$

$$\text{mixture of } N(\theta_1, 1) \text{ and } N(3+\theta_1, 36).$$

The values θ_1 and θ_2 are found so that the power of the Mann-Whitney test in each of these cases is equal to approximately 0.95 for equal sample size of 20. The values θ_1 and θ_2 are then divided by 5, and power comparisons of the Mann-Whitney test and the median test are made for each of the following distributions:

a. mixture of $N(0, 1)$ and $N(3, 36)$ *versus*

mixture of $N(\theta_1/5, 1)$ and $N(3+\theta_1/5, 36)$,

b. mixture of $N(0, 1)$ and $N(3, 36)$ *versus*

mixture of $N(2\theta_1/5, 1)$ and $N(3+2\theta_1/5, 36)$,

c. mixture of $N(0, 1)$ and $N(3, 36)$ *versus*

mixture of $N(3\theta_1/5, 1)$ and $N(3+3\theta_1/5, 36)$,

d. mixture of $N(0, 1)$ and $N(3, 36)$ *versus*

mixture of $N(4\theta_1/5, 1)$ and $N(3+4\theta_1/5, 36)$, and

e. mixture of $N(0, 1)$ and $N(3, 36)$ *versus*

mixture of $N(\theta_1, 1)$ and $N(3+\theta_1, 36)$.

The next type of distribution to be considered under the equal variance assumption and then when the equal variance assumption is relaxed is the uniform distribution with the following cases considered:

$$U(0, 1) \textit{ versus } U(\theta_1, 1).$$

The values θ_1, θ_2, θ_3, and θ_4 are again chosen so that the power of the Mann-Whitney test in each of these cases is equal to approximately 0.95 for equal sample sizes of 20. The values θ_1, θ_2, θ_3, and θ_4 are divided by five, and power comparisons of the Mann-Whitney test and the median test are made for each of the following:

a. $U(0, 1)$ *versus* $U(\theta_1/5, 1)$,

b. $U(0, 1)$ *versus* $U(2\theta_1/5, 1)$,

c. $U(0, 1)$ *versus* $U(3\theta_1/5, 1)$,

d. $U(0, 1)$ *versus* $U(4\theta_1/5, 1)$, and

e. $U(0, 1)$ *versus* $U(\theta_1, 1)$.

We examine the relative power of the two tests in the same way for equal sample sizes of 10.

The exponential distribution is next considered under the equal variance assumption and then when the equal variance assumption is relaxed. The following cases are considered:

$$\text{Exp}(0, 1) \textit{ versus } \text{Exp}(\theta_1, 1).$$

The values θ_1 and θ_2 are found so that the power of the Mann-Whitney test in each of these cases is equal to approximately 0.95 for equal sample sizes of 20. The values θ_1 and θ_2 are then divided by five, and power comparisons of the two tests are made in each of the following cases:

a. Exp $(0, 1)$ *versus* Exp $(\theta_1/5, 1)$,

b. Exp $(0, 1)$ *versus* Exp $(2\theta_1/5, 1)$,

c. Exp $(0, 1)$ *versus* Exp $(3\theta_1/5, 1)$,

d. Exp $(0, 1)$ *versus* Exp $(4\theta_1/5, 1)$, and

e. Exp $(0, 1)$ *versus* Exp $(\theta_1, 1)$.

We also examine the relative power of the Mann-Whitney test and the median test in the same way for equal sample sizes of 10.

The relative powers of the two tests are also examined for the Cauchy distribution. The value, θ_1, was found so that the Mann-Whitney test had a power of 0.95 when testing Cauchy(0) versus Cauchy(θ_1) with equal sample sizes. The value θ_1 is then divided by five, and power comparisons of the Mann-Whitney test and the median test were made in each of the following cases:

a. Cauchy (0) *versus* Cauchy $(\theta_1/5)$,

b. Cauchy (0) *versus* Cauchy $(2\theta_1/5)$,

c. Cauchy (0) *versus* Cauchy $(3\theta_1/5)$,

d. Cauchy (0) *versus* Cauchy $(4\theta_1/5)$, and

e. Cauchy (0) *versus* Cauchy (θ_1).

We use the SAS package program to generate random samples from each distribution. The subroutines RANNOR, RANUNI, RANEXP, and RANCAU are used to generate random numbers from normal, uniform, exponential, and Cauchy distributions, respectively. To simulate the samples from each population 5,000 times and to perform the Mann-Whitney test and the median test each time, the SAS/Macro was used. The power of each test is simulated by the proportion of times each test rejected the null hypothesis out of 5,000 times.

4. Simulation Results

The goals of this paper is to compare the powers of the Mann-Whitney test and the median test for a variety of circumstances. The powers are compared under both the equal variance assumption and when the equal variance assumption is relaxed for the following types of distributions: Normal, Uniform, Exponential, and Cauchy. Mixed normal distributions are considered under the equal variance assumption. Equal sample sizes of 10 and 20 are used. The alpha value is always 0.05. The power ratio is calculated in each case where the power ratio is defined as the power of the Mann-Whitney test divided by the power of the median test. A power ratio less than 1 indicates the median test is better.

Normal distribution case

According to the results of Table 1 and 2, the Mann-Whitney test is found to be more powerful than the median test for both equal sample sizes of 10 and 20 when the equal variance assumption is true. The power ratio is between 1.2047 and 1.9167 for equal sample sizes of 10 and between 1.2211 and 1.8908 for equal sample sizes of 20.

Table 1. Estimated Powers of Mann-Whitney Test and the Median Test for N(0,1) With Equal Samples of Size 10 and $\alpha=0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0436	0.0246	0.0190	1.7724
0.3586	0.1012	0.0528	0.0484	1.9167
0.7172	0.2886	0.1600	0.1286	1.8038
1.0758	0.5674	0.3492	0.2182	1.6249
1.4344	0.8122	0.5862	0.2260	1.3855
1.7930	0.9500	0.7886	0.1614	1.2047

* These powers are estimated based on 5,000 simulations.

** Power Ratio is power of Mann-Whitney test / power of Median test.

Table 2. Estimated Powers of Mann-Whitney Test and the Median Test for N(0,1) With Equal Samples of Size 20 and $\alpha=0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0484	0.0246	0.0238	1.9675
0.2420	0.1108	0.0586	0.0522	1.8908
0.4840	0.3006	0.1614	0.1392	1.8625
0.7260	0.5780	0.3422	0.2358	1.6891
0.9680	0.8248	0.5754	0.2494	1.4334
1.2100	0.9500	0.7780	0.1720	1.2211

* These powers are estimated based on 5,000 simulations.

** Power Ratio is power of Mann-Whitney test / power of Median test.

Mixed normal distribution case

We compare the powers of the two tests for equal sample sizes of 10 and 20 from the mixture population 75% $N(0,1)$ and 25% $N(3,36)$. The power ratio were all grater than 1 for equal samples of size 20, indicating the Mann-Whitney test is more powerful than the median test. When samples are of 10(not given here), the power close to 1, indicating both tests performed about the same. those results are given in Table 3 and Table 4.

Table 3. Estimated Powers of Mann-Whitney Test and the Median Test of Mixture of 75% of N(0,1) and 25% of N(3,36) With Equal Samples of Size 20 and $\alpha=0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0382	0.0204	0.0178	1.8725
0.4860	0.1488	0.0972	0.0516	1.5309
0.9720	0.4572	0.3468	0.1104	1.3183
1.4580	0.7462	0.6772	0.0690	1.1019
1.9440	0.8926	0.8852	0.0110	1.0084
2.4300	0.9502	0.9592	-0.0090	0.9906

* These powers are estimated based on 5,000 simulations.

** Power Ratio is power of Mann-Whitney test / power of Median test.

Table 4. Estimated Powers of Mann-Whitney Test and the Median Test of Mixture of 90% of N(0,1) and 10% of N(33,100) With Equal Samples of Size 20 and $\alpha=0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0244	0.0158	0.0086	1.5443
0.3004	0.0796	0.0502	0.0294	1.5857
0.6008	0.2788	0.1716	0.1072	1.6247
0.9012	0.5642	0.3904	0.1738	1.4452
1.2016	0.8254	0.6424	0.1830	1.2847
1.5020	0.9500	0.8488	0.1012	1.1192

* These powers are estimated based on 5,000 simulations.

** Power Ratio is power of Mann-Whitney test / power of Median test.

Uniform distribution case

We examine the powers of the two tests when the equal variance assumption is true and then when variance assumption is violated. The power ratios are between 1.4230 and 2.8690 when sample sizes are found in 10 and when variances are equal. The results for both sample sizes are found in Table 5 and 6. These results indicate that the Mann-Whitney test is more powerful in this case.

Table 5. Estimated Powers of Mann-Whitney Test and the Median Test for U(0,1) With Equal Samples of Size 10 and $\alpha = 0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0430	0.0222	0.0208	1.9370
0.1092	0.1034	0.0410	0.0624	2.5220
0.2184	0.2892	0.1008	0.1884	2.8690
0.3275	0.5808	0.2300	0.3508	2.5252
0.4376	0.8280	0.4366	0.3914	1.8965
0.5459	0.9500	0.6676	0.2824	1.4230

* These powers are estimated based on 5,000 simulations.
** Power Ratio is power of Mann-Whitney test / power of Median test.

Table 6. Estimated Powers of Mann-Whitney Test and the Median Test for U(0,1) With Equal Samples of Size 20 and $\alpha = 0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0568	0.0272	0.0296	2.0882
0.0744	0.1208	0.0434	0.0774	2.7834
0.1488	0.3264	0.1040	0.2224	3.1385
0.2233	0.6026	0.2182	0.3844	2.7617
0.2977	0.8276	0.3962	0.4314	2.0888
0.3721	0.9500	0.5960	0.3540	1.5940

* These powers are estimated based on 5,000 simulations.
** Power Ratio is power of Mann-Whitney test / power of Median test.

Exponential distribution case

The Mann-Whitney test is found to be more powerful than the median test when the equal variance assumption is true. The maximum power ratio is 2.7759 and the minimum power ratio is 1.2673 when samples are of size 20, and the power ratios are between 1.1038 and 2.5731 when samples are of size 10. Table 7 and 8 show the results.

Table 7. Estimated Powers of Mann-Whitney Test and the Median Test for Exp(1) With Equal Samples of Size 10 and $\alpha=0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0430	0.0222	0.0208	1.9370
0.3440	0.1760	0.0684	0.1076	2.5731
0.6880	0.4806	0.2486	0.2320	1.9332
1.0320	0.7358	0.5044	0.2314	1.4588
1.3760	0.8792	0.7288	0.1504	1.2064
1.7200	0.9510	0.8616	0.0894	1.1038

* These powers are estimated based on 5,000 simulations.
** Power Ratio is power of Mann-Whitney test / power of Median test.

Table 8. Estimated Powers of Mann-Whitney Test and the Median Test for Exp(1) With Equal Samples of Size 20 and $\alpha=0.05$.

	Estimated Powers *			
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0568	0.0272	0.0296	1.9190
0.1918	0.1610	0.0580	0.1030	2.7759
0.3836	0.4130	0.1590	0.2540	2.5975
0.5854	0.6774	0.3482	0.3292	1.9454
0.7672	0.8656	0.5582	0.3074	1.5572
0.9590	0.9500	0.7496	0.2004	1.2673

* These powers are estimated based on 5,000 simulations.
** Power Ratio is power of Mann-Whitney test / power of Median test.

Cauchy distribution case

When the location shifts are 0.00 and 0.605 for equal sample sizes of 20 and the location shifts are 0.00 and 1.464 for equal sample sizes of 10, the Mann-Whitney test are found to be better than the median test. However, for most of the location shifts considered in this case, the power of the median test is slightly higher. The results are given in Table 9 and 10.

Table 9. Estimated Powers of Mann-Whitney Test and the Median Test for Cauchy(0,1) With Equal Samples of Size 20 and $\alpha=0.05$.

		Estimated Powers *		
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0524	0.0260	0.0264	2.0154
0.6050	0.1802	0.1438	0.0364	1.2531
1.2100	0.4724	0.4756	-0.0032	0.9933
1.8150	0.7476	0.7788	-0.0312	0.9599
2.4200	0.8892	0.9232	-0.0340	0.9632
3.0250	0.9500	0.9744	-0.0244	0.9750

* These powers are estimated based on 5,000 simulations.
** Power Ratio is power of Mann-Whitney test / power of Median test.

Table 10. Estimated Powers of Mann-Whitney Test and the Median Test for Cauchy(0,1) With Equal Samples of Size 10 and $\alpha=0.05$.

		Estimated Powers *		
Location Difference Between Pop 1 and Pop 2	Mann-W	Median	Difference in Powers [(2)-(3)]	Power Ratio** [(2)-(3)]
(1)	(2)	(3)	(4)	(5)
0.0000	0.0454	0.0234	0.0220	1.9402
1.4640	0.3206	0.3000	0.0206	1.0687
2.9280	0.6714	0.6976	-0.0262	0.9624
4.3920	0.8386	0.8786	-0.0400	0.9545
5.8560	0.9150	0.9474	-0.0324	0.9658
7.3200	0.9500	0.9744	-0.0244	0.9750

* These powers are estimated based on 5,000 simulations.
** Power Ratio is power of Mann-Whitney test / power of Median test.

6. Conclusions and Future Research

The powers are calculated by counting the number of times a test resulted in a rejection divided by 5,000. The test is simulated 5,000 times for each situation. The test is simulated 5,000 times for each situation. The powers of the Mann-Whitney test and the median test are simulated for equal samples of 10 and 20 taken from four different types of populations. Cases are considered for when the equal variance assumption is true and for when it is relaxed.

When the equal variance assumption is true, the results of the simulation study indicated that the Mann-Whitney test is generally more powerful than the median test when the underlying distributions are normal, uniform, and exponential. This is not an unexpected result, based on past research. When the underlying distribution is Cauchy, the median test is almost always more powerful than the Mann-Whitney test.

The Mann-Whitney test and the median test generally do about the same for both samples of size 10 and 20 for the mixture population 75% $N(0,1)$ and 25% $N(3,36)$. The Mann-Whitney test has larger powers for samples of size 20.

The median test does better in comparison to the Mann-Whitney test when samples sizes are 10 instead of 20. That is, the power ratios which are calculated when sample sizes are 10 are smaller than the power ratios which are calculated when sample sizes are 20.

When the equal variance assumption is relaxed, the median test is more conservative. The actual α value for the median test always stayes below 0.05 for all cases considered. This is not true for the Mann-Whitney test. In one case, it is as high as 0.074.

Future research recommended is to compare the powers of the Mann-Whitney test and the median test for more mixture populations. The median test does well in comparison to the Mann-Whitney test for both mixture populations included in the study.

6. References

[1] Andrews, F.C. (1954). Asymptotic Behavior of Some Rank Tests for Analysis of Variance. *Annals of Mathematical Statistics*, 25, 724-736.

[2] Blair, R.C., and Higgins, J.J. (1980). The power of t and Wilcoxon statistics. *Evaluation Review*, Vol. 4, No. 5, 645-656.

[3] Conover, W.J. (1980). *Pratical Nonparametric Statistics.* 2nd edition. John Wiley & Sons Publishing Company, New York.

[4] Conover, W.J., Wegmanen, O., and Ramsey, R.L. (1978). A Note on the Small Sample Power Functions for Nonparametric Tests of Location in the Double Exponential Family. *Journal of the American Statistical Association,* 73, 188-190 (5.1).

[5] Daniel, W.W. (1990). *Applied Nonparametric Statistics.* 2nd edition. PWS-Kent Publishing Company, Boston.

[6] Gibbons, J.D.(1971). *Nonpapametric Statistical Inference.* McGraw-Hill Book Company, New York.

[7] Gibbons, J.D., and Chakraborti, S. (1990). Comparisons of the Mann-Whitney, Student's, and Alternate t Tests for Means of Normal Distribution. *Journal of Experimental Education,* 258-267.

[8] Hodges, J.L., and Legmann, E.L. (1956). The Efficiency of Some Nonparametric Competiters of the t Test. *Annals of Mathematical Statistics,* 27, 324-335.

[9] Kruskal, W.H., and Wallis, W.A. (1952). Use of Ranks in One-Criterion Variance Analysis. *Journal of American Statistical Association,* 47, 583-621.

[10] Mann, H.B., and Whitney, D.R. (1947). On a Test of Whether One of Two Random Variables is Stochastically Larger Than the Other. *Annals of Mathematical Statistics*, 18, 50-60.

[11] Mood, A.M. (1950). *Introduction to the Theory of Statistics*. McGraw-Hill, New York.

[12] Rasmussen, J.L. (1985). The Power of student's t and Wilcoxon w Statistics. *Evaluation Review*, Vol. 19, 505-510.

[13] Wilcoxon, F. (1945). Individual Comparisons by Ranking Methods. *Biometrics*, 1, 80-83.

[14] Zimmerman, D.W. (1987). Comparative Power of Student t Test and Mann-Whitney U Test for Unequal Sample Sizes and Variances. *Journal of Experimental Education*, 55. 171-174.