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DISTRIBUTION-FREE TWO-SAMPLE TEST ON RANKED-SET SAMPLES

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Abstract.

In this paper, we propose the two-sample test statistic using Wilcoxon signed rank test on ranked-set sampling(RSS) and obtain the asymptotic relative efficiencies(ARE) of the proposed test statistic with respect to Mann-Whitney-Wilcoxon statistic on simple random sampling(SRS), the Mann-Whitney-Wilcoxon statistic on RSS, sign statistic on RSS and Wilcoxon signed rank test on SRS. From the simulation works, we compare the powers of the proposed test staistic, Mann-Whitney-Wilcoxon statistic on RSS, the usual two-sample t statistic, sign statistic on RSS, where the underlying distributions are uniform, normal, double exponential, logistic and Cauchy distributions.

1. Introduction

Many sample designs use simple random sampling(SRS) but ranked-set sampling (RSS) can take the place of SRS under certain conditions. The concept of RSS solves the problems in many other sampling methods. In the situation that measurements of the sample data are difficult or costly expensive but ranking is easy, we can use the RSS.

RSS is a sampling method designed by McIntyre (1952). Many authors have studied the properties of RSS. Dell and Clutter (1972), Stokes and Sager (1988) studied the properties of the empirical distribution function based on RSS. Bohn and Wolfe (1992, 1994) considered the Mann-Whitney-Wilcoxon statistic on RSS using perfect and imperfect rankings. Kvam and Samaniego (1993, 1994) considered the setting in which RSS need not be balanced. Hettmansperger (1995) has considered one sample RSS sign test statistic and its asymptotic properties. Two-sample problem using sign test on RSS is considered in Kim and Kim (1998). Bohn (1996) has reviewed RSS methodology.

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Now we show the process of RSS. From the first simple random sample, we judge the smallest item among the k items. From the second simple random sample, we find the second smallest item among the k items. And from the last simple random sample, the largest item can be found among the k items. This process independently continues until a complete cycle is finished from each of the n classes. The entire cycle is repeated nk^2 times from nk preranking sample items.

In section 2, we propose the test statistic and deal with the asymptotic properties of the proposed test statistic. Section 3 gives asymptotic relative efficiencies of the proposed test statistic with respect to the other competitors. Simulation design and results under several underlying distributions are given in section 4.

2. Test Statistic

Let $X_{(1)1}, \cdots, X_{(1)m}, \cdots, X_{(k)1}, \cdots, X_{(k)m}$ be a ranked-set sample of size mk from a continuous distribution with pdf $f(x)$ and cdf $F(x)$ satisfying $F(0) = \frac{1}{2}$. Let $Y_{(1)1}, \cdots, X_{(1)n}, \cdots, Y_{(q)1}, \cdots, X_{(q)n}$ be a ranked-set sample of size nq from a continuous distribution with pdf $g(y)$ and cdf $G(y)$, where $G(y) = F(y-\theta)$. Throughout this thesis, we deal with the special case of $m=n$ and $k=q$. We denote the pdf of $X_{(j)i}$, $i = 1, \cdots, n$, $j = 1, \cdots, k$ by $f_{(j)}(t)$. This pdf is the j-th order statistic from a distribution with $F(x)$ and is given by

$$f_{(j)}(t) = \frac{k!}{(j-1)!(k-j)!}[F(t)]^{j-1}[1-F(t)]^{k-j}f(t).$$

In this paper, we consider the testing problem for testing $H_0 : \theta = 0$ against $H_1 : \theta > 0$. First, we think Wilcoxon signed rank test statistic based on each RSS. Under alternatives $H_1 : \theta > 0$, we can expect that Y ranked-set samples have larger Wilcoxon signed rank test statistic than X ranked-set samples, so we propose the following test statistic, which is given by

$$W_{RSS}^{+} = \sum_{j=1}^{k}(W_{2(j)} - W_{1(j)}).$$

where

$$W_{1(j)} = \sum_{i=1}^{n} iT_{1(j)i},$$

$$\begin{aligned} T_{1(j)i} &= 1 \text{ if } X_{(j)i} \text{ corresponds to a positive measurement.} \\ &= 0, \text{ otherwise.} \end{aligned}$$

$$W_{2(j)} = \sum_{i=1}^{n} iT_{2(j)i},$$

$T_{2(j)i} = 1$ if $Y(j)i$ corresponds to a positive measurement.

$= 0$, otherwise.

Then we reject H_0 in favor of H_1 for large values of W^{+}_{RSS}. Since $W_{1(j)}$ and $W_{2(j)}$ are distribution-free, the proposed statistic W^{+}_{RSS} is also distribution-free.

To obtain the asymptotic properties of the proposed statistic, for convenience, let

$$V_{1(j)} = \frac{1}{\binom{n}{2}} (\text{ number of } (i, i') \text{ such that } 1 \le i \le i' \le n,\ X_{(j)i} + X_{(j)i'} \ge 0)$$

$$= \frac{1}{\binom{n}{2}} W_{1(j)},$$

so we consider $V_{1(j)}$ instead of $W_{1(j)}$. $V_{1(j)}$ can be written as

$$V_{1(j)} = \frac{1}{\binom{n}{2}} \sum_{i \le i'}^{n} \Psi(X_{(j)i} + X_{(j)i'})$$

$$= \frac{1}{\binom{n}{2}} \sum_{i=i'}^{n} \Psi(X_{(j)i}) + \frac{1}{\binom{n}{2}} \sum_{i<i'}^{n} \Psi(X_{(j)i} + X_{(j)i'}),$$

where $\Psi(x) = 1$ if $x > 0$ and 0 otherwise.

So as to obtain the expectation of the proposed statistic W^{+}_{RSS}, we calculate the expectations of $V_{1(j)}$ and $V_{2(j)}$ as follows.

$$E(V_{1(j)}) = \frac{2}{n-1}(1 - F_{(j)}(0)) + \int_{-\infty}^{\infty} [1 - F_{(j)}(-x)] dF_{(j)}(x)$$

Since $V_{2(j)}$ is given by,

$$V_{2(j)} = \frac{1}{\binom{n}{2}} \sum_{i=i'}^{n} \Psi(Y_{(j)i}) + \frac{1}{\binom{n}{2}} \sum_{i<i'}^{n} \Psi(Y_{(j)i} + Y_{(j)i'}),$$

we can obtain the expectation of $V_{2(j)}$.

$$E(V_{2(j)}) = \frac{2}{n-1}(1 - F_{(j)}(0)) + \int_{-\infty}^{\infty} [1 - F_{(j)}(-y - 2\theta)] dF_{(j)}(y),$$

where the last equality holds by the transformation of $t = y - \theta$.

Then we have,

$$E_\theta(W^{+}_{RSS}) = n \sum_{j=1}^{k} [F_{(j)}(0) - G_{(j)}(0)]$$

$$+ \binom{n}{2} \sum_{j=1}^{k} \left\{ \int_{-\infty}^{\infty} [1 - G_{(j)}(-y)] dG_{(j)}(y) - \int_{-\infty}^{\infty} [1 - F_{(j)}(-x)] dF_{(j)}(x) \right\}.$$

Let $f(t) = \frac{1}{k}\sum_{j=1}^{k} f_{(j)}(t)$ and $F(t) = \frac{1}{k}\sum_{j=1}^{k} F_{(j)}(t)$, $g(t) = \frac{1}{k}\sum_{j=1}^{k} g_{(j)}(t)$ and $G(t) = \frac{1}{k}\sum_{j=1}^{k} G_{(j)}(t)$.

Then,

$$E_\theta(W_{RSS}^{+}) = nk[F_{(j)}(0) - G_{(j)}(0)] + \binom{n}{2}\sum_{j=1}^{k}\left\{\int_{-\infty}^{\infty}[1-F_{(j)}(-y-2\theta)]dF_{(j)}(y) - \int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right\}.$$

Now, we compute the variance of $W_{1(j)}$, $W_{2(j)}$.

$$Var(\sum_{j=1}^{k} W_{1(j)}) = \frac{nk}{4}\delta_1^2 + \binom{n}{2}\sum_{j=1}^{k}\left\{\left(\int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right)\left(1 - \int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right)\right\}$$

and

$$Var(\sum_{j=1}^{k} W_{2(j)}) = nk\,G(0)(1-G(0))\delta_2^2$$

$$+ \binom{n}{2}\sum_{j=1}^{k}\left\{\left(\int_{-\infty}^{\infty}[1-F_{(j)}(-y-2\theta)]dF_{(j)}(y)\right)\left(1 - \int_{-\infty}^{\infty}[1-F_{(j)}(-y-2\theta)]dF_{(j)}(y)\right)\right\},$$

where

$$\delta_1^2 = 1 - \frac{4}{k}\sum_{j=1}^{k}(F_{(j)}(0) - \frac{1}{2})^2,$$

$$\delta_2^2 = 1 - \frac{\sum_{j=1}^{k}(G_{(j)}(0) - G(0))^2}{kG(0)(1-G(0))}.$$

Then we have the variance of W_{RSS}^{+}.

$$Var_\theta(W_{RSS}^{+}) = nk\left\{\frac{\delta_1^2}{4} + G(0)(1-G(0))\delta_2^2 + \frac{n-1}{2k}\sum_{j=1}^{k}\left(\int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right)\left(1 - \int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right) + \frac{n-1}{2k}\sum_{j=1}^{k}\left(\int_{-\infty}^{\infty}[1-G_{(j)}(-y)]dF_{(j)}(y)\right)\left(1 - \int_{-\infty}^{\infty}[1-G_{(j)}(-y)]dG_{(j)}(y)\right)\right\}.$$

When $\theta = 0$, $\delta_1^2 = \delta_2^2 = \delta_0^2$, with k fixed and $n \to \infty$, by central limit theorem,

$$\frac{W_{RSS}^{+} - E_0(W_{RSS}^{+})}{\sqrt{Var_0(W_{RSS}^{+})}}$$

has the standard normal distribution,
where

$$Var_0(W^{+}_{RSS})$$
$$= \frac{nk}{2}\delta_0^2 + n(n-1)\sum_{j=1}^{k}\left(\int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right)\left(1-\int_{-\infty}^{\infty}[1-F_{(j)}(-x)]dF_{(j)}(x)\right)$$

Theorem Under $H_0 : \theta = 0$,

(i) $F(0) = G(0) = \frac{1}{2}$.

(ii) $F_{(j)}(0) = \frac{k!}{(j-1)!(k-j)!}\int_0^{\frac{1}{2}} u^{j-1}(1-u)^{k-j}du.$

(iii) $E_0(W^{+}_{RSS}) = 0$

(iv) $Var_0(W^{+}_{RSS}) = \frac{nk(n+1)(2n+1)}{12}\delta_0^2,$

where $\delta_0^2 = 1 - \frac{4}{k}\sum_{j=1}^{k}(F_{(j)}(0) - \frac{1}{2})^2.$

Proof The expectation and variance formulas follow from the fact that

$$F(0) = \frac{1}{k}\sum_{j=1}^{k}F_{(j)}(0) = \frac{1}{2},\quad G(0) = \frac{1}{k}\sum_{j=1}^{k}G_{(j)}(0) = \frac{1}{2}.$$

(ii) By the change variable $u = F(t)$, note that

$$F_{(j)}(0) = \frac{k!}{(j-1)!(k-j)!}\int_{-\infty}^{0}[F(t)]^{j-1}[1-F(t)]^{k-j}dF(t).$$

3. Asymptotic Relative Efficiencies

We compare the asymptotic relative efficiencies (ARE) of the RSS two-sample Wilcoxon signed rank test statistic W^{+}_{RSS} with respect to Mann-Whitney-Wilcoxon statistic U_{SRS} based on SRS, Mann-Whitney-Wilcoxon statistic U_{RSS} based on RSS and the RSS two-sample sign test statistic T^{+}_{RSS}. Both U_{SRS} statistic and U_{RSS} statistic are considered in Bohn and Wolfe (1992).

We consider the Pitman efficiency of $\sum_{j=1}^{k}(V_{2(j)} - V_{1(j)})$. For this, we have

$$E_\theta(\sum_{j=1}^{k}(V_{2(j)} - V_{1(j)})) = \sum_{j=1}^{k}\left\{\frac{2}{n-1}[F_{(j)} - F_{(j)}(-\theta)]\right.$$

$$+ \sum_{j=1}^{k} \int_{-\infty}^{\infty} [1 - F_{(j)}(-y - 2\theta)] dF_{(j)}(y) - \int_{-\infty}^{\infty} [1 - F_{(j)}(-x)] dF_{(j)}(x) \Big\}.$$

The derivative of $E_\theta(\sum_{j=1}^{k}(V_{2(j)} - V_{1(j)}))$ evaluated at $\theta = 0$ is

$$\begin{aligned} &\frac{\partial}{\partial\theta} E_\theta\Big(\sum_{j=1}^{k}(V_{2(j)} - V_{1(j)})\Big)\Big|_{\theta=0} \\ &= \sum_{j=1}^{k}\Big\{\frac{2}{n-1} f_{(j)}(-\theta) + \int_{-\infty}^{\infty} f_{(j)}(y) f_{(j)}(-y-2\theta) dy\Big\}\Big|_{\theta=0} \\ &= \sum_{j=1}^{k}\Big\{\frac{2}{n-1} f_{(j)}(-0) + 2\int_{-\infty}^{\infty} f^2_{(j)}(y) dy\Big\}. \end{aligned}$$

By the definition 5.2.14 of Randles and Wolfe (1979), we have the following efficacy (*eff*) of W^+_{RSS}.

$$eff^2(W^+_{RSS}) = \frac{6\Big[\sum_{j=1}^{k}\int_{-\infty}^{\infty} f^2_{(j)}(y)dy\Big]^2}{k^2\delta_0^2},$$

where

$$f^2_{(j)}(y) = \Big(\frac{k!}{(j-1)!(k-j)!}\Big)^2 [F(y)]^{2(j-1)}[1-F(y)]^{2(k-j)} f^2(y), \; j = 1, \cdots, k.$$

Let W^+_{SRS} be the statistic based on SRS, which has the same structure of W^+_{RSS}. The efficacy of W^+_{SRS} is given by

$$eff^2(W^+_{SRS}) = 6\Big[\int_{-\infty}^{\infty} f^2(y)dy\Big]^2.$$

The test statistic U_{SRS} and U_{RSS} in Bohn and Wolfe (1992) are

$$U_{SRS} = \sum_{s=1}^{q}\sum_{t=1}^{n}\sum_{i=1}^{k}\sum_{j=1}^{m} \Psi(Y_{st} - Y_{ij})$$

and

$$U_{RSS} = \sum_{s=1}^{q}\sum_{t=1}^{n}\sum_{i=1}^{k}\sum_{j=1}^{m} \Psi(Y_{(s)t} - Y_{(i)j}).$$

Test statistic T^+_{RSS} given by Kim and Kim (1998) is

$$T^+_{RSS} = \sum_{j=1}^{k}(T_{2(j)} - T_{1(j)})$$

where $T_{1(j)} = \sum_{i=1}^{n} I(X_{(j)i} > 0)$ has a binomal distribution with parameter n and $1 - F_{(j)}(0)$ and $T_{2(j)} = \sum_{i=1}^{n} I(Y_{(j)i} > 0)$ as a binomal distribution with parameter n and $1 - G_{(j)}(0)$. Note that $I(x)$ is the indicator function. Here, we calculate the only case with $k = q$ and $n = m$. Then we obtain efficacies of Mann-Whitney-Wilcoxon statistic U_{SRS}

based on SRS and Mann-Whitney-Wilcoxon statistic U_{RSS} based on RSS by Bohn and Wolfe (1992, 1994). We obtain the efficacy of RSS two-sample sign test statistic T^{+}_{RSS} by Kim and Kim (1998).

$$eff^2(U_{SRS}) = 3k\left[\int_{-\infty}^{\infty} f^2(y)\,dy\right]^2$$

$$eff^2(U_{RSS}) = \frac{2\tau^2}{\zeta_{0,1}+\zeta_{1,0}}$$

$$eff^2(T^{+}_{RSS}) = 2\left[\frac{\sum_{j=1}^{k} f_{(j)}(0)}{k\delta_0}\right]^2$$

where τ, $\zeta_{1,0}$ $\zeta_{0,1}$ are given by Bohn and Wolfe (1992).

Then we obtain the asymptotic relative efficiencies of RSS two-sample Wilcoxon signed rank test statistic W^{+}_{RSS} with respect to Mann-Whitney-Wilcoxon statistic U_{SRS} based on SRS, Mann-Whitney-Wilcoxon statistic U_{RSS} based on RSS, the RSS two-sample sign test statistic T^{+}_{RSS} and the SRS two-sample Wilcoxon signed rank test statistic W^{+}_{SRS}.

$$ARE(W^{+}_{RSS}, U_{SRS}) = \frac{2}{k}\left[\frac{\sum_{j=1}^{k}\int_{-\infty}^{\infty} f^2_{(j)}(y)\,dy}{k\delta_0\int_{-\infty}^{\infty} f^2(y)\,dy}\right]^2,$$

$$ARE(W^{+}_{RSS}, U_{RSS}) = 3(\zeta_{0,1}+\zeta_{1,0})\left[\frac{\sum_{j=1}^{k}\int_{-\infty}^{\infty} f^2_{(j)}(y)\,dy}{k\delta_0\tau}\right]^2,$$

$$ARE(W^{+}_{RSS}, T^{+}_{RSS}) = 3\left[\frac{\sum_{j=1}^{k}\int_{-\infty}^{\infty} f^2_{(j)}(y)\,dy}{\sum_{j=1}^{k} f_{(j)}(0)}\right]^2,$$

$$ARE(W^{+}_{RSS}, W^{+}_{SRS}) = \left[\frac{\sum_{j=1}^{k}\int_{-\infty}^{\infty} f^2_{(j)}(y)\,dy}{k\delta_0\int_{-\infty}^{\infty} f^2(y)\,dy}\right]^2.$$

Bohn and Wolfe (1992) considered the ARE of U_{RSS} and U_{SRS} for the special case of $k = q = 2$ because of the difficulty to calculate, but we calculate $ARE(W^{+}_{RSS}, U_{RSS})$, $ARE(W^{+}_{RSS}, T^{+}_{RSS})$ in Table3.1 for the case of $k = q = 2, 3, 4$, where the underlying distribution are uniform, double exponential, logistic distributions.

For all distributions, Bohn and Wolfe statistic U_{RSS} based on Mann-Whitney-Wilcoxon statistic is superior to the W^+_{RSS} and T^+_{RSS} because U_{RSS} use more informations than the other statistics. We also find that the statistic W^+_{RSS} is more efficient than the statistic T^+_{RSS}.

Table 1. Asymptotic Relative Efficiencies of W^+_{RSS} with respect to U_{RSS}, U_{SRS}, T^+_{RSS} and W^+_{SRS}

k	ARE	(W^+_{RSS} , U_{RSS})	(W^+_{RSS} , U_{SRS})	(W^+_{RSS} , T^+_{RSS})	(W^+_{RSS} , W^+_{SRS})
2	Uniform	0.40	2.37	5.33	2.37
	Double Exponential	0.30	1.81	1.02	1.81
	Logistic	0.32	1.92	1.92	1.92
3	Uniform	0.34	2.73	7.68	4.10
	Double Exponential	0.24	1.90	1.33	2.84
	Logistic	0.25	1.97	2.47	2.96
4	Uniform	0.31	3.06	10.02	6.12
	Double Exponential	0.20	2.00	1.64	4.00
	Logistic	0.21	2.14	3.12	4.28

4. Simulation Design and Results

This simulation is shown to compare the powers of W^+_{RSS}, U_{RSS}, T^+_{RSS} and usual two-sample t statistic T. The powers are obtained from the uniform, normal, double exponential, logistic and Cauchy distributions on which these simulations are based. The location parameter θ has 0.0 (0.2) 0.8. We consider the case of k = 3. The cycle of sampling, denoted by n, sets at 5, 10 and 20. And the results of this simulation have 1,000 replications. The uniform and normal random variates are generated by using the IMSL (Internation Mathematics and Statistical Library) program GGUBS and GGNML, respectively, and double exponential and Cauchy random variates are generated by using the GGUBS and the probability integral transformation.

Table 2 gives empirical powers of tests for k = 3, and α = 0.05 in underlying distributions. From the simulation results, we can know that the powers of U_{RSS} are particularly greater than those of the competitors for uniform, normal, logistic distributions because U_{RSS} use more informations than the other statistics. For uniform, normal and logistic distributions, the empirical powers of W^+_{RSS} are higher than those of the statistic T^+_{RSS} at n = 10, 20. For double exponential distribution, the powers of W^+_{RSS} are larger than those of T. For normal distribution, T has smaller powers than U_{RSS} because T is based on SRS.

Table 2. Empirical Powers of Tests (k = 3, α = 0.05)

	n	Statistic	θ = 0.0	0.2	0.4	0.6	0.8
Uniform (-1/2, 1/2)	5	W_{RSS}^{+}	.045	.100	.204	.343	.523
		U_{RSS}	.071	.182	.446	.698	.897
		T	.038	.133	.285	.444	.688
		T_{RSS}^{+}	.062	.120	.223	.357	.486
	10	W_{RSS}^{+}	.022	.098	.267	.534	.793
		U_{RSS}	.037	.270	.650	.925	.989
		T	.063	.191	.468	.758	.936
		T_{RSS}^{+}	.031	.105	.227	.442	.695
	20	W_{RSS}^{+}	.032	.171	.536	.871	.979
		U_{RSS}	.046	.430	.892	.994	1.000
		T	.055	.298	.681	.956	.998
		T_{RSS}^{+}	.054	.181	.476	.764	.941

	n	Statistic	θ = 0.0	0.2	0.4	0.6	0.8
Normal	5	W_{RSS}^{+}	.045	.128	.272	.463	.627
		U_{RSS}	.057	.214	.445	.709	.901
		T	.046	.147	.303	.486	.707
		T_{RSS}^{+}	.040	.120	.267	.464	.629
	10	W_{RSS}^{+}	.022	.142	.377	.696	.897
		U_{RSS}	.041	.254	.682	.926	.991
		T	.062	.192	.493	.764	.926
		T_{RSS}^{+}	.034	.141	.390	.672	.872
	20	W_{RSS}^{+}	.032	.232	.679	.944	1.000
		U_{RSS}	.054	.471	.912	1.000	1.000
		T	.056	.292	.677	.953	.998
		T_{RSS}^{+}	.045	.262	.656	.922	.994

	n	Statistic	$\theta =$ 0.0	0.2	0.4	0.6	0.8
Double Exponential	5	W^+_{RSS}	.021	.133	.336	.548	.719
		U_{RSS}	.055	.252	.552	.810	.951
		T	.058	.144	.305	.504	.717
		T^+_{RSS}	.043	.218	.467	.686	.788
	10	W^+_{RSS}	.031	.205	.563	.837	.947
		U_{RSS}	.056	.337	.827	.988	.998
		T	.045	.189	.466	.760	.917
		T^+_{RSS}	.037	.275	.660	.879	.956
	20	W^+_{RSS}	.021	.369	.858	.990	1.000
		U_{RSS}	.043	.608	.986	1.000	1.000
		T	.054	.310	.715	.942	.998
		T^+_{RSS}	.031	.505	.895	.996	1.000

	n	Statistic	$\theta =$ 0.0	0.2	0.4	0.6	0.8
Logistic	5	W^+_{RSS}	.024	.116	.296	.516	.681
		U_{RSS}	.051	.188	.457	.767	.924
		T	.050	.117	.307	.500	.706
		T^+_{RSS}	.039	.169	.360	.516	.738
	10	W^+_{RSS}	.023	.164	.466	.780	.934
		U_{RSS}	.056	.308	.717	.957	.994
		T	.047	.172	.470	.736	.926
		T^+_{RSS}	.035	.186	.431	.752	.905
	20	W^+_{RSS}	.015	.263	.727	.961	1.000
		U_{RSS}	.051	.467	.946	1.000	1.000
		T	.052	.329	.716	.951	.996
		T^+_{RSS}	.036	.310	.735	.955	.994

	n	Statistic	θ = 0.0	0.2	0.4	0.6	0.8
Cauchy	5	W^{+}_{RSS}	.021	.118	.279	.468	.583
		U_{RSS}	.058	.196	.456	.674	.838
		T	.040	.064	.106	.138	.212
		T^{+}_{RSS}	.044	.185	.389	.610	.735
	10	W^{+}_{RSS}	.031	.173	.455	.713	.851
		U_{RSS}	.057	.281	.690	.911	.987
		T	.025	.052	.108	.159	.214
		T^{+}_{RSS}	.030	.231	.564	.803	.908
	20	W^{+}_{RSS}	.021	.285	.734	.948	.987
		U_{RSS}	.049	.480	.910	.994	1.000
		T	.028	.066	.097	.140	.202
		T^{+}_{RSS}	.054	.433	.863	.985	.998

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