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Hybrid Group-Sequential Conditional-Bayes Approaches to the Double Sampling Plans[1]

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Abstract

This research aims here to develop a certain extended double sampling plan, EDS^*, which is an extension of ordinary double sampling plan in the sense that the second-stage sampling effort and second-stage critical value are allowed to depend on the point at which the first-stage continuation region is traversed. For purpose of comparison, single sampling plan, optimal ordinary double sampling plan(ODS^*) and sequential probability ratio test are considered with the same overall error rates, respectively. It is observed that the EDS^* idea allows less sampling effort than the optimal ODS.

1. Introduction

Determining or adjusting second-stage sample size and critical region based on first-stage outcome constitutes an extension of classical double sampling plans as originally proposed (Dodge and Romig(1941)) and subsequently developed(Hald(1975) and Spurrier and Hewett(1975)). In this study, we consider double sampling plans with variable second-stage, that is, at the end of the first-stage an interim analysis is performed with the objective of deciding whether or not to continue the study based on results of the interim analysis. If the study is continued, the first-stage information is systematically put to work in conducting the second-stage, including its sample size and critical region, with the goal of achieving agreed-upon overall, as well as stage-specific operating characteristics. Where the proposed approach differs from previous work is in our casting of the design of extended double sampling(EDS) plans in the form of constrained optimization problems. The constraints of the optimization problems borrow from ''group sequential'' formulations idea of ''allocating'' error between the two stages; indeed we go beyond the usual group sequential formulation in that both the errors of the first and of the second kind are so allocated. The objective functions for optimization problem borrows from Bayesian analysis the feature that a certain average of measures of sampling effort is minimized.

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Section 2 provides the formulations of *EDS* plan and details their optimization. Section 3 develops three plans alternative to EDS^*. These include the single sampling(*SS*) plan for testing μ_0 against μ_1, the sequential probability ratio test (*SPRT*) for testing μ_0 against μ_1, and the optimal (with respect to first-stage sampling time) ordinary double sampling plan (ODS^*) with overall error rates α and β equally allocated between the two stages, as in EDS^* derived in section 2.

Section 4 compares the four plans SS, ODS^*, EDS^* and $SPRT$, and discusses certain methodological features of EDS^*, and of a certain possible alternate version of it.

2. The plan EDS^*: Detailing the optimization

We have chosen to initially study the analytically tractable Wiener process (Kac(1959)) which, of course, is relevant to the case of normally distributed measurements, and, by the arcsine transformation, to the binomial case. Specifically, we consider discriminating between the Wiener process with drift parameter μ_0 and the Wiener process with drift parameter μ_1, $\mu_1 > \mu_0$. In either case, we assume a unit scale parameter.

The discrimination is to be based on a two-stage procedure with initial sampling time T_0 and continuation interval, $[l, u]$. At time T_0, μ_1(respectively, μ_0) is to be accepted if the process exceeds u (respectively, below l). Further, if the process equals an intermediate value s at time T_0, $l \leq s \leq u$, then a predetermined second-stage sampling plan is implemented, with sampling time T_s and critical value k_s depending on s. Thus our plan is determined by three numbers (T_0, l , u), and the two functions T_s and k_s, $l \leq s \leq u$.

Predetermined overall error rates α and β (of the first and second kind) are equally allocated between the two stages. Under this restriction, the plan (T_0, l , u, $\{T_s: l \leq s \leq u\}$, $\{k_s: l \leq s \leq u\}$) is to be chosen in such a way as to minimize a certain average $\overline{AST}$, the unweighted average of expected sampling times for the values of μ , AST_μ, between μ_0 and μ_1.

$$\overline{AST} = (\mu_1 - \mu_0)^{-1} \int_{\mu_0}^{\mu_1} AST_\mu \, d\mu \tag{2.1}$$

where

$$AST_\mu = T_0 + \int_l^u \frac{T_s}{\sqrt{2\pi}\sqrt{T_0}} \exp\left[-\frac{1}{2}\left(\frac{s-\mu T_0}{\sqrt{T_0}}\right)^2\right] ds.$$

Now, we give an account of the manner in which we identify the optimal extended double sampling plan by minimizing the objective function (2.1), in the case of parameter values $(\alpha, \beta, \mu_0, \mu_1)$ equal to $(0.05, 0.10, 0, 0.50)$.

We begin by fixing first-stage sampling time T_0 such that the first-stage error rates are $\alpha/2$ and $\beta/2$. This determines a continuation interval, $[l, u]$. Given T_0, l and u, we then minimize $\overline{AST}$ (or, equivalently, $\overline{AST}^{(2)} \equiv \overline{AST} - T_0$) subject to the restriction that the second-stage error rates equal $\alpha/2$ and $\beta/2$. This minimization is with respect to the second-stage sampling time function T_s, $l \le s \le u$, and second-stage critical value function k_s, $l \le s \le u$.

For given T_s and k_s,

$$\overline{AST}^{(2)} = (\mu_1 - \mu_0)^{-1} \int_{\mu_0}^{\mu_1} \left[\int_l^u \phi(s|\mu T_0, \sqrt{T_0}) \cdot T_s \, ds \right] d\mu \tag{2.2}$$

where $\phi(s|\mu T_0, \sqrt{T_0})$ is the normal density with mean μT_0 and standard deviation $\sqrt{T_0}$. Interchanging the order of integration, the restricted minimization of (2.2) is explicitly given in the form :

$$\text{minimize}_{\{T_s\},\{k_s\}}$$

$$[(\mu_1-\mu_0)\cdot T_0]^{-1}\int_l^u[\Phi(\frac{\mu_1 T_0-s}{\sqrt{T_0}})-\Phi(\frac{\mu_0 T_0-s}{\sqrt{T_0}})]\cdot T_s ds$$

subject to

$$\int_l^u \phi(s|\,\mu_0 T_0,\sqrt{T_0})\int_{k_s}^{\infty}\phi(x|\,\mu_0 T_s,\sqrt{T_s})dxds \;=\; \alpha/2$$

and $$\int_l^u \phi(s|\,\mu_1 T_0,\sqrt{T_0})\int_{-\infty}^{k_s}\phi(x|\,\mu_1 T_s,\sqrt{T_s})dxds \;=\; \beta/2$$

where $\Phi(y)=\int_{-\infty}^{y}\phi(x|0,1)dx.$

We implement this restricted minimization by subdividing the continuation interval $[l,\ u]$, as determined by the first-stage sampling time T_0, into a grid of 2^r grid points, $l+\frac{(u-l)}{2^{r+1}},\ l+\frac{3(u-l)}{2^{r+1}},\ \cdots,\ u-\frac{(u-l)}{2^{r+1}}$. In view of this appropriate value r which provides a sufficiently accurate approximate formulation could be determined and our problem is recasted as follows with $m\ =\ 2^r$:

$$\text{minimize}_{\{T_i\}_{i=1}^m,\{k_i\}_{i=1}^m} f(T_1,\cdots,T_m,k_1,\cdots,k_m) \tag{2.3}$$

$$\text{subject to}\quad g_0(T_1,\cdots,T_m,k_1,\cdots,k_m)-\alpha/2=0$$

$$\text{and}\quad g_1(T_1,\cdots,T_m,k_1,\cdots,k_m)-\beta/2=0.$$

We address this problem via a standard *Kuhn-Turker* argument (see for example, Ko(1994)) involving the Lagrangian kernel

$$\begin{aligned} L(\boldsymbol{T},\boldsymbol{k};\rho_0,\rho_1) &= f(\boldsymbol{T},\boldsymbol{k}) + \rho_0\cdot g_0(\boldsymbol{T},\boldsymbol{k}) + \rho_1\cdot g_1(\boldsymbol{T},\boldsymbol{k}) \\ &\equiv \sum_{i=1}^m[\eta_i T_i + \rho_0\cdot g_0(T_i,k_i) + \rho_1\cdot g_1(T_i,k_i)] \\ &\equiv \sum_{i=1}^m h_i(T_i,k_i), \end{aligned}$$

where $\boldsymbol{T}=(T_1,\cdots,T_m)$ and $\boldsymbol{k}=(k_1,\cdots,k_m)$.

The *Kuhn-Turker* argument for solving (2.3) now proceeds as follows :

Denote the $2m$-dimensional vector $(\boldsymbol{T}, \boldsymbol{k})$ by $\boldsymbol{x}$, and observe that $\boldsymbol{x}^*$ will minimize $f(\boldsymbol{x})$, subject to $g_0(\boldsymbol{x})-\alpha/2=g_1(\boldsymbol{x})-\beta/2=0$ if $\boldsymbol{\rho}^*=(\rho_0^*, \rho_1^*)$ can be found, $-\infty < \rho_0^*, \rho_1^* < \infty$, such that

(1) $\boldsymbol{x}^*$ minimizes $L(\boldsymbol{x}, \boldsymbol{\rho}^*)$ over range (2.4)

$$0 \leq T_i, \quad -\infty < k_i < \infty; \; i=1,2,\cdots,m$$

(2) $g_0(\boldsymbol{x}^*)-\alpha/2 = g_1(\boldsymbol{x}^*)-\beta/2=0.$

The search for $\boldsymbol{x}^*$ is made feasible by the following two features of the problem :

First, the classical first-order Lagrangian formulation, based on differentiating the Lagrangian kernel L, provides candidate values for (ρ_0^*, ρ_1^*), for the $2m$ values $\{T_i^*\}_{i=1}^m$ and $\{k_i^*\}_{i=1}^m$, so that our *Kuhn-Turker* analysis merely provides verification; the candidate values are iteratively derived in the following steps: i) identify a trial pair ρ_0, ρ_1; ii) for each i, obtain two relations for (T_i, k_i), by setting the two derivatives of the ensuring $h_i(T_i, k_i)$, with respect to T_i and k_i, equal to zero; iii) for each i, obtain trial values for (T_i, k_i), by solving these two relations; iv) using all $2m$ trial values, check the two restrictions of (2.3); if these are not satisfied to within sufficient accuracy (six decimal figures in our case), select new trial pair (ρ_0, ρ_1). The pattern of discrepancies developed after several iterations allows achieving the desired accuracy, and hence identifying a candidate (ρ_0^*, ρ_1^*), and simultaneously, candidate $T_i{}^*$ and $k_i{}^*$, in manageable time.

Second, given (ρ_0^*, ρ_1^*) and T_i^* and k_i^*, the additive separability of our objective function allows verifying (2.4) by sufficient condition that, for each i, $h_i(T_i, k_i)$ is indeed minimum at (T_i^*, k_i^*). It remains only to detail those minimization. To that end, the following lemma is useful.

Lemma Suppose that both ρ_0 and ρ_1 are positive. Then, for fixed T_i,

$$h_i(T_i, k(T_i)) \leq h_i(T_i, k_i), \quad -\infty < k_i < \infty, \quad \text{where}$$

$$k(T_i) = \left(\frac{\mu_0 + \mu_1}{2}\right) \cdot (T_i + T_0) - \left[s + \frac{\ln\left(\frac{\rho_0}{\rho_1}\right)}{(\mu_1 - \mu_0)}\right].$$

Proof :

In the purpose of the proof, we delete the subscript i.

Setting the derivative of $h(T, k)$ with respect to k equal to zero yields the relation

$$\rho_1 \phi(s|\ \mu_1 T_0, \sqrt{T_0}) \phi(k(T)|\ \mu_1 T, \sqrt{T}) = \rho_0 \phi(s|\ \mu_0 T_0, \sqrt{T_0}) \phi(k(T)|\ \mu_0 T, \sqrt{T}). \qquad (2.5)$$

Further, the derivative $h'(T, k)$ of $h(T, k)$ with respect to k, evaluated at $k(T) + \delta$, $\delta > 0$ equals

$$\rho_1 \phi(s|\ \mu_1 T_0, \sqrt{T_0}) \phi(k(T) + \delta|\ \mu_1 T, \sqrt{T}) - \rho_0 \phi(s|\ \mu_0 T_0, \sqrt{T_0}) \phi(k(T) + \delta|\ \mu_0 T, \sqrt{T})$$

$$= \rho_1 \phi(s|\ \mu_1 T_0, \sqrt{T_0}) \phi(k(T)|\ \mu_1 T, \sqrt{T}) \exp\left[-\left(\frac{\delta k(T)}{\sqrt{T}} + \frac{\delta^2}{2T}\right)\right] \exp(\delta \mu_1 \sqrt{T})$$

$$- \rho_0 \phi(s|\ \mu_0 T_0, \sqrt{T_0}) \phi(k(T)|\ \mu_0 T, \sqrt{T}) \exp\left[-\left(\frac{\delta k(T)}{\sqrt{T}} + \frac{\delta^2}{2T}\right)\right] \exp(\delta \mu_0 \sqrt{T})$$

$$= \rho_1 \phi(s|\mu_1 T_0, \sqrt{T_0}) \phi(k(T)|\ \mu_1 T, \sqrt{T}) \exp\left[-\left(\frac{\delta k(T)}{\sqrt{T}} + \frac{\delta^2}{2T}\right)\right] \{\exp(\delta \mu_1 \sqrt{T}) - \exp(\delta \mu_0 \sqrt{T})\}$$

$$> 0,$$

where the last equality follows from (2.5), and the inequality follows from $\mu_1 > \mu_0$.

Finally, an analogous argument shows that $h'(T, k) < 0$ for $k = k(T) - \delta$. This completes the proof.

ρ_0^* and ρ_1^* being positive through(Ko and David(1996)), we are able to verify (2.4) for each i, by empirically verifying that $h_i(T_i, k(T_i))$ is minimized at T_i^*, with $k(T_i^*) = k_i^*$.

It is possible to carry out the entire above program for certain range of T_0- values $(0, \overline{T_0})$. Unfortunately, $\overline{T_0}$ is not largest possible T_0-value for which it is possible to satisfy our error restrictions. This is most easily seen by the fact that $\overline{T_0}$ is exceeded by the first-stage sampling time $\widehat{T}_0$ of ordinary double sampling plans with largest possible first-stage sampling time. However, the optimized $\overline{AST}$,

call it now $\overline{AST}(T_0)$, appears convex over $(0, \overline{T}_0)$, and, in any event, achieves a minimum over that interval, at an interior point T_0^*. With $l^*, u^*, \{T_i^*\}_{i=1}^m, \{k_i^*\}_{i=1}^m$, the solutions found for $T_0 = T_0^*$, we call the plan $(T_0^*, l^*, u^*, \{T_i^*\}_{i=1}^m, \{k_i^*\}_{i=1}^m)$ the optimal extended double sampling plan, EDS^*.

3. Alternative plans

The main procedures alternative to extended double sampling plans are single sampling plans(SS), and ordinary double sampling plans(ODS), and possibly sequential probability ratio test($SPRT$) plans, though the latter typically are difficult to implement in practical settings, which calls for developing group sequential(O'Blein and Fleming(1979), Pocock(1982)) approaches, specially in clinical trials.

3.1 Single Sampling Plan (SS)

The simplest sampling plan, single sampling, with error rates α and β at hypothesis points μ_0 and μ_1, is determined as follows for the Wiener process :

$$\Pr\{\phi(x|\mu_0 T, \sqrt{T}) \geq k\} = \alpha \text{ and } \Pr\{\phi(x|\mu_1 T, \sqrt{T}) < k\} = \beta. \tag{3.1}$$

The AST_μ of SS is the constant value T^* satisfying (3.1) and the OC_μ of SS is given by the function $\Pr\{\phi(x|\mu T, \sqrt{T}) < k^*\}$ of μ, where k^* satisfies (3.1).

3.2 Ordinary Double Sampling Plans (ODS)

We define such plans by fixing second-stage sampling time and critical value, i.e., the plans EDS without flexibility of second-stage. An unique ordinary double sampling plan, ODS, with allocation of the two error rates α and β to the two-stages, exists for all

first-stage sampling time T_0 in $(0, \widehat{T}_0]$, where $\widehat{T}_0$ is a value for first-stage sampling time that happens to exceed $\overline{T}_0$, upper limit of first-stage sampling time for EDS^*. It's kind of obvious when we consider restrictions of two plans. The AST_μ and OC_μ function of ODS^* are determined as follows for the Wiener process :

$$AST_\mu = T_0 + \int_l^u \phi(s|\mu T_0, \sqrt{T_0}) \cdot T_2\, ds$$

(3.2)

and

$$OC_\mu = \int_{\infty}^{u} \phi(x|\mu T_0, \sqrt{T_0})dx + \int_l^u \phi(s|\mu T_0, \sqrt{T_0}) \cdot [\int_{\infty}^{k_2} \phi(x|\mu T_2, \sqrt{T_2})dx]ds$$

where T_2 and k_2 satisfy

$$\alpha = \int_u^\infty \phi(x|\mu_0 T_0, \sqrt{T_0})dx + \int_l^u \phi(s|\mu_0 T_0, \sqrt{T_0}) \cdot [\int_{k_2}^{\infty} \phi(x|\mu_0 T_2, \sqrt{T_2})dx]ds$$

and

$$\beta = \int_{-\infty}^{l} \phi(x|\mu_1 T_0, \sqrt{T_0})dx + \int_l^u \phi(s|\mu_1 T_0, \sqrt{T_0}) \cdot [\int_{-\infty}^{k_2} \phi(x|\mu_1 T_2, \sqrt{T_2})dx]ds.$$

AST_μ is computed according to (3.2) for all first-stage sampling times T_0 in $(0, \widehat{T}_0]$, and further first-stage optimization over $(0, \widehat{T}_0]$ gives minimum, $\widehat{T}_0^*$, also corresponding continuation region $(\widehat{l^*}, \widehat{u^*})$ and the second-stage parameters $(\widehat{T}_2^*, \widehat{k}_2^*)$ to constitute ODS^*.

3.3 Sequential Probability Ratio Test (SPRT)

The $SPRT$ to be compared with EDS^* is one with error rates α and β, at hypothesis point μ_0 and μ_1. The standard $SPRT$ theory for Wiener process gives the following formulas for OC and AST :

$$OC_\mu = \begin{cases} (\acute{A}-1)/(\acute{A}-\acute{B}) & \text{for } \mu < \bar{\mu} = (\mu_0+\mu_1)/2 \\ \log(A)/[\log(A)-\log(B)] & \text{for } \mu = \bar{\mu} \\ (1-\acute{B})/(\acute{A}-\acute{B}) & \text{for } \mu > \bar{\mu} \end{cases}$$

where $\acute{A} = A^{|2\mu-(\mu_0+\mu_1)|}$, $\acute{B} = B^{|2\mu-(\mu_0+\mu_1)|}$, $A=(1-\beta)/\alpha$ and $B=\beta/(1-\alpha)$;

$$AST_\mu = \begin{cases} \dfrac{\log(A)\cdot(1-OC_\mu)+\log(B)\cdot OC_\mu}{(\mu_1-\mu_0)(\mu-\bar{\mu})} & \text{for } \mu \neq \bar{\mu} \\ \dfrac{-\log(B)\cdot\log(A)}{(\mu_1-\mu_0)^2} & \text{for } \mu = \bar{\mu} \end{cases}$$

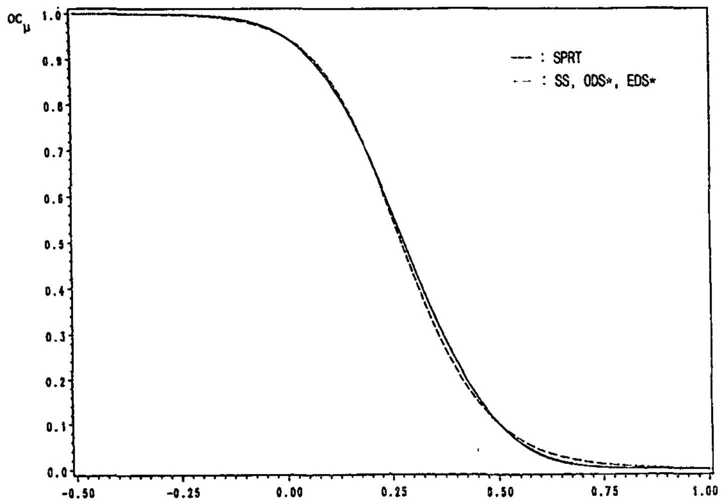
4. Comparisons and concluding remarks

As discussed in Section 2, we implement the minimization to derive EDS^* by subdividing the continuation interval into a grid of 2^r equally spaced points. This was in fact done for both $r = 5$ and $r = 6$, and it was found that the computation yielded essentially the same solutions. In view of this we decided that $r = 6$ provided a sufficiently accurate approximated formulation. Table 1 gives the second-stage sampling time and critical value depending on equally spaced 2^r grid points, s, in $[l^*, u^*]$. The comparable plan ODS^* has $\widehat{T_0^*} = 19.9$, $\widehat{T_2^*} = 22.01824$, $\widehat{k_2^*} = 6.05654$ and $[\widehat{l^*}, \widehat{u^*}] = [2.41260, 8.74329]$.

Table 1 : Optimal second-stage sampling time T_s^* and critical value k_s^* in $[l^*, u^*]$ for $T_0^* = 18.9$ and $(\alpha, \beta, \mu_0, \mu_1) = (0.05, 0.10, 0, 0.50)$.

s[1)]	T_s^*	k_s^*	s	T_s^*	k_s^*
2.34775	14.60943	7.15922	5.45857	22.49736	6.02038
2.44496	15.30639	7.23624	5.55578	22.50024	5.92389
2.54217	15.92306	7.29320	5.65299	22.49214	5.82465
2.63939	16.47847	7.33484	5.75020	22.47308	5.72267
2.73660	16.98459	7.36416	5.84742	22.44310	5.61796
2.83381	17.44959	7.38319	5.94463	22.40217	5.51052
2.93103	17.87927	7.39340	6.04184	22.35029	5.40033
3.02824	18.27802	7.39587	6.13906	22.28741	5.28740
3.12545	18.64916	7.39144	6.23627	22.21350	5.17171
3.22266	18.99533	7.38077	6.33348	22.12846	5.05324
3.31988	19.31867	7.36440	6.43070	22.03225	4.93197
3.80594	20.64445	7.20978	6.91676	21.37886	4.28256
3.90316	20.85883	7.16616	7.01397	21.21251	4.14376
4.00037	21.05807	7.11876	7.11119	21.03371	4.00185
4.09758	21.24276	7.06771	7.20840	20.84217	3.85675
4.19480	21.41340	7.01316	7.30561	20.63755	3.70838
4.29201	21.57042	6.95520	7.40283	20.41945	3.55664
4.38922	21.71421	6.89394	7.50004	20.18742	3.40142
4.48643	21.84513	6.82945	7.59725	19.94095	3.24259
4.58365	21.96347	6.76182	7.69447	19.67942	3.07999
4.68086	22.06949	6.69112	7.79168	19.40215	2.91346
4.77807	22.16343	6.61739	7.88889	19.10834	2.74280
4.87529	22.24551	6.54070	7.98611	18.79703	2.56776
4.97250	22.31590	6.46108	8.08332	18.46712	2.38807
5.06971	22.37475	6.37858	8.18053	18.11725	2.20339
5.16693	22.42221	6.29323	8.27775	17.74582	2.01332
5.26414	22.45841	6.20507	8.37496	17.35090	1.81737
5.36135	22.48344	6.11411	8.47217	16.92999	1.61493

1) $s = l^* + \xi/2, l^* + 3\xi/2, \ldots, u^* - \xi/2$ where $\xi = (u^* - l^*)/2^6$.



note : OC_{μ} functions of SS, ODS^* and EDS^* coincide to within the third decimal place.

Figure 1 : OC_{μ} of $SPRT, EDS^*$, ODS^* and SS, for the Wiener processes $\omega(0,1)$ and $\omega(0.5,1)$ with $(\alpha, \beta) = (0.05, 0.10)$.

As shown in Figure 1, the OC's for all plans are essentially the same, with $SPRT$ a bit more discrimination than EDS^* between μ_0 and μ_1, and the reverse true elsewhere.

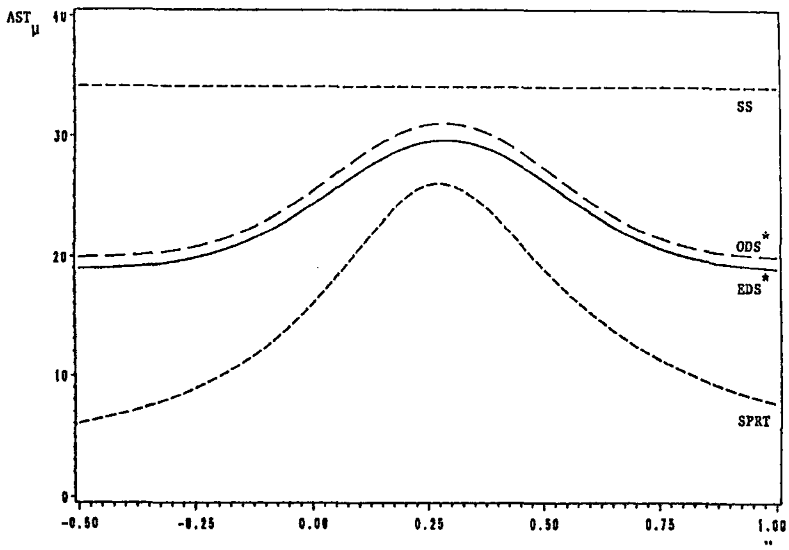


Figure 2 : AST_{μ} of $SPRT, EDS^*$, ODS^* and SS, for the Wiener processes $\omega(0,1)$ and $\omega(0.5,1)$ with $(\alpha, \beta) = (0.05, 0.10)$.

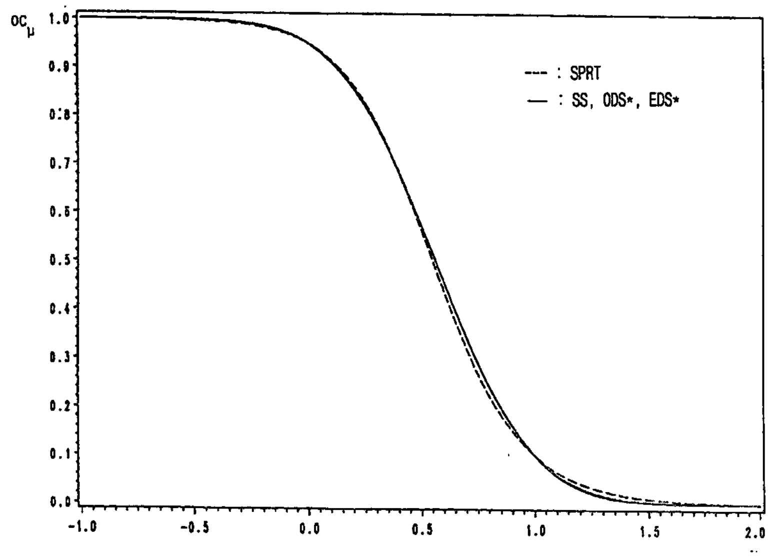
The four AST functions shown in Figure 2 are strictly ordered, with the AST function of EDS^* uniformly about 4% smaller than that of ODS^*.

That the domination of ODS^* by EDS^* is uniform certainly is good news. The bad news is that it is only by about 4%, suggesting a clear limitation to what can be achieved with adaptive second stages. While the $SPRT$ holds a substantial edge in AST over all three other procedures, it is often hard to implement in practice as mentioned before.

On the other hand, AST reduction may not be the entire story. The possibility of adjusting the second stage in response to the initial evidence may of itself have methodological merit : EDS^* possesses the feature that T_s is largest for values of s well within the continuation region as indicated in Table 1, where, in other words, the first-stage evidence for discrimination between μ_0 and μ_1 is least conclusive. That feature can be construed methodologically as adducing greater discrimination power when the initial returns are inconclusive; or as adducing greater evidence for estimation μ when neither hypothesis is likely to hold. On the other hand, we conjecture that, had we minimized not the average $\overline{AST}$ of AST_μ over the interval $[\mu_0, \mu_1]$, but rather had minimized $AST_{\bar{\mu}}$, $\bar{\mu} = (\mu_0 + \mu_1)/2$, we would have found the reverse future; that is , we would have obtained a plan with values of T_s smallest for values of s well within the continuation region. That alternative feature could be attractive to a researcher *not* willing to expend unnecessary sampling effort in marginal situations. Thus, EDS^*, beyond improving on ODS^* with regard to sampling effort, may allow connection to the details of optimizing plan to broad methodological considerations regarding the allocation of sampling effort.

It is further possible to consider other objective functions according to what the researchers make plans for, for example, if a researcher would like to minimize the sampling efforts on the alternative hypothesis point for some reasons, such as cost of experiments or ethical merit in clinical trials, he could consider the expected sampling time for $\mu = \mu_1$, i.e., AST_{μ_1}. Also, it is equally possible to develop such designs for the other weights over parameter space or hypotheses points and different allocations of error rates.

We have repeated the entire analysis for $\mu_1 = 1$, rather than $\mu_1 = 0.5$, and, as expected, have come similar conclusions, regarding both the behavior of (T_s^*, k_s^*) and the OC and AST functions. For the sake of completeness, the results for $\mu_1 = 1$ are shown in Figures 3 and 4.



note : OC_μ functions of *SS*, ODS^* and EDS^* coincide to within the third decimal place.

Figure 3 : OC_μ of *SPRT*, EDS^*, ODS^* and *SS*, for the Wiener processes $\omega(0,1)$ and $\omega(1,1)$ with $(\alpha, \beta) = (0.05, 0.10)$.

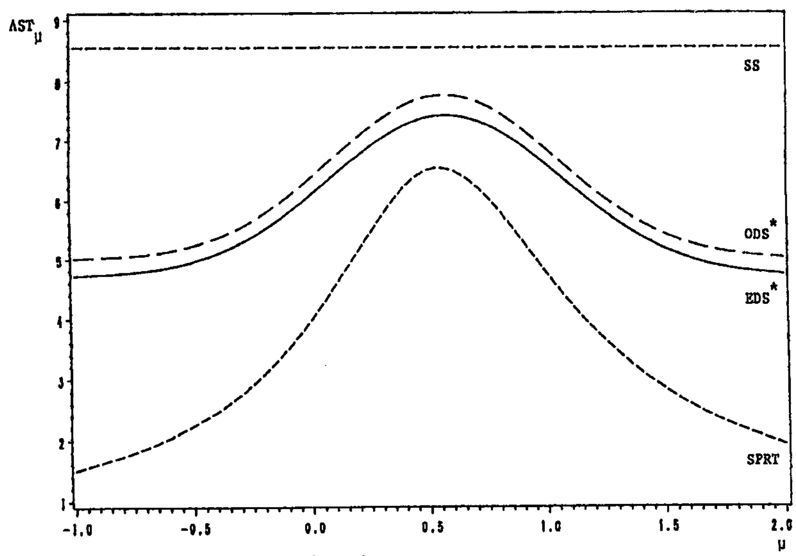


Figure 4 : AST_μ of *SPRT*, EDS^*, ODS^* and *SS*, for the Wiener processes $\omega(0,1)$ and $\omega(1,1)$ with $(\alpha, \beta) = (0.05, 0.10)$.

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