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Review of Screening Procedure as Statistical Hypothesis Testing

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Abstract

A screening procedure, where one or more correlated variables are used for screening, is reviewed from the point of statistical hypothesis testing. Without assuming a specific probability model for the joint distribution of the performance and screening variables, some principles are provided to establish the best screening region. Application examples are provided for two cases; i) the case where the performance variable is dichotomous and ii) the case where the performance variable is continuous. In case i), a normal model is assumed for the conditional distribution of the screening variable given the performance variable. In

case ii), the performance and screening variables are assumed to be jointly normally distributed.

1. Introduction

Due to the recent advances in inspection systems and the increasing requirements of the marketplace, 100% inspection (screening) becomes very popular at one or more stages of a manufacturing process. In a screening procedure, every item is inspected with one or more screening variables. The screening variable may be the major quality characteristic (performance variable) of the item itself or a surrogate variable which is highly correlated with the performance variable. In many practical situations, screening is performed with a surrogate variable. There have been a number of studies concerning the screening procedure. The previous works on screening may be classified into two groups; i) those which aim at attaining a target outgoing quality after screening and ii) those which aim at minimizing the expected cost relevant to screening. The former includes Owen et al.(1975), Owen and Boddie(1976), Owen and Su(1977), Li and Owen(1979), Haas et al.(1985), Wong et al.(1985), Boys and Dunsmore(1986, 1987), Turkman and Turkman(1989). The latter includes Riew and Bai(1985), Tang(1987,1988a,1988b), Moskowitz and Tsai(1988), Tang and Tang(1989), Bai et al.(1990), Kim and Bai(1990, 1992), Bai and Hong(1992), Bai and Lee(1993). For more detailed literature review, see Tang and Tang(1994) and see Bai and Kwon(1995), Bai et al.(1995), Boys et al.(1996), Kwon and Bai(1996), Bai and Kwon(1997), and Hong et al.(1998) for recent works.

When a correlated variable used for screening, there will be two types of misclassification if the correlation is not perfect; i) rejection of a conforming item (type I error) and ii) acceptance of a nonconforming item (type II error). While the economic models attempt to minimize the total expected cost by balancing these two errors economically, the other models aim at attaining a desirable level of the outgoing quality. In this article, we review the screening problem from a slightly different point of view. We take screening as testing whether an item is conforming or not and find the best screening region. This approach provides a pretty general tool to analyze the screening problem. Whether the performance variable is continuous or dichotomous, it may be applicable consistently. Attainability of the target outgoing quality of a screening procedure can be decided in advance.

In Section 2, we present a procedure for screening without assuming any specific probability distribution for the performance and screening variable. In Section 3, we consider the case where the performance variable is dichotomous and the screening variable is continuous. The screening variable given the performance variable is assumed to be normally distributed. In Section 4, we consider the case where the performance variable is continuous. The performance and screening variables are assumed to be jointly normally distributed. For given values of the correlation coefficient, the maximum proportions of conforming items attainable by screening are also provided when only one screening variable is used.

2. Screening Procedure

Let Y be the performance variable of an item. An item is considered to be conforming if the measured value y of Y for that item belongs to a prespecified set S_y. When Y is difficult or expensive to measure directly, one or more surrogate variables are often used to decide whether an item is acceptable or not. The surrogate variables are usually highly correlated with the performance variable. We will call the surrogate variable the screening variable and denote a set of the screening variables by a random vector $\underline{X}$. Then the screening procedure can be described as follows:

i) For every incoming item from the manufacturing process, obtain the observed or measured value $\underline{x}$ of $\underline{X}$.

ii) Accept the item with $\underline{x} \in S_{\underline{x}}$. Any items with $\underline{x} \notin S_{\underline{x}}$ are rejected.

In the previous works, many authors considered the problem to find the set $S_{\underline{x}}$ such that the proportion of conforming items can rise from the current level

$$\gamma = P(Y \in S_y) \tag{1}$$

to a predetermined higher level δ after screening. Thus, the design problem of a screening procedure is to find $S_{\underline{x}}$ satisfying

$$\delta = P(Y \in S_y \mid \underline{X} \in S_{\underline{x}}). \tag{2}$$

We consider here the screening problem as testing
$H_0 : y \in S_y$ versus $H_1 : y \notin S_y$

statistically. In a screening procedure, there can be two types of errors as in the statistical hypothesis testing; a conforming item may be screened out (type I error) or a nonconforming item may be accepted for the next process or shipment (type II error). To achieve the objective of screening, these errors must be minimized. However, it is not possible to reduce these two errors at the same time and we need to find a reasonable screening procedure for practical use. For development of a procedure which attains the aimed result of screening, we employ some definitions similar to those in the statistical hypothesis testing.

Definition 1. *A screening region of size* α *is defined as the set* C_x *such that an item with the observed value of the screening variable belonging to this set is screened out (rejected) with the probability of type I misclassification error* $P(\underline{X} \in C_x \mid Y \in S_y) = \alpha$.

Definition 2. C_x^* *is called the best screening region of size* α *if*

i) $P(\underline{X} \in C_x^* \mid Y \in S_y) = \alpha$

ii) $P(\underline{X} \in C_x^* \mid Y \notin S_y) \geq P(\underline{X} \in C_x \mid Y \notin S_y)$ *for any other screening region* C_x *with* $P(\underline{X} \in C_x \mid Y \in S_y) = \alpha$.

Based on these definitions, we construct the following lemma and theorem which can be used to find the best screening region.

Lemma 1. *Let* $p(\underline{x} \mid Y \in S_y)$ *and* $p(\underline{x} \mid Y \notin S_y)$ *be the conditional probability density functions of* $\underline{x}$, *given* $Y \in S_y$ *and* $Y \notin S_y$, *respectively. Then* C_x^* *is the best screening region of size* α *if*

i) $\dfrac{p(\underline{x} \mid Y \in S_y)}{p(\underline{x} \mid Y \notin S_y)} \leq k$, *for all* $\underline{x} \in C_x^*$,

ii) $\dfrac{p(\underline{x} \mid Y \in S_y)}{p(\underline{x} \mid Y \notin S_y)} \leq k$, *for all* $\underline{x} \notin C_x^*$,

iii) $P(\underline{X} \in C_x^* \mid Y \in S_y) = \alpha$, *for some* $k > 0$.

See Appendix for proof of Lemma 1.

Theorem 1. *The best screening region of size is of the form*

$$C_x^* = \{x : P(Y \in S_y \mid x) \leq q\} \tag{3}$$

where $q(0 \leq q \leq 1)$ *is a number satisfying* $P(X \in C_x^* \mid Y \in S_y) = \alpha$.

Proof) From Lemmal, $C_x^* = \{x : p(x \mid Y \in S_y)/p(x \mid Y \notin S_y) \leq k\}$. But we have

$$\frac{p(x \mid Y \in S_y)}{p(x \mid Y \notin S_y)} = \frac{p(Y \in S_y \mid x)}{1 - p(Y \in S_y \mid x)} \times \frac{1-\gamma}{\gamma} \leq k,$$

which can be reduced to $P(Y \in S_y \mid x) \leq q$, where $q = (k\gamma)/(1-\gamma+k\gamma)$.

The conditional probability $P(Y \in S_y \mid x)$ is the probability that an item will be conforming given the observed value of the screening variable for this item is $X = x$. When all the parameters of the distribution of (X, Y) are known, $P(Y \in S_y \mid x)$ depends on x only. Thus, if we denote $P(Y \in S_y \mid x)$ by $h(x)$, the best screening region C_x^* is determined by $h(x)$ and α. If $x \notin C_x^*$, then $h(x)$ is the probability that an item accepted by screening based on C_x^* is conforming. Often, we may want the outgoing proportion of conforming items to be equal to or greater than a prespecified value δ after screening. In this case, we choose the screening region

$$C_x^* = \{x : h(x) < \delta\}, \text{ that is, } S_x = \{x : h(x) \geq \delta\}. \tag{4}$$

This, however, is not always possible since there may be cases where $\sup_x h(x) < \delta$. It will be discussed in Section 4.

3. Case with Dichotomous Performance Variable

Suppose that the performance variable Y is a Bernoulli random variable, that is, the item is conforming if $y = 0$ and nonconforming if $y = 1$, and its probability function is given by

$$P(Y=y)=(-\gamma)^{y}\gamma^{1-y},\quad y=0,1. \tag{5}$$

Assume that the screening variable X is continuous and its conditional distribution given $Y=i$ is $N(\mu_i, \sigma^2), i=0,1(\mu_0<\mu_1)$. Here, we consider normal model only and assume an equal variance for simplicity. We obtain $h(x)=P(Y=0 \mid x)$ as

$$h(x)=\frac{\gamma\exp\left[-\frac{(x-\mu_0)^2}{2\sigma^2}\right]}{\gamma\exp\left[-\frac{(x-\mu_0)^2}{2\sigma^2}\right]=(1-\gamma)\gamma\exp\left[-\frac{(x-\mu_1)^2}{2\sigma^2}\right]} \tag{6}$$

Let $\Phi(.)$ be the standard normal distribution function and $Z_\alpha=\Phi^{-1}(1-\alpha)$. Then, the best screening region of size is obtained as

$$C_x^{*}=\left\{x:\frac{\mu_0+\mu_1}{2}-\frac{\sigma^2}{\mu_1-\mu_0}\ln\frac{q(1-\gamma)}{\gamma(1-q)}\le x\right\}, \tag{7}$$

where $q=\dfrac{\frac{\gamma}{1-\gamma}\exp\left[\left(\frac{\mu_1-\mu_0}{\sigma^2}\right)\left\{\frac{\mu_0+\mu_1}{2}+Z_\alpha\sigma\right\}\right]}{1+\frac{\gamma}{1-\gamma}\exp\left[\left(\frac{\mu_1-\mu_0}{\sigma^2}\right)\left\{\frac{\mu_0+\mu_1}{2}+Z_\alpha\sigma\right\}\right]}$. By replacing q with δ in the best screening region of (7), the prespecified outgoing quality δ can be attained. In this case, the probability of type I misclassification error is obtained by

$$\alpha=1-\Phi\left(\frac{\mu_1-\mu_0}{2\sigma}-\frac{\sigma}{\mu_1-\mu_0}\ln\frac{\delta(1-\gamma)}{\gamma(1-\delta)}\right). \tag{8}$$

Example 1. Consider a nozzle that is incorporated into a fuel injection equipment. Its major quality characteristic (performance variable) is whether or not it sprays the fuel properly so that the injection equipment can function well. To observe the performance variable, the injection equipment may be tested after installing each nozzle. But this may require a time-consuming work and may be costly. Instead of observing the performance variable, the amount of air flow through the nozzle (screening variable) is measured in liters per minute. Assume that the conditional distribution of the screening variable given the performance variable is normal with distribution parameters $\mu_0=10$, $\mu_1=13$, and $\sigma=1$. Suppose that the proportion

of conforming items is γ=0.8 before screening and is desired to be δ=0.975 after screening. Using formula (7), we obtain the best screening region C_x^*=[10.74, ∞). Thus, any item whose observed value of the screening variable is greater than or equal to 10.74 is screened out (rejected).

4. Case with Continuous Performance Variable

Assume that both of the screening variable X and the performance variable Y are continuous and have a joint distribution $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ with positive correlation coefficient. The specification limit of Y may have one of three forms; i) $S_y=[L, \infty)$, ii) $S_y=(-\infty, U]$ and iii) $S_y=[L, U]$. Let $V=(X-\mu_x)/\sigma_x$, $W=(Y-\mu_y)/\sigma_y$, $w_1=(L-\mu_y)/\sigma_y$, $w_u=(U-\mu_y)/\sigma_y$ and S_w be the specification limit of W which corresponds to S_y. Then V and W have a joint distribution $N(0, 0, 1, 1, \rho)$. Since W, given $V=v$, is normally distributed with mean ρv and variance $(1-\rho^2)$, $h(v)$ is given by

$$
\begin{aligned}
h(v) &= 1-\Phi\left(\frac{w_1-\rho v}{\sqrt{1-\rho^2}}\right), && \text{if } S_w=[w_l, \infty)\\
&= \Phi\left(\frac{w_u-\rho v}{\sqrt{1-\rho^2}}\right), && \text{if } S_w=(-\infty, w_u]\\
&= \Phi\left(\frac{w_u-\rho v}{\sqrt{1-\rho^2}}\right)-\Phi\left(\frac{w_l-\rho v}{\sqrt{1-\rho^2}}\right), && \text{if } S_w=[w_l, w_u]. \qquad (9)
\end{aligned}
$$

Note that $h(v)$ is i) an increasing function of v when $S_w=[w_l, \infty)$, ii) a decreasing function of v when $S_w=(-\infty, w_u]$, and iii) a unimodal function of v with its maximum value

$$
h(v^*) = 2\Phi\left(\frac{w_u-w_l}{2\sqrt{1-\rho^2}}\right)-1 \qquad (10)
$$

at $v=v^*=(w_l+w_u)/(2\rho)$ when $S_w=[w_l, w_u]$. Thus, the best screening region is of the form $(-\infty, v_L]$ if $S_w=[w_l, \infty)$, or $[v_U, \infty)$ if $S_w=(-\infty, w_u]$, or $(-\infty, v_L]\cup[v_U, \infty)$ if $S_w=[w_l, w_u]$. Suppose that the proportion of conforming

items is desired to be greater than or equal to δ after screening. The screening region based on V is

$$\begin{aligned} C_v^* &= \{v: v \le v_l\}, && \text{if } S_w = [w_l, \infty), \\ &= \{v: v \ge v_u\}, && \text{if } S_w = (-\infty, w_u], \\ &= \{v: v \le v_l' \text{ or } v \ge v_u'\}, && \text{if } S_w = [w_l, w_u], \end{aligned} \tag{11}$$

where $v_l = (w_l - Z_\delta\sqrt{1-\rho^2})/\rho$, $v_u = (w_u + Z_\delta\sqrt{1-\rho^2})/\rho$, and v_l' and v_u' are two values of v satisfying $h(v) = \delta$. The type I misclassification error of the procedure is

$$\begin{aligned} \alpha &= \frac{1}{\gamma}\{\Phi(v_l) - \Psi(v_l, w_l; \rho)\}, && \text{if } S_w = [w_l, \infty), \\ &= \frac{1}{\gamma}\{\gamma - \Psi(v_l, w_u; \rho)\}, && \text{if } S_w = (-\infty, w_u], \\ &= \frac{1}{\gamma}\{\gamma - \Psi(v_u', w_u; \rho) + \Psi(v_u', w_l; \rho) + \Psi(v_l', w_u; \rho) - \Psi(v_l', w_l; \rho)\}, \\ & && \text{if } S_w = [w_l, w_u], \end{aligned} \tag{12}$$

where $\Psi(.,.;\rho)$ is the bivariate standard normal distribution function with correlation coefficient ρ. The screening region based on X is

$$\begin{aligned} C_v^* &= \{x: x \le x_l\}, && \text{if } S_y = [L, \infty), \\ &= \{x: x \ge x_u\}, && \text{if } S_y = (-\infty, U], \\ &= \{x: x \le x_l' \text{ or } x \ge x_u'\}, && \text{if } S_y = [L, U], \end{aligned} \tag{13}$$

where $x_l = \mu_x + \sigma v_l$, $x_u = \mu_x + \sigma v_u$, $x_l' = \mu_x + \sigma v_l'$ and $x_u' = \mu_x + \sigma v_u'$.

If $S_w = [w_l, w_u]$, the target proportion δ can be attained only when $h(v^*) \ge \delta$. In this case, $h(v^*)$ is the maximum proportion of conforming items attainable by screening. <Table 1> shows $h(v^*)$ for $\rho = 0.50(0.05)0.95$ and $\gamma = 0.80(0.05)0.95$.

< Table 1 > $h(v^*)$ for $\rho = 0.50(0.05)0.95$ and $\gamma = 0.80(0.05)0.95$

ρ γ	0.80	0.85	0.90	0.95
0.50	0.860	0.904	0.943	0.976
0.55	0.875	0.915	0.952	0.981
0.60	0.891	0.928	0.962	0.986
0.65	0.908	0.942	0.970	0.990
0.70	0.927	0.956	0.979	0.994
0.75	0.947	0.971	0.987	0.997
0.80	0.967	0.984	0.994	0.999
0.85	0.985	0.994	0.998	1.000
0.90	0.997	0.999	1.000	1.000
0.95	1.000	1.000	1.000	1.000

Example 2. Consider an electronic device whose major quality characteristic Y is the voltage at an internal point. The lower specification limit of the performance variable Y is 8 volts. Instead of measuring Y, we may use the voltage X at an external point which is easy to measure. X and Y are known to be approximately normally distributed with $\mu_x = 8$ volts, $\mu_y = 10$ volts, $\sigma_x = \sigma_y = 2$ volts, and $\rho = 0.90$. Suppose that the proportion of conforming items after screening is specified to be $\delta = 0.975$. Since $w_l = (8-10) / 2 = -1$, $S_w = [-1, \infty)$. Using (11), we obtain $C_v^* = (-\infty, -0.16]$ and thus, $C_v^* = (-\infty, 7.68]$. Every item with $x \leq 7.68$ is screened out.

5. Concluding Remarks

The screening procedure is reviewed as a statistical hypothesis testing procedure. Without assuming a specific probability model for the performance and screening variable, some principles are provided to obtain the best screening region. It provides a pretty general tool to analyze the screening problem.

The procedure may be applicable consistently regardless of the type of the performance variable. Attainability of the target outgoing quality of a screening procedure can be decided in advance. Cases of both dichotomous and continuous performance variables are studied assuming normal probability model with known parameters. Examples are also provided for illustration.

The result shows that each screening procedure may be properly converted to a corresponding test procedure. This may enable us to employ the existing theory of statistical testing in solving some screening problems.

Appendix

Proof of Lemma 1.

Let C_x^* be the screening region satisfying i), ii), and iii) of Lemma 1 and C_x be any screening region of size α. We must show that

$$P(\underline{X} \in C_x^* \mid Y \notin S_y) \geq P(\underline{X} \in C_x \mid Y \notin S_y)$$

to prove C_x^* is the best screening region of size α. We will prove only for the case where $p(\underline{x} \mid Y \in S_y)$ is continuous. The discrete case can be proved similarly. Denote the complements of C_x^* and C_x by C_x^{*c} and C_x^c, respectively. Then

$$P(\underline{X} \in C_x^* \mid Y \notin S_y) - P(\underline{X} \in C_x \mid Y \notin S_y) = \int_{C_x^*} p(\underline{x} \mid Y \notin S_y)\, d\underline{x} - \int_{C_x} p(\underline{x} \mid Y \notin S_y)\, d\underline{x}$$

$$= \int_{C_x^* \cap C_x^c} p(\underline{x} \mid Y \notin S_y)\, d\underline{x} + \int_{C_x^* \cap C_x} p(\underline{x} \mid Y \notin S_y)\, d\underline{x} - \int_{C_x \cap C_x^*} p(\underline{x} \mid Y \notin S_y)\, d\underline{x} - \int_{C_x \cap C_x^{*c}} p(\underline{x} \mid Y \notin S_y)\, d\underline{x}$$

$$= \int_{C_x^* \cap C_x^c} p(\underline{x} \mid Y \notin S_y)\, d\underline{x} - \int_{C_x \cap C_x^{*c}} p(\underline{x} \mid Y \notin S_y)\, d\underline{x}$$

$$\geq \int_{C_x^* \cap C_x^c} \frac{1}{k}\, p(\underline{x} \mid Y \in S_y)\, d\underline{x} - \int_{C_x \cap C_x^{*c}} \frac{1}{k}\, p(\underline{x} \mid Y \in S_y)\, d\underline{x}$$

$$= \frac{1}{k}\left[\int_{C_x^* \cap C_x^c} p(\underline{x} \mid Y \in S_y)\, d\underline{x} + \int_{C_x^* \cap C_x} p(\underline{x} \mid Y \in S_y)\, d\underline{x} - \int_{C_x \cap C_x^*} p(\underline{x} \mid Y \in S_y)\, d\underline{x} - \int_{C_x \cap C_x^{*c}} p(\underline{x} \mid Y \in S_y)\, d\underline{x} \right]$$

$$= \frac{1}{k}\left[\int_{C_x^*} p(\underline{x} \mid Y \in S_y)\, d\underline{x} - \int_{C_x} p(\underline{x} \mid Y \in S_y)\, d\underline{x} \right]$$

$$= \frac{1}{k}\,[\alpha - \alpha]$$

$$= 0.$$

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