⊠연구논문

Serially Correlated Process Monitoring Using Forward and Backward Prediction Errors from Linear Prediction Lattice Filter

Sungwoon Choi · Sanghoon Lee
Dept. of Industrial Engineering, Kyung Won University

Abstract

We propose an adaptive monitoring approach for serially correlated data. This algorithm uses the adaptive linear prediction lattice filter (ALPLF) which makes it compute process parameters in real time and recursively update their estimates. It involves computation of the forward and backward prediction errors. CUSUM control charts are applied to prediction errors simultaneously in both directions as an omnibus method for detecting changes in process parameters. Results of computer simulations demonstrate that the proposed adaptive monitoring approach has great potentials for real-time industrial applications, which vary frequently in their control environment.

1. Introduction

Statistical process control (SPC) techniques have been widely applied in industry for process improvement and for estimating parameters or monitoring variability of a given process. In the typical application of the SPC charts, it is traditionally assumed that the observations are uncorrelated. However, this assumption is generally invalid in many industrial processes. The presence of autocorrelation in the processes gives a profound effect on control charts developed for identically and independently distributed (IID) observations, thereby resulting in increasing the frequency of false signals. Approaches for dealing with autocorrelated data in the

SPC environment have been developed by fitting an appropriate time series model to the observations and applying control charts to the stream of residuals from this model. These methods are based on the assumption that the residuals are white noise when there is no special cause in the process and can then utilize any of the conventional tools for SPC. Alwan and Roberts [1] proposed two separate charts to monitor the process: common-cause chart and special-cause chart. The common cause chart is a plot of fitted values using the autoregressive integrated moving average (ARIMA) model and provides information on the systematic variation of the process. The special cause chart is to apply a conventional Shewhart chart to the residuals. English et al. [2] proposed a similar approach using the forecasted errors from Kalman filtering to monitor a continuous flow process.

Lee and Choi [3] presented a control chart scheme for continuous flow processes, which employes the adaptive linear prediction lattice filter (ALPLF) [4]. The filter is designed for adaptive prediction of time series as an on-line process by computing the predictor coefficients from the correlation sequence of the process. It involves computation of the forward and backward prediction errors. The scale CUSUM procedure, which was introduced by Hawkins [5] for controlling the variance for IID normal processes, was used to detect changes in serial correlation parameters. The control chart was applied to the forward prediction errors which are recursively obtained by the ALPLF. This study extends the ALPLF scheme to use the prediction errors in both directions for SPC and investigates performance of the adaptive chart for various cases of the change in the process mean.

2. Adaptive Linear Prediction Lattice Filter

A serially-correlated processes $\{y(t)\}$ can be modeled with a discrete AR zero-mean time series of order n if the process mean is known:

$$y(t) = -\sum_{i=1}^{n} A_i^{(n)} y(t-i) + \varepsilon(t)$$

where $A_i^{(n)}$is the ith AR coefficient and $\varepsilon(t) \sim N(0, \sigma^2)$. The forward linear predictor and prediction error of the pth order are then written:

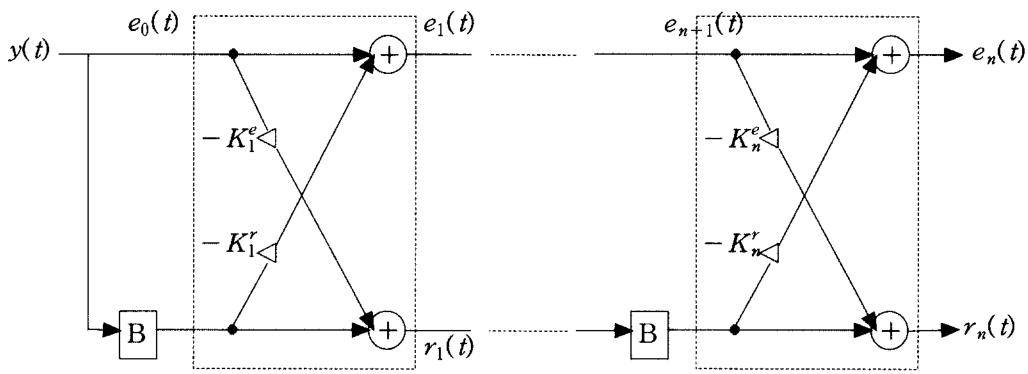
$$\hat{y}_p(t) = -\sum_{i=1}^{p} A_i^{(p)} y(t-i)$$

$$e_p(t) = y(t) - \hat{y}_p(t)$$

and the backward predictors and prediction error are also defined:

$$\hat{y}_p(t-p-1) = -\sum_{i=1}^{p} B_{(p-i+1)}^{(p)} \, y(t-i)$$

$$r_p(t-1) = y(t-p-1) - \hat{y}_p(t-p-1)$$



< Figure 1 > Linear Prediction Lattice Filter

where $1 \le p \le n$. The coefficients of the optimal predictor are uniquely determined by the second order statistics of the process, the autocorrelation coefficients $\{R_i\}$ where $R_i = E[y(t)y(t-i)]$. Given the second order statistics by the forward and backward mean-square errors, that is, $R_p^e = E[e_p^2(t)]$ and $R_p^r = E[r_p^2(t-1)]$, the predictor coefficients can be efficiently computed from the correlation sequence of the process using the Levinson algorithm [4]. <Figure 1> outlines the ALPLF algorithm. The transfer function of the lattice filter in Figure 1 is determined by the values of the parameters $\{K_p\}$ which are referred to as reflection coefficients:

$$K_{p+1}^{e} = E[e_{p(t)} r_p(t-1)] / R_p^{e}$$

$$K_{p+1}^{r} = E[e_{p(t)} r_p(t-1)] / R_p^{r}$$

The reflection coefficients are determined by the autocorrelation sequence $\{R_i\}$ as a cross correlation of the forward and backward prediction errors.

3. Adaptive Process Monitoring

Fitting of the AR model makes it possible by study of its residuals to isolate the departures from control that may be traceable to special causes. If the adaptive filter estimates the appropriate model, the sequence of prediction errors from the filter will then behave as white noise. Therefore, conventional control charts can be applied to the stream of the prediction errors.

If $z_t \sim N(0,1)$, then $|z_t|^{1/2}$ closely approximates an $N(.822, .349^2)$ distribution, and that changes in the scale of z_t affect the location of $|z_t|^{1/2}$. Based on this fact, CUSUM control of location and scale parameters was suggested by Hawkins [5]. This study extended the Hawkins' CUSUM scheme to use the forward and backward prediction errors generated from ALPLF. Let e_t and r_t be the forward and backward errors from the ALPLF at time t respectively, the CUSUM can be operated for a given reference value k by forming the cumulative sums as the followings:

$$w_t^e = (|e_t^e|^{1/2} - 0.822)/0.349, \quad w_t^r = (|r_t^r|^{1/2} - 0.822)/0.349$$

$$L_{et}^{+} = \max\{0, L_{e,t-1}^{+} + e_t - k\}, \quad L_{et}^{-} = \min\{0, L_{e,t-1}^{-} + e_t + k\}$$

$$L_{rt}^{+} = \max\{0, L_{r,t-1}^{+} + r_t - k\}, \quad L_{rt}^{-} = \min\{0, L_{r,t-1}^{-} + r_t + k\}.$$

$$S_{et}^{+} = \max\{0, S_{e,t-1}^{+} + w_t^e - k\}, \quad S_{et}^{-} = \min\{0, S_{e,t-1}^{-} + w_t^e + k\}$$

$$S_{rt}^{+} = \max\{0, S_{r,t-1}^{+} + w_t^r - k\}, \quad S_{rt}^{-} = \min\{0, S_{r,t-1}^{-} + w_t^r + k\}$$

The CUSUMs $\{L_t^+\}$ and $\{L_t^-\}$ test for shifts in process mean in the upward and downward directions, respectively. The CUSUMs $\{S_t^+\}$ and $\{S_t^-\}$ test for shifts in variability in the upward and downward directions, respectively. For a sequence of prediction errors, $\{e_t, r_t\}$, given control limits h_m for mean shift and h_v, for variance shift, the control chart signals an out-of-control condition against shifts in location and scale parameters of the process when

$$MCX_e = \max\{L_{et}^+, L_{et}^-\}, \quad MCX_r = \max\{L_{rt}^+, L_{rt}^-\}$$

$$VCX_e = \max\{S_{et}^+, S_{et}^-\}, \quad VCX_r = \max\{S_{rt}^+, S_{rt}^-\}$$

$$MCX_B = \max\{MCX_e, MCX_r\} > h_m$$

$$VCX_B = \max\{VCX_e, VCX_r\} > h_v$$

The VCX_e and VCX_r are the scale CUSUM (SCUSUM) charts, which were proposed by Hawkins [6], using the forward prediction errors and the backward prediction errors of the ALPLF respectively.

4. Experiments

In this section, we considered a sequence of IID standard normal data for the target process and all the results were obtained by 10,000 Monte Carlo simulation runs. For each case, we simulated 10,000 independent data series by Monte Carlo method and each average run length (ARL) was computed by averaging the run lengths of the applied control scheme for these 10,000 series. Serially correlated data were generated using the first order AR model described in Section 2. We chose the reference values, k = 0.5 for $\{L_t\}$ and k = 0.25 for $\{S_t\}$ as used in Hawkins [5][6]. First, we applied the SCUSUM chart to IID standard normal data with no perturbation in error variance, that is, $\sigma^2 = 1$. The SCUSUM has an ARL of approximately 200 when using and h = 6.8460. Using h = 0.6840 and k = 0.25, we applied the SCUSUM scheme to the simulated sequences of IID normal data with various perturbations of process. The ARL results are shown in <Table 1>.

< Table 1 > ARLs according to perturbation in error variance σ^2

Perturbation in σ^2	SCUSUM	VCX_e			VCX_r		
	ρ = 0	ρ = 0	ρ = 0.5	ρ = -0.5	ρ = 0	ρ = 0.5	ρ = -0.5
-50%	16	16	16	16	15	15	15
-40%	24	24	24	24	22	23	23
-30%	39	40	40	40	37	38	38
-20%	77	77	77	77	74	75	76
-10%	162	162	161	162	158	162	162
0	200	203	202	202	202	202	202
10%	111	115	114	115	117	115	114
20%	58	61	61	61	62	61	62
30%	37	39	38	38	40	39	40
40%	26	27	27	27	28	28	28
50%	20	21	21	21	22	22	22

<Table 1> also contains the results when the SCUSUM scheme was applied to the prediction errors generated from the ALPLF of the first order for six simulation series with the serial correlations of ρ = 0, 0.5 and -0.5 respectively. After initiating the ALPLF for 200 steps by initially setting the first order coefficient to an arbitrary value of 0.1, we started to apply the chart to the prediction errors. The results of adaptive approach for serially correlated data are almost same with the direct application of the SCUSUM for IID normal data. It indicates that the prediction errors from the ALPLF are IID normal. Next, the SCUSUM was applied to the prediction errors from the ALPLF for the simulated sequences, which were changed in noise variance and serial correlation after 200 time steps from IID standard normal data. <Table 2> shows the ARLs for detecting a change in variability of the process. The chart schemes VCX_e and VCX_B used h_v = 6.814 and 6.959 corresponding to in-control ARL = 200 respectively. When the data change to the positive relation of serially-correlated processes or reversely, to the negative, the detecting performance of the adaptive scheme is almost invariable if the absolute levels are same. The VCX_B scheme using the prediction errors in both directions shows better performance in ARL for relatively small changes in process variation than one using only the forward prediction error. Next, the MCX scheme was applied to the simulated data series which change from IID normal data to serially-correlated data with various shifts in process mean at the 201th time step. <Table 3> illustrates the ARL results

with k = 0.5. We used the control limits h_m = 4.224 and 6.662 for MCX_e and MCX_B, which result in ARL = 200 for in-control processes. The ARL performance of the MCX scheme using the forward prediction error is better than when using the prediction errors in both directions. The control limit of MCX_B for in-control ARL = 200 has a relatively larger value than that of MCX_e. It makes the chart give slower signal when the process mean is shifted from the target.

< Table 2 > ARLs of signaling out-of-control from change-point when IID standard normal data change to serially-correlated processes with various perturbation in σ^2 after 200 time steps.

Perturbation in σ^2	VCX_e						VCX_B					
	ρ= 0.75	ρ= 0.5	ρ= 0.25	ρ= -0.25	ρ= -0.5	ρ= -0.75	ρ= 0.75	ρ= 0.5	ρ= 0.25	ρ= -0.25	ρ= -0.5	ρ= -0.75
-50%	31	21	17	17	20	30	30	20	16	15	19	28
-40%	44	33	26	25	32	43	42	31	24	23	29	41
-30%	51	57	44	42	55	53	48	54	41	39	50	50
-20%	44	96	84	82	95	50	43	89	80	76	85	48
-10%	34	123	160	161	132	38	33	109	149	144	113	37
0	24	83	170	180	95	27	25	76	156	159	82	27
10%	19	45	91	97	51	21	19	44	88	91	48	21
20%	16	30	50	53	33	17	16	30	50	51	31	17
30%	14	22	33	34	23	14	14	22	33	34	23	14
40%	12	18	24	25	19	12	12	18	24	25	18	12
50%	11	14	19	19	15	11	11	15	19	19	15	11

< Table 3 > ARL results of signaling out-of-control from change-point when IID normal data change into serially-correlated processes with various shifts in mean.

Mean Shift	MCX_e			MCX_B		
	ρ= 0.75	ρ= 0.5	ρ= 0.25	ρ= 0.75	ρ= 0.5	ρ= 0.25
0.5	12	18	24	18	26	29
1.0	11	10	9.4	15	14	13
1.5	8.8	6.8	5.6	12	9.1	8.0
2.0	7.4	5.0	4.0	9.4	6.8	5.7
2.5	6.2	4.1	3.2	7.9	5.5	4.5
3.0	5.4	3.4	2.7	6.8	4.6	3.8

5. Conclusions

This paper presented an adaptive monitoring approach for the detection of changes in variability in IID normal processes by serial correlation with mean shift and noise variance perturbation. This scheme employs the adaptive linear prediction lattice filter and CUSUM control charts. Although the lattice filter is conceptually easy, its implementation is quite simple and the algorithm is computationally efficient to eliminate the systematic pattern and generate white noise prediction error. In our experiments, the proposed scheme demonstrates a good prospect of monitoring both common and special causes simultaneously.

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