한국퍼지 및 지능 시스템학회 논문지 1998, Vol. 8, No. 4. (1998) 73~77

Fuzzy Pairwise Almost Open Mapping and P*-fuzzy Almost Open Mapping

Kuo-Douk Park* and Young-Bin Im**

**Department of Mathematics, Dongguk University*
***Department of Mathematics, Seonam University*

1. Introduction

Azad[3] introduced a fuzzy semiopen set and a fuzzy regular open set, and investigated a fuzzy semicontinuous mapping, a fuzzy almost continuous mapping and a fuzzy weakly continuous mapping on fuzzy topological spaces. Mukherjee and Sinha[5] studied a fuzzy almost open mapping on fuzzy topological spaces. In[7], Park, Im and Soung further studied a fuzzy almost continuous mapping and a fuzzy weakly continuous mapping on fuzzy topological spaces. Kandil[4] introduced a fuzzy bitopological space as a natural generalization of a fuzzy topological space. Park, Han and Im[6] studied a (T_i, T_j)-fuzzy regular open set and a fuzzy pairwise almost continuous mapping on fuzzy bitopological spaces. In [2], Allam and Zahran introduced a P*-fuzzy weakly continuous mapping and a P*-fuzzy almost open mapping on fuzzy bitopological spaces.

In this paper, we further investigate some properties of a fuzzy pairwise almost open mapping and a P*-fuzzy almost open mapping on fuzzy bitopological spaces. And we show that a fuzzy pairwise open mapping is a fuzzy pairwise almost open mapping, but the converse is not true in general. We also show that a fuzzy pairwise almost open mapping and a P*-fuzzy almost open mapping do not have any specific relations.

2. Preliminaries

A systems (X, T_1, T_2) consisting of a set X with two fuzzy topologies T_1 and T_2 on X is called a **fuzzy bitopological space** [*fbts*, for shorts][4]. Throughout this paper, induces i, j take values in $\{1,2\}$ with $i \neq j$.

A mapping $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is said to be **fuzzy pairwise continuous** [*fpc*] (respectively **fuzzy pairwise open** [*fp open*]) if and only if the induced mappings $f: (X, T_k) \to (Y, T_k^*)$ $(k=1,2)$ are fuzzy continuous (respectively fuzzy open) [1].

Definition 2.1 [2,8,9] Let μ be any fuzzy set of a *fbts* X. Then μ is called;

(i) a (T_i, T_j)**-fuzzy semiopen** [(T_i, T_j)-*fso*] **set** of X if there exists a T_i-fuzzy open [T_i-*fo*] set ν of X such that $\nu \leq \mu \leq T_j\text{-Cl}\nu$,

(ii) a (T_i, T_j)**-fuzzy semiclosed** [(T_i, T_j)-*fsc*] **set** of X if there exists a T_i-fuzzy closed [T_i-*fc*] set ν of X such that $T_j\text{-Int}\ \nu \leq \mu \leq \nu$,

(iii) a (T_i, T_j)**-fuzzy preopen** [(T_i, T_j)-*fpo*] **set** of X if $\mu \leq T_i\text{-Int}(T_j\text{-Cl}\mu)$,

(iv) a (T_i, T_j)**-fuzzy preclosed** [(T_i, T_j)-*fpc*] **set** of X if $T_i\text{-Cl}(T_j\text{-}\mu) \leq \mu$,

(v) a (T_i, T_j)**-fuzzy regular open** [(T_i, T_j)-*fro*] **set** of X if $\mu = T_i\text{-Int}(T_j\text{-Cl}\mu)$,

(vi) a (T_i, T_j)**-fuzzy regular closed** [(T_i, T_j)-*frc*] **set** of X if $\mu = T_i\text{-Cl}(T_j\text{-Int}\mu)$.

Lemma 2.2 [9] Let μ be any fuzzy set of a *fbts* X. Then the following statements are equivalent:

(i) μ is a (T_i, T_j)-*fsc* set.
(ii) μ^c is a (T_i, T_j)-*fso* set.
(iii) $T_j\text{-Int}(T_i\text{-Cl}\mu) \leq \mu$.
(iv) $T_j\text{-Cl}(T_i\text{-Int}\mu^c) \geq \mu^c$.

Definition 2.3 [9] A mapping $f: (X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is said to be **fuzzy pairwise semicontinuous** [*fpsc*], if $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X for each T_i^*-*fo* set ν of Y.

From the above definition we know that every *fpc* is *fpsc*, but the converse is not true in general [9].

Definition 2.4. [6] A mapping $f: (X, T_1, T_2) \to (Y,$

T_1^*, T_2^*) is said to be **fuzzy pairwise almost continuous** [*fpac*], if $f^{-1}(v)$ is a T_i-*fo* set of X for each (T_i^*, T_j^*)-*fro* set v of Y.

Clearly every *fpc* is *fpac*. But the converse need not be true [6].

Definition 2.5 [2] A mapping f: $(X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is said to be **P*-fuzzy weakly continuous** [*P*FW-continuous*], if $f^{-1}(v) \leq T_i\text{-Int}(f^{-1}(T_j^*\text{-Cl}v))$ for each T_i^*-*fo* set v of Y.

Obviously every *fpac* is *P*FW-continuous*. But the converse need not be true by the following example.

Example 2.6 Let λ, μ and v be fuzzy sets of I, defined as follows; for each $x \in I$,

$$\lambda(x) = x,\ \mu(x) = 1 - x \quad \text{and} \quad v(x) = \begin{cases} x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \frac{1}{2} < x \leq 1. \end{cases}$$

Consider fuzzy topologies $T_1 = \{0_I, \lambda \vee \mu, \lambda \wedge \mu, 1_I\}$, $T_2 = \{0_I, v, \lambda \vee \mu, \lambda \wedge \mu, 1_I\}$, $T_1^* = \{0_I, \lambda, \lambda \wedge \mu, 1_I\}$ and $T_2^* = \{0_I, \lambda, v, \lambda \wedge \mu, 1_I\}$ on I and the identity mapping i_I: $(I, T_1, T_2) \to (I, T_1^*, T_2^*)$ defined by $i_I(x) = x$ for each $x \in I$. Then i_I is *P*FW-continuous*, but i_I is not *fpac*. □

3. Fuzzy Pairwise Almost Open Mapping

Definition 3.1 Let f: $(X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be a mapping. Then f is called;

(i) a **fuzzy pairwise almost open** [*fpa open*] if $f(\mu)$ is a T_i^*-*fo* set of Y for each (T_i, T_j)-*fro* set μ of X,

(ii) a **fuzzy pairwise almost closed** [*fpa closed*] if $f(\mu)$ is a T_i^*-*fc* set of Y for each (T_i, T_j)-*frc* set μ of X.

Clearly every *fp open* is *fpa open*. But the converse need not be true by the following example.

Example 3.2 Let λ, μ and v be fuzzy sets of I, defined in Example 2.6. Consider fuzzy topologies $T_1 = \{0_I, v, \lambda \vee \mu, \lambda \wedge \mu, 1_I\}$, $T_2 = \{0_I, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1_I\}$, $T_1^* = \{0_I, \lambda, \mu, \lambda \wedge \mu, \lambda \vee \mu, 1_I\}$ and $T_2^* = \{0_I, \lambda \vee \mu, \lambda \wedge \mu, 1_I\}$ on I and the identity mapping i_I: $(I, T_1, T_2) \to (I, T_1^*, T_2^*)$ defined by $i_I(x) = x$ for each $x \in I$. Then i_I is *fpa open*, but i_I is not *fp open*. □

Theorem 3.3 A mapping f: $(X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is *fpa open* if and only if for each T_i-*fo* set μ of X, $f(\mu) \leq T_i^*\text{-Int}(f[T_i\text{-Int}(T_j\text{-Cl}\mu)])$.

Proof. Let μ be a T_i-*fo* set of X. Then $\mu \leq T_i\text{-Int}(T_j\text{-Cl}\mu)$ and so $f(\mu) \leq f[T_i\text{-Int}(T_j\text{-Cl}\mu)]$. Since f is *fpa open* and $T_i\text{-Int}(T_j\text{-Cl}\mu)$ is (T_i, T_j)-*fro* set of X, $f[T_i\text{-Int}(T_j\text{-Cl}\mu)]$ is a T_i^*-*fo* set of Y. Hence we have

$$f(\mu) \leq f[T_i\text{-Int}(T_j\text{-Cl}\mu)] = T_i^*\text{-Int}(f[T_i\text{-Int}(T_j\text{-Cl}\mu)]).$$

Conversely, let μ be a (T_i, T_j)-*fro* set of X. Then $\mu = T_i\text{-Int}(T_j\text{-Cl}\mu)$ and by hypothesis,

$$f(\mu) \leq T_i^*\text{-Int}(f[T_i\text{-Int}(T_j\text{-Cl}\mu)]) = T_i\text{-Int}\mu.$$

Hence $f(\mu)$ is a T_i^*-*fo* set of Y. Therefore f is *fpa open*. □

Theorem 3.4 A mapping f: $(X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ is *fpa open* if and only if for each (T_j, T_i)-*fsc* set μ of X, $f(T_i\text{-Int}\mu) \leq T_i^*\text{-Int}(f(\mu))$.

Proof. Let f be *fpa open* and let μ be a (T_j, T_i)-*fsc* set μ of X. Since μ is a (T_j, T_i)-*fsc* set of X, $T_i\text{-Int}(T_j\text{-Cl}\mu)$ is a (T_i, T_j)-*fro* set of X and hence $f[T_i\text{-Int}(T_j\text{-Cl}\mu)]$ is a T_i^*-*fo* set of Y. Then

$$T_i\text{-Int}\mu = T_i\text{-Int}(T_j\text{-Cl}\mu) \leq \mu.$$

Now, $f(T_i\text{-Int}\mu) = f[T_i\text{-Int}(T_j\text{-Cl}\mu)] \leq f(\mu)$, and hence

$$\begin{aligned} f(T_i\text{-Int}\mu) &= f[T_i\text{-Int}(T_j\text{-Cl}\mu)] \\ &= T_i^*\text{-Int}(f[T_i\text{-Int}(T_j\text{-Cl}\mu)]) \\ &\leq T_i^*\text{-Int}(f(\mu)). \end{aligned}$$

Therefore $f(T_i\text{-Int}\mu) \leq T_i^*\text{-Int}(f(\mu))$.

Conversely, let μ be a (T_i, T_j)-*fro* set of X. Then μ is a (T_j, T_i)-*fsc* set of X and hence $f(T_i\text{-Int}\mu) \leq T_i^*\text{-Int}(f(\mu))$. Since μ is a T_i-*fo* set of X, $\mu = T_i\text{-Int}\mu$ and thus $f(\mu) = f(T_i\text{-Int}\mu) \leq T_i^*\text{-Int}(f(\mu))$. Hence $f(\mu)$ is a T_i^*-*fo* set of Y. Therefore f is *fpa open*. □

Theorem 3.5 Let f: $(X, T_1, T_2) \to (Y, T_1^*, T_2^*)$ be both *fpsc* and *fpa open*. Then $f^{-1}(v)$ is a (T_j, T_i)-*fsc* set of X for each (T_j^*, T_i^*)-*fsc* set v of Y.

Proof. Let f be both *fpsc* and *fpa open* and let v be a (T_j^*, T_i^*)-*fsc* set of Y. Then $T_i^*\text{-Int}(T_j^*\text{-Cl}v) \leq v$.

Since f is a *fpsc* mapping, $f^{-1}(T_j^*\text{-Cl}\nu)$ is a (T_j, T_i)-*fsc* set of X. Hence

$$T_i\text{-Int}(T_j\text{-Cl}(f^{-1}[T_j^*\text{-Cl}\nu])) \leq T_i\text{-Int}(f^{-1}[T_j^*\text{-Cl}\nu]).$$

And since f is *fpa open*,

$$\begin{aligned} f[T_i\text{-Int}(f^{-1}[T_j^*\text{-Cl}\nu])] &\leq T_i^*\text{-Int}(f[f^{-1}[T_j^*\text{-Cl}\nu]]) \\ &\leq T_i^*\text{-Int}(T_j^*\text{-Cl}\nu) \\ &\leq \nu, \end{aligned}$$

and thus $T_i\text{-Int}(f^{-1}[T_j^*\text{-Cl}\nu]) \leq f^{-1}(\nu)$. Now,

$$\begin{aligned} T_i\text{-Int}(T_j\text{-Cl}(f^{-1}(\nu))) &\leq T_i\text{-Int}(T_j\text{-Cl}(f^{-1}[T_i^*\text{-Cl}\nu])) \\ &\leq T_i\text{-Int}(f^{-1}[T_j^*\text{-Cl}\nu]) \\ &\leq f^{-1}(\nu). \end{aligned}$$

Therefore $f^{-1}(\nu)$ is a (T_j, T_i)-*fsc* set of X. □

Theorem 3.6 A mapping f: $(X, T_1, T_2) \rightarrow (Y, T_1^*, T_2^*)$ is *fpa open* if and only if for any fuzzy set ν of Y and any (T_i, T_j)-*frc* set μ of X with $f^{-1}(\nu) \leq \mu$, there exists a T_i^*-*fc* set λ of Y containing ν such that $f^{-1}(\lambda) \leq \mu$.

Proof. Let f be *fpa open* and let ν be any fuzzy set of Y, and let μ be a (T_i, T_j)-*frc* set of X such that $f^{-1}(\nu) \leq \mu$. Let $\lambda = (f(\mu^c))^c$. Then λ is a T_i^*-*fc* set of Y and $\nu \leq \lambda$. Therefore

$$\begin{aligned} f^{-1}(\lambda) &= (f^{-1}(f(\mu^c)))^c \\ &\leq (\mu^c)^c = \mu. \end{aligned}$$

Conversely, let δ be a (T_i, T_j)-*fro* set of X. Let $\nu = (f(\delta))^c$ and $\mu = \delta^c$. Then

$$\begin{aligned} f^{-1}(\nu) &= f^{-1}((f(\delta))^c) \\ &= (f^{-1}(f(\delta)))^c \\ &\leq \delta^c = \mu. \end{aligned}$$

Hence there exists a T_i^*-*fc* set λ of Y such that $\nu \leq \lambda$ and $f^{-1}(\lambda) \leq \mu = \delta^c$. Thus $\delta \leq (f^{-1}(\lambda))^c = f^{-1}(\lambda^c)$ and $f(\delta) \leq f(f^{-1}(\lambda^c)) \leq \lambda^c$. Since $\nu \leq \lambda$, $f(\delta) = \nu^c \geq \lambda^c$. Therefore $f(\delta) = \lambda^c$ is T_i^*-*fo* set of Y and consequently, f is *fpa open*. □

Theorem 3.7 Let f: $(X, T_1, T_2) \rightarrow (Y, T_1^*, T_2^*)$ be bijection. Then the following statements are equivalent:

(i) f is *fpa open*.

(ii) f is *fpa closed*.

(iii) f^{-1} is *fpac*.

Proof. (i) implies (ii): Let ν be a (T_i, T_j)-*frc* set of X. Then ν^c is a (T_i, T_j)-*fro* set of X, and $f(\nu^c)$ is a T_i^*-*fo* set of Y. Since f is bijection, $(f(\nu))^c$ is a T_i^*-*fo* set of Y. Hence $f(\nu)$ is a T_i^*-*fc* set of Y.

(ii) implies (iii): Let ν be a (T_i, T_j)-*frc* set of X. Then $f(\nu)$ is a T_i^*-*fc* set of Y. Since $f(\nu) = (f^{-1})^{-1}(\nu)$ and f^{-1}: $(Y, T_1^*, T_2^*) \rightarrow (X, T_1, T_2)$, we find that the inverse image of (T_i, T_j)-*frc* set ν of X is a T_i^*-*fc* set of Y. Hence f^{-1} is *fpac*.

(iii) implies (i): Let μ be a (T_i, T_j)-*fro* set of X. Since f^{-1} is *fpac*, $(f^{-1})^{-1}(\mu) = f(\mu)$ is a T_i^*-*fo* set of Y. Hence f is *fpa open*. □

Theorem 3.8 Let f: $(X, T_1, T_2) \rightarrow (Y, T_1^*, T_2^*)$ and g: $(Y, T_1^*, T_2^*) \rightarrow (Z, G_1, G_2)$ be two surjection. If f is *fpac* and if $g \circ f$ is *fp open* (respectively *fp closed*), then g is *fpa open* (respectively *fpa closed*).

Proof. Let f be *fpac* and let $g \circ f$ be *fp open* (respectively *fp closed*), and let μ be a (T_i, T_j)-*fro* (respectively (T_i, T_j)-*frc*) set of Y. Then $f^{-1}(\mu)$ is a T_i-*fo* (respectively T_i-*fc*) set of X. Now $g \circ f$ is *fp open* (respectively *fp closed*), and $(g \circ f)(f^{-1}(\mu))$ is a G_i-*fo* (respectively G_i-*fc*) set of Z. Since f is surjection,

$$(g \circ f)(f^{-1}(\mu)) = g(f(f^{-1}(\mu))) = g(\mu).$$

Thus $g(\mu)$ is a G_i-*fo* (respectively G_i-*fc*) set of Z. Therefore g is *fpa open* (respectively *fpa closed*). □

4. P*-fuzzy Almost Open Mapping

Definition 4.1 [2] A mapping f: $(X, T_1, T_2) \rightarrow (Y, T_1^*, T_2^*)$ is said to be **P*-fuzzy almost open** [*P*FA-open*], if $f^{-1}(T_j^*\text{-Cl}\nu) \leq T_j\text{-Cl}(f^{-1}(\nu))$ for each T_i^*-*fo* set ν of Y.

The following examples show that *fpa open* and *P*FA-open* do not have any specific relations.

Example 4.2 Let μ_1, μ_2 and μ_3 be fuzzy sets of $X = \{a, b, c\}$, defined as follows;

$\mu_1(a)=0.3$, $\mu_1(b)=0.2$, $\mu_1(c)=0.2$,
$\mu_2(a)=0.4$, $\mu_2(b)=0.3$, $\mu_2(c)=0.3$,
$\mu_3(a)=0.2$, $\mu_3(b)=0.1$, $\mu_3(c)=0.1$.

Consider fuzzy topologies $T_1 = \{0_X, \mu_1, 1_X\}$, $T_2 = \{0_X, \mu_1, \mu_3, 1_X\}$, $T_1^* = \{0_X, \mu_1, \mu_3^c, 1_X\}$ and $T_2^* = \{0_X, \mu_1, \mu_2, 1_X\}$. Then the identity mapping i_X: $(X, T_1, T_2) \rightarrow (X, T_1^*,$

T_2^*) is *fpa open*, but i_X is not *P*FA-open*. □

Example 4.3 Let μ_1, μ_2 and μ_3 be fuzzy sets of $X=\{a, b, c\}$, defined in Example 4.2. Consider fuzzy topologies $T_1=\{0_X, \mu_1, 1_X\}$, $T_2=\{0_X, \mu_1, \mu_3, 1_X\}$, $T_1^*=\{0_X, \mu_2, 1_X\}$ and $T_2^*=\{0_X, \mu_2^c, 1_X\}$. Then the identity mapping i_X: $(X, T_1, T_2)\rightarrow(X, T_1^*, T_2^*)$ is *P*FA-open*, but i_X is not *fpa open*. □

Theorem 4.4 Let f: $(X, T_1, T_2)\rightarrow(Y, T_1^*, T_2^*)$ be both *fpsc* and *P*FA-open*. Then $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X for each (T_i^*, T_j^*)-*fso* set ν of Y.

Proof. Let ν be a (T_i^*, T_j^*)-*fso* set of Y. Then there exists T_i^*-*fo* set μ of Y such that $\mu\le\nu\le T_j^*\text{-Cl}\mu$. Since f is *P*FA-open*,

$$f^{-1}(\mu)\le f^{-1}(\nu)\le f^{-1}(T_j^*\text{-Cl}\mu)\le T_j\text{-Cl}(f^{-1}(\mu)).$$

And since f is *fpsc*, $f^{-1}(\mu)$ is (T_i, T_j)-*fso* set of X. Hence there exists a T_i-*fo* set ω of X such that $\omega\le f^{-1}(\mu)\le T_j\text{-Cl}\omega$. Now,

$$T_j\text{-Cl}\omega\le T_j\text{-Cl}(f^{-1}(\mu))\le T_j\text{-Cl}\omega,$$

and thus $\omega\le f^{-1}(\mu)\le f^{-1}(\nu)\le T_j\text{-Cl}(f^{-1}(\mu))\le T_j\text{-Cl}\omega$.

Therefore $f^{-1}(\nu)$ is a (T_i, T_j)-*fso* set of X. □

Theorem 4.5 Let f: $(X, T_1, T_2)\rightarrow(Y, T_1^*, T_2^*)$ be a mapping. Then the following statements are equivalent:

(i) f is *P*FA-open*.

(ii) $f^{-1}(T_j^*\text{-Cl}(T_i^*\text{-Int}\nu))\le T_j\text{-Cl}(f^{-1}(\nu))$ for any fuzzy set ν of Y.

(iii) $f(T_j\text{-Int}\mu)\le T_j^*\text{-Int}(T_i^*\text{-Cl}(f(\mu)))$ for any fuzzy set μ of X.

(iv) $f(\mu)\le T_j^*\text{-Int}(T_i^*\text{-Cl}(f(\mu)))$ for any T_j-*fo* set μ of X.

(v) For any fuzzy set ν of Y and T_j-*fc* set μ of X containing $f^{-1}(\nu)$, there exists a (T_j^*, T_i^*)-*fpc* set λ of Y containing ν such that $f^{-1}(\lambda)\le\mu$.

Proof. (i) implies (ii): Let ν be any fuzzy set of Y. Then

$$f^{-1}(T_j^*\text{-Cl}(T_i^*\text{-Int }\nu))\le T_j\text{-Cl}(f^{-1}(T_i^*\text{-Int }\nu))$$
$$\le T_j\text{-Cl}(f^{-1}(\nu)).$$

(ii) implies (i): Obvious.

(ii) implies (iii): Let μ be any fuzzy set of X. Then

$$f^{-1}[T_j^*\text{-Cl}(T_i^*\text{-Int }((f(\mu))^c))]\le T_j\text{-Cl}(f^{-1}[(f(\mu))^c]).$$

Now,

$$f^{-1}[T_j^*\text{-Cl}((T_i^*\text{-Cl}(f(\mu)))^c)]\le T_j\text{-Cl}(f^{-1}[(f(\mu))^c]),$$

and hence $f^{-1}[(T_j^*\text{-Int }(T_i^*\text{-Cl}(f(\mu))))^c]\le(T_j\text{-Int }\mu)^c$. Thus

$$(f(T_j\text{-Int }\mu))^c\ge(T_j^*\text{-Int }(T_i^*\text{-Cl}(f(\mu))))^c,$$

and therefore, $f(T_j\text{-Int }\mu)\le T_j^*\text{-Int}(T_i^*\text{-Cl }(f(\mu)))$.

(iii) implies (iv): Straight forward.

(iv) implies (v): Let ν be any fuzzy set of Y and let μ be a T_j-*fc* set of X such that $f^{-1}(\nu)\le\mu$. Then μ^c is a T_j-*fo* set of X and

$$f(\mu^c)\le T_j^*\text{-Int }(T_i^*\text{-Cl }(f(\mu^c))).$$

Thus $f(\mu^c)$ is a (T_j^*, T_i^*)-*fpo* set of Y. If $\lambda=(f(\mu^c))^c$, then λ is a (T_j^*, T_i^*)-*fpc* set and $\nu\le\lambda$. Hence $f^{-1}(\lambda)=f^{-1}((f(\mu^c))^c)\le\mu$.

(v) implies (ii): Let ν be any fuzzy set of Y. Then $\mu=T_j\text{-Cl}(f^{-1}(\nu))$ is a T_j-*fc* set of X and $f^{-1}(\nu)\le\mu$. Therefore there exists a (T_j^*, T_i^*)-*fpc* set λ of Y containing ν such that $f^{-1}(\lambda)\le\mu$ and $f^{-1}(\lambda)\le T_j\text{-Cl}(f^{-1}(\nu))$. Thus $\nu\le\lambda$. Since λ is a (T_j^*, T_i^*)-*fpc* set of Y,

$$f^{-1}[T_j^*\text{-Cl }(T_i^*\text{-Int }\nu)]\le f^{-1}[T_j^*\text{-Cl }(T_i^*\text{-Int }\lambda)]\le f^{-1}(\lambda).$$

Hence $f^{-1}[T_j^*\text{-Cl }(T_i^*\text{-Int }\nu)]\le T_j^*\text{-Cl }(f^{-1}(\nu))$.

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박 거 덕(Kuo-Douk Park) 정회원
제 7권 제 3호 참조
현재 : 동국대학교 수학과

임 영 빈(Young-Bin Im) 정회원
제 7권 제 3호 참조
현재 : 서남대학교 수학과