Journal of the Korean
Statistical Society
Vol. 27, No. 1, 1998

# Durbin-Watson Type Unit Root Test Statistics [†]

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## Abstract

In the analysis of time series it is an important issue to determine whether a time series under study is stationary. For the test of the stationarity of the time series the Dickey-Fuller (DF) type tests have been mainly used. In this paper, we consider the regular unit root tests and seasonal unit root tests based on the generalized Durbin-Watson (DW) statistics when the errors are independent. The limiting distributions of the proposed DW-type test statistics are the functionals of standard Brownian motions. We also obtain the finite distributions and powers of the DW-type test statistics and compare the performances with the DF-type tests. It is observed that the DW-type test statistics have good behaviors against the DF-type test statistics especially in the nonzero (seasonal) mean model.

**KEY WORDS** : Durbin-Watson statistics; Regular unit root; Seasonal unit root; Brownian motions; Imhof routine.

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[†]The study is supported by the academic research fund(BSRI-97-1418) of Ministry of Education, Republic of Korea.

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## 1. INTRODUCTION

Let $\{Y_t\}$ be a univariate time series under consideration satisfying

$$Y_t = \mathbf{x}_t'\beta + u_t, \tag{1.1}$$

where $\{\mathbf{x}_t\}$ is a deterministic sequence. If $\{u_t\}$ satisfies

$$u_t = \phi u_{t-1} + \epsilon_t, \tag{1.2}$$

where $\epsilon_t$'s are independently and identically distributed(i.i.d.) random variables with mean zero and variance $\sigma_\epsilon^2$, the models (1.1) and (1.2) are jointly represented by

$$Y_t = \phi Y_{t-1} + (\mathbf{x}_t - \phi \mathbf{x}_{t-1})'\beta + \epsilon_t. \tag{1.3}$$

The traditional Dickey-Fuller (DF) regular unit root test procedures test whether $\phi = 1$ or $|\phi| < 1$ in (1.3). For the seasonal unit root tests, the model of $\{u_t\}$ is given by

$$u_t = \Phi u_{t-s} + \epsilon_t.$$

We test whether $\Phi = 1$ or $|\Phi| < 1$.

Since Durbin and Watson (1950, 1951) proposed the Durbin-Watson (DW) statistic, it has long been used to detect the first order autocorrelation of the disturbances in the regression model (1.1). The test has been generalized for the autocorrelation of any order by Vinod (1973). The generalized DW statistics are

$$d_k = \frac{\sum_{t=k+1}^{n}(\hat{u}_t - \hat{u}_{t-k})^2}{\sum_{t=1}^{n}\hat{u}_t^2}, \quad k = 1, \ldots, n-1,$$

where $\hat{u}_t$ are the residuals of the regression model (1.1). Dickey and Fuller (1981) considered various statistics including the DW statistic for the test of unit roots. Sargan and Bhargava (1983), Bhargava (1986), and Nabeya and Tanaka (1990) developed DW-type test statistics for the unit root tests.

In this paper we develop the unit root tests based on the generalized DW statistics. The advantages of the DW-type test statistics against DF-type test statistics for unit root tests are that the former is easier to calculate the exact distributions and powers and is readily extended to the general model and wide class of tests.

## 2. REGULAR UNIT ROOT TESTS

Imhof (1961) provided a method to obtain the distribution of the quadratic form

$$Q = \sum_{r=1}^{m} \lambda_r \chi^2_{h_r;\delta_r^2}, \tag{2.1}$$

where $\lambda_r$'s are distinct and non-zero, and $\chi^2_{h_r;\delta_r^2}$ are independent $\chi^2$-variables with $h_r$ degrees of freedom and the non-centrality parameter $\delta_r^2$.

Let $\{Y_t\}$ satisfy the following univariate time series model

$$Y_t = u_t, \quad u_t = \phi u_{t-1} + \epsilon_t, \tag{2.2}$$

or

$$Y_t = \mu + u_t, \quad u_t = \phi u_{t-1} + \epsilon_t, \tag{2.3}$$

where $\epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$. For the test of

$$H_0 \; : \; \phi = 1 \;\; vs \quad H_1 \; : \; |\phi| < 1,$$

we define DW-type test statistics for models (2.2) and (2.3), respectively,

$$R_1 = \frac{\sum_{t=1}^n (Y_t - Y_{t-1})^2}{\sum_{t=1}^n Y_t^2}, \quad R_2 = \frac{\sum_{t=2}^n (Y_t - Y_{t-1})^2}{\sum_{t=1}^n (Y_t - \bar{Y})^2},$$

where $Y_0 = 0$ and $\bar{Y} = n^-1 \sum_{t=1}^n Y_t$. Note that $R_2$ is independent of $Y_1$ under $H_0$.

The limiting distributions of $nR_1$ and $nR_2$ under $H_0$ are given by Tanaka (1996).

**Theorem 2.1 (Tanaka, 1996)** Under $H_0 : \phi = 1$, $nR_1$ and $nR_2$ have the limiting distributions, respectively,

$$nR_1 \;\Rightarrow\; \frac{1}{\int W^2(r)dr},$$
$$nR_2 \;\Rightarrow\; \frac{1}{\int\{W(r) - \int W(r)\}^2 dr},$$

where $W(r)$ is a standard Brownian motion.

Since both the denominators and the numerators of $R_1$ and $R_2$ are $O_p(n)$ under $H_1$, these statistics are $O_p(1)$. More precisely, $R_1$ and $R_2$ converge

**Table 1.** Distributions of $nR_1$ and $nR_2$ for the regular unit root tests[a]

| | Probability of a smaller value | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 |
| | | | | | $nR_1$ | | | | | |
| 25 | .374 | .473 | .595 | .806 | 1.232 | 7.825 | 11.579 | 15.363 | 19.104 | 23.909 |
| 50 | .366 | .470 | .599 | .820 | 1.268 | 8.221 | 12.287 | 16.471 | 20.693 | 26.267 |
| 100 | .362 | .469 | .602 | .828 | 1.286 | 8.431 | 12.666 | 17.073 | 21.572 | 27.593 |
| 200 | .361 | .469 | .603 | .832 | 1.296 | 8.539 | 12.864 | 17.386 | 22.034 | 28.296 |
| 500 | .359 | .469 | .603 | .835 | 1.302 | 8.605 | 12.985 | 17.581 | 22.319 | 28.721 |
| | | | | | $nR_2$ | | | | | |
| 25 | 1.421 | 1.761 | 2.164 | 2.814 | 3.975 | 14.354 | 18.884 | 23.143 | 27.162 | 32.139 |
| 50 | 1.381 | 1.740 | 2.165 | 2.846 | 4.059 | 15.176 | 20.231 | 25.115 | 29.846 | 35.889 |
| 100 | 1.362 | 1.731 | 2.166 | 2.862 | 4.102 | 15.614 | 20.962 | 26.195 | 31.340 | 38.013 |
| 200 | 1.354 | 1.726 | 2.167 | 2.871 | 4.123 | 15.841 | 21.341 | 26.763 | 32.128 | 39.141 |
| 500 | 1.348 | 1.724 | 2.167 | 2.876 | 4.136 | 15.979 | 21.575 | 27.114 | 32.615 | 39.844 |

[a] The entries are obtained using the Imhof routine.

**Table 2.** Powers of the DW-type[a] and the DF-type[b] tests for the regular unit root at the 5% level[c]

| | zero mean | | | | | | | nonzero mean | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $\phi =$ | .995 | .99 | .98 | .95 | .90 | .80 | $\phi =$ | .995 | .99 | .98 | .95 | .90 | .80 |
| 25 | $nR_1$ | .053 | .056 | .063 | .087 | .143 | .326 | $nR_2$ | .051 | .053 | .056 | .068 | .095 | .187 |
| | $\hat{\rho}$ | .053 | .056 | .063 | .088 | .145 | .333 | $\hat{\rho}_\mu$ | .051 | .053 | .056 | .067 | .092 | .176 |
| 50 | $nR_1$ | .056 | .063 | .078 | .141 | .313 | .765 | $nR_2$ | .053 | .056 | .064 | .095 | .184 | .501 |
| | $\hat{\rho}$ | .056 | .063 | .079 | .144 | .323 | .781 | $\hat{\rho}_\mu$ | .053 | .056 | .063 | .092 | .174 | .471 |
| 100 | $nR_1$ | .063 | .078 | .116 | .307 | .749 | .999 | $nR_2$ | .056 | .064 | .083 | .182 | .488 | .966 |
| | $\hat{\rho}$ | .063 | .079 | .119 | .319 | .768 | .999 | $\hat{\rho}_\mu$ | .056 | .063 | .081 | .173 | .459 | .954 |
| 200 | $nR_1$ | .078 | .116 | .229 | .740 | .999 | 1.00 | $nR_2$ | .064 | .083 | .141 | .481 | .962 | 1.00 |
| | $\hat{\rho}$ | .079 | .119 | .238 | .762 | .999 | 1.00 | $\hat{\rho}_\mu$ | .063 | .081 | .135 | .453 | .949 | 1.00 |
| 500 | $nR_1$ | .139 | .301 | .735 | 1.00 | 1.00 | 1.00 | $nR_2$ | .095 | .181 | .477 | .996 | 1.00 | 1.00 |
| | $\hat{\rho}$ | .143 | .315 | .758 | 1.00 | 1.00 | 1.00 | $\hat{\rho}_\mu$ | .092 | .172 | .450 | .994 | 1.00 | 1.00 |

[a] $nR_1$ and $nR_2$. [b] $\hat{\rho}$ and $\hat{\rho}_\mu$. [c] The entries are obtained using the Imhof routine.

to $2(1-\phi)$ almost surely. These properties of $nR_1$ and $nR_2$ under $H_0$ and $H_1$ can be used to construct the test statistics for a regular unit root. After some algebra we may rewrite $R_1$ and $R_2$ as quadratic forms of (2.1) under the assumption that $\epsilon_t$'s follow normal distribution, so we can obtain the exact distributions and powers of $nR_1$ and $nR_2$ using the Imhof routine.

Table 1 shows the exact distributions of $nR_1$ and $nR_2$ numerically calculated by the Imhof routine for various sample sizes under $H_0 : \phi = 1$ for models (2.2) and (2.3). Without loss of generality, for the generation of $\{Y_t\}$, we assume that $Y_0 = 0$ for the zero mean model (2.2) and $Y_1 = 0$ for the nonzero mean model (2.3). Table 1 shows the asymmetric nature of the distributions of $nR_1$ and $nR_2$.

We obtain the exact powers of the DW-type test statistics and the DF-type test statistics for the zero mean model (2.2) and the nonzero mean model (2.3) in Table 2. For the power comparisons we consider the DF-type test statistics $\hat{\rho} = n(\hat{\phi} - 1)$ and $\hat{\rho}_\mu = n(\hat{\phi}_\mu - 1)$ due to Dickey and Fuller (1979), where $\hat{\phi}$ and $\hat{\phi}_\mu$ are the OLS estimates of $\phi$ for models (2.2) and (2.3), respectively, and $nR_1$ and $nR_2$ for the DW-type test statistics. $Y_1 \sim N(0,1)$ is assumed for the zero mean model and $Y_1 \sim N(0, 1/(1-\phi^2))$ is assumed for the nonzero mean model. The significant level is 0.05 and the sample sizes considered are $n = 25, 50, 100, 200$, and 500. The distributions and powers of the DF-type test statistics $\hat{\rho}$ and $\hat{\rho}_\mu$ are also obtained using the Imhof routine. The critical values for the DW-type test statistics are obtained from Table 1. The obtained critical values of the DF-type test statistics are $-7.371$, $-7.692$, $-7.862$, $-7.949$, and $-8.003$ for $\hat{\rho}$, and $-12.487$, $-13.252$, $-13.663$, $-13.877$, and $-14.006$ for $\hat{\rho}_\mu$, respectively.

For the simulation of the zero mean model we assume $Y_1 \sim N(0,1)$ instead of $Y_1 \sim N(0, 1/(1-\phi^2))$. This is because, when we use $Y_1 \sim N(0, 1/(1-\phi^2))$ for $|\phi| < 1$, the values of $Y_t$ are often too much different from zero, which makes one to use the nonzero mean model instead the zero mean model. Pantula *et al.* (1994) studied the effects of the initial values and observed that $Y_1 \sim N(0,1)$ and $Y_1 \sim N(0, 1/(1-\phi^2))$ yield different powers.

It is observed that the DF-type test performs better than the DW-type test for the zero mean model (2.2). But for the nonzero mean model (2.3) the DW-type test performs better than the DF-type test.

It should be remarked that though the OLS estimates are used for the comparison in this paper other types of the DF-type test statistics based on the GLS or ML estimates can also be used for the comparison as indicated in Pantula *et al.* (1994).

## 3. SEASONAL UNIT ROOT TESTS

Let $\{Y_t\}$ satisfy the following univariate seasonal time series model with seasonal period $s$,

$$Y_t = u_t, \quad u_t = \Phi u_{t-s} + \epsilon_t, \tag{3.1}$$

or

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + u_t, \quad u_t = \Phi u_{t-s} + \epsilon_t, \tag{3.2}$$

where $\epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$ and $\delta_{jt}$ is the indicator function such that $\delta_{jt} = 1$ if $t$ is in the $j$-th season and 0 otherwise. For the test of

$$H_0 \ : \ \Phi = 1 \ \ vs \quad H_1 \ : \ |\Phi| < 1,$$

we define DW-type test statistics for (3.1) and (3.2), respectively,

$$S_1 = \frac{\sum_{t=1}^n (Y_t - Y_{t-s})^2}{\sum_{t=1}^n Y_t^2}, \quad S_2 = \frac{\sum_{t=s+1}^n (Y_t - Y_{t-s})^2}{\sum_{t=1}^n (Y_t - \sum_{j=1}^s \bar{Y}_j \delta_{jt})^2},$$

where $Y_{-s+1} = \cdots = Y_0 = 0$ and $\bar{Y}_j = m^{-1} \sum_{l=1}^m Y_{(l-1)s+j}$'s are the OLS estimates of $\beta_j$'s which are the means of $Y_t$ for the $j$-th seasons. We may assume that $n = ms$ for simplicity, where $m$ is a positive integer. Note that $S_2$ does not depend on $(Y_1, \cdots, Y_s)$ under $H_0$.

Then we can obtain the limiting distributions of $nS_1$ and $nS_2$ as follows.

**Theorem 3.1** Under $H_0 : \Phi = 1$, $nS_1$ and $nS_2$ have the limiting distributions, respectively,

$$nS_1 \ \Rightarrow \ \frac{s^2}{\sum_{j=1}^s \int W_j^2(r) dr},$$
$$nS_2 \ \Rightarrow \ \frac{s^2}{\sum_{j=1}^s \int \{W_j(r) - \int W_j(r)\}^2 dr},$$

where $W_j(r)$ are the mutually independent standard Brownian motions.

The proof can be easily obtained from Theorem 2.1 and the independency of $\epsilon_t$. Under $H_1$, $S_1$ and $S_2$ are $O_p(1)$, respectively, thus we can construct the test statistics for seasonal unit root. After some algebra we may rewrite $S_1$ and $S_2$ as quadratic forms of (2.1) under the assumption that $\epsilon_t$'s follow normal distribution, so we can obtain the finite exact distributions and powers of $nS_1$ and $nS_2$ using the Imhof routine.

Table 3 and 4 show the exact distributions of $nS_1$ and $nS_2$ for $s = 4$ and $s = 12$ numerically calculated by the Imhof routine for various sample sizes under $H_0 : \Phi = 1$ for the zero mean seasonal model (3.1) and the nonzero seasonal mean model (3.2). Without loss of generality, for the generation of $\{Y_t\}$, we assume that $Y_{-s+1} = \cdots = Y_0 = 0$ for model (3.1) and $Y_1 = \cdots = Y_s = 0$ for model (3.2). Table 3 and 4 show the asymmetric nature of distributions of $nS_1$ and $nS_2$. It is observed that the percentiles of the distributions of $nS_1$ and $nS_2$ for $s = 12$ are larger than those for $s = 4$.

**Table 3.** Distributions of $nS_1$ and $nS_2$ for the seasonal unit root tests and$s = 4^a$

| | Probability of a smaller value | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 |
| | | | | | $nS_1$ | | | | | |
| 40 | 2.794 | 3.198 | 3.626 | 4.241 | 5.211 | 13.185 | 17.020 | 20.854 | 24.655 | 29.596 |
| 60 | 2.783 | 3.214 | 3.672 | 4.326 | 5.359 | 13.847 | 17.954 | 22.090 | 26.226 | 31.664 |
| 80 | 2.779 | 3.225 | 3.697 | 4.371 | 5.436 | 14.198 | 18.453 | 22.753 | 27.072 | 32.778 |
| 100 | 2.777 | 3.231 | 3.712 | 4.400 | 5.484 | 14.416 | 18.763 | 23.167 | 27.602 | 33.477 |
| 200 | 2.774 | 3.246 | 3.744 | 4.458 | 5.583 | 14.868 | 19.408 | 24.034 | 28.716 | 34.967 |
| 400 | 2.773 | 3.253 | 3.761 | 4.488 | 5.634 | 15.102 | 19.745 | 24.487 | 29.301 | 35.746 |
| | | | | | $nS_2$ | | | | | |
| 40 | 10.173 | 11.393 | 12.641 | 14.349 | 16.875 | 32.406 | 38.065 | 43.175 | 47.888 | 53.636 |
| 60 | 10.039 | 11.364 | 12.714 | 14.558 | 17.283 | 34.157 | 40.432 | 46.174 | 51.541 | 58.192 |
| 80 | 9.985 | 11.359 | 12.759 | 14.672 | 17.497 | 35.086 | 41.699 | 47.795 | 53.528 | 60.692 |
| 100 | 9.954 | 11.360 | 12.790 | 14.743 | 17.630 | 35.663 | 42.490 | 48.806 | 54.778 | 62.269 |
| 200 | 9.902 | 11.366 | 12.857 | 14.892 | 17.902 | 36.861 | 44.140 | 50.939 | 57.413 | 65.610 |
| 400 | 9.881 | 11.374 | 12.893 | 14.970 | 18.043 | 37.484 | 45.004 | 52.059 | 58.804 | 67.395 |

[a] The entries are obtained using the Imhof routine.

**Table 4.** Distributions of $nS_1$ and $nS_2$ for the seasonal unit root tests and$s = 12^a$

| | Probability of a smaller value | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 |
| | | | | | $nS_1$ | | | | | |
| 60 | 11.435 | 12.381 | 13.307 | 14.526 | 16.254 | 26.472 | 30.415 | 34.168 | 37.814 | 42.513 |
| 120 | 11.771 | 12.905 | 14.012 | 15.466 | 17.524 | 29.665 | 34.364 | 38.851 | 43.228 | 48.905 |
| 180 | 11.906 | 13.108 | 14.282 | 15.822 | 18.001 | 30.875 | 35.870 | 40.649 | 45.320 | 51.389 |
| 240 | 11.980 | 13.216 | 14.424 | 16.008 | 18.251 | 31.512 | 36.664 | 41.600 | 46.428 | 52.719 |
| 300 | 12.026 | 13.282 | 14.511 | 16.123 | 18.405 | 31.905 | 37.155 | 42.188 | 47.112 | 53.538 |
| 600 | 12.119 | 13.420 | 14.692 | 16.360 | 18.723 | 32.717 | 38.171 | 43.406 | 48.536 | 55.243 |
| 1200 | 12.168 | 13.492 | 14.785 | 16.482 | 18.886 | 33.136 | 38.697 | 44.037 | 49.274 | 56.129 |
| | | | | | $nS_2$ | | | | | |
| 60 | 40.258 | 42.653 | 44.938 | 47.856 | 51.823 | 71.878 | 78.399 | 84.131 | 89.330 | 95.597 |
| 120 | 40.529 | 43.654 | 46.613 | 50.369 | 55.453 | 81.118 | 89.568 | 97.088 | 103.991 | 112.448 |
| 180 | 40.773 | 44.132 | 47.317 | 51.360 | 56.835 | 84.600 | 93.819 | 102.066 | 109.677 | 119.047 |
| 240 | 40.919 | 44.396 | 47.696 | 51.885 | 57.560 | 86.432 | 96.064 | 104.702 | 112.697 | 122.574 |
| 300 | 41.010 | 44.564 | 47.932 | 52.210 | 58.007 | 87.562 | 97.452 | 106.336 | 114.575 | 124.774 |
| 600 | 41.217 | 44.917 | 48.425 | 52.881 | 58.928 | 89.897 | 100.326 | 109.729 | 118.476 | 129.340 |
| 1200 | 41.333 | 45.102 | 48.680 | 53.228 | 59.403 | 91.103 | 101.815 | 111.493 | 120.510 | 131.726 |

[a] The entries are obtained using the Imhof routine.

Table 5 and 6 compare the exact powers of the DW-type test statistics, $nS_1$ and $nS_2$, and the DF-type test statistics, $\hat{\rho}_s = n(\hat{\Phi} - 1)$ and $\hat{\rho}_{\mu s} = n(\hat{\Phi}_\mu - 1)$ due to Dickey *et al.* (1984), where $\hat{\Phi}$ and $\hat{\Phi}_\mu$ are the OLS estimates of $\Phi$ for models (3.1) and (3.2) for various sample sizes. As in the regular unit root tests case, we assume that for $1 \leq j \leq s$, $Y_j \sim i.i.d.\ N(0,1)$ for the zero mean model and that for $1 \leq j \leq s$, $Y_j \sim i.i.d.\ N(0, 1/(1-\Phi^2))$ for the nonzero seasonal mean model. The significant level is 0.05 and the sample sizes con-

**Table 5.** Powers of the DW-type[a] and the DF-type[b] tests for the seasonal unit root at the 5% level for $s = 4$[c]

| | zero mean | | | | | | | nonzero seasonal mean | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $\Phi =$ | .995 | .99 | .98 | .95 | .90 | .80 | $\Phi =$ | .995 | .99 | .98 | .95 | .90 | .80 |
| 40 | $nS_1$ | .054 | .058 | .067 | .101 | .187 | .465 | $nS_2$ | .052 | .053 | .057 | .070 | .098 | .188 |
| | $\hat{\rho}_s$ | .054 | .059 | .068 | .105 | .200 | .500 | $\hat{\rho}_{\mu s}$ | .052 | .053 | .057 | .068 | .093 | .170 |
| 60 | $nS_1$ | .056 | .062 | .077 | .138 | .305 | .754 | $nS_2$ | .053 | .055 | .061 | .084 | .139 | .336 |
| | $\hat{\rho}_s$ | .056 | .063 | .080 | .148 | .335 | .797 | $\hat{\rho}_{\mu s}$ | .052 | .055 | .060 | .080 | .128 | .298 |
| 80 | $nS_1$ | .058 | .067 | .088 | .183 | .448 | .927 | $nS_2$ | .054 | .057 | .066 | .100 | .192 | .523 |
| | $\hat{\rho}_s$ | .059 | .069 | .092 | .200 | .495 | .950 | $\hat{\rho}_{\mu s}$ | .053 | .057 | .064 | .095 | .174 | .464 |
| 100 | $nS_1$ | .060 | .072 | .100 | .237 | .598 | .987 | $nS_2$ | .054 | .059 | .071 | .119 | .259 | .708 |
| | $\hat{\rho}_s$ | .061 | .074 | .106 | .262 | .656 | .992 | $\hat{\rho}_{\mu s}$ | .054 | .059 | .069 | .111 | .231 | .641 |
| 200 | $nS_1$ | .072 | .100 | .181 | .588 | .984 | 1.00 | $nS_2$ | .060 | .071 | .101 | .260 | .701 | .999 |
| | $\hat{\rho}_s$ | .074 | .106 | .200 | .652 | .992 | 1.00 | $\hat{\rho}_{\mu s}$ | .059 | .069 | .095 | .232 | .636 | .996 |
| 400 | $nS_1$ | .100 | .180 | .432 | .983 | 1.00 | 1.00 | $nS_2$ | .071 | .101 | .194 | .698 | .999 | 1.00 |
| | $\hat{\rho}_s$ | .106 | .200 | .490 | .992 | 1.00 | 1.00 | $\hat{\rho}_{\mu s}$ | .069 | .096 | .176 | .633 | .996 | 1.00 |

[a] $nS_1$ and $nS_2$. [b] $\hat{\rho}_s$ and $\hat{\rho}_{\mu s}$. [c] The entries are obtained using the Imhof routine.

**Table 6.** Powers of the DW-type[a] and the DF-type[b] tests for the seasonal unit root at the 5% level for $s = 12$[c]

| | zero mean | | | | | | | nonzero seasonal mean | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $\Phi =$ | .995 | .99 | .98 | .95 | .90 | .80 | $\Phi =$ | .995 | .99 | .98 | .95 | .90 | .80 |
| 60 | $nS_1$ | .054 | .058 | .068 | .105 | .199 | .501 | $nS_2$ | .052 | .053 | .057 | .069 | .094 | .167 |
| | $\hat{\rho}_s$ | .055 | .059 | .070 | .112 | .220 | .553 | $\hat{\rho}_{\mu s}$ | .052 | .053 | .056 | .068 | .090 | .155 |
| 120 | $nS_1$ | .059 | .068 | .091 | .198 | .491 | .955 | $nS_2$ | .054 | .058 | .066 | .099 | .184 | .475 |
| | $\hat{\rho}_s$ | .060 | .071 | .099 | .227 | .567 | .976 | $\hat{\rho}_{\mu s}$ | .053 | .057 | .065 | .094 | .166 | .413 |
| 180 | $nS_1$ | .063 | .079 | .120 | .327 | .789 | 1.00 | $nS_2$ | .056 | .062 | .077 | .140 | .320 | .812 |
| | $\hat{\rho}_s$ | .065 | .084 | .134 | .389 | .864 | 1.00 | $\hat{\rho}_{\mu s}$ | .055 | .061 | .074 | .128 | .278 | .732 |
| 240 | $nS_1$ | .068 | .091 | .155 | .484 | .949 | 1.00 | $nS_2$ | .058 | .067 | .089 | .191 | .492 | .969 |
| | $\hat{\rho}_s$ | .071 | .099 | .178 | .572 | .978 | 1.00 | $\hat{\rho}_{\mu s}$ | .057 | .065 | .084 | .170 | .423 | .932 |
| 300 | $nS_1$ | .074 | .105 | .196 | .643 | .993 | 1.00 | $nS_2$ | .060 | .072 | .102 | .254 | .670 | .998 |
| | $\hat{\rho}_s$ | .078 | .116 | .231 | .741 | .998 | 1.00 | $\hat{\rho}_{\mu s}$ | .059 | .070 | .095 | .221 | .584 | .991 |
| 600 | $nS_1$ | .105 | .195 | .478 | .993 | 1.00 | 1.00 | $nS_2$ | .072 | .103 | .194 | .674 | .998 | 1.00 |
| | $\hat{\rho}_s$ | .117 | .232 | .575 | .998 | 1.00 | 1.00 | $\hat{\rho}_{\mu s}$ | .070 | .096 | .172 | .587 | .991 | 1.00 |

[a] $nS_1$ and $nS_2$. [b] $\hat{\rho}_s$ and $\hat{\rho}_{\mu s}$. [c] The entries are obtained using the Imhof routine.

sidered are $n = 40, 60, 80, 100, 200$, and $400$ for $s = 4$ and $n = 60, 120, 180, 240$, $300$, and $600$ for $s = 12$. The distributions and powers of the DF-type test statistics are also obtained using the Imhof routine. The critical values for the DW-type test statistics are obtained from Table 3 and 4. The obtained critical values of the DF-type test statistics are $-8.719$, $-8.829$, $-8.887$, $-8.923$, $-8.997$, and $-9.036$ for $\hat{\rho}_4$, $-11.681$, $-11.609$, $-11.595$, $-11.590$, $-11.587$, and $-11.583$ for $\hat{\rho}_{12}$, $-25.255$, $-26.059$, $-26.485$, $-26.747$, $-27.286$, and $-27.562$ for $\hat{\rho}_{\mu 4}$, and $-54.494$, $-56.561$, $-57.387$, $-57.815$, $-58.078$, and $-58.609$ for $\hat{\rho}_{\mu 12}$, respectively.

From the power comparisons of two types of test statistics, we conclude that in the zero seasonal mean model (3.1) the DF-type test statistics perform better than the DW-type statistics, and in the nonzero seasonal mean model (3.2) the DW-type statistics are more powerful than the DF-type test statistics.

## 4. CONCLUSIONS

We develop the DW-type unit root test statistics for regular unit root tests and also for seasonal unit root tests as well. The exact finite distributions and powers are obtained using the Imhof routine.

Power comparisons with the traditional DF-type test statistics show that in the zero (seasonal) mean models the DF-type tests perform better than the DW-type tests, but in the nonzero (seasonal) mean models the relations are reversed.

## REFERENCES


(1) Bhargava, A. (1986). On the theory of testing for unit roots in observed time series, *Review of Economic Studies*, **53**, 309-384.

(2) Dickey, D. A. and Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *Journal of the American Statistical Association*, **74**, 427-431.

(3) Dickey, D. A. and Fuller, W. A. (1981). Likelihood Ratio Test Statistics for Autoregressive Time Series with a Unit Root, *Econometrica*, **49**, 1057-1072.

(4) Dickey, D. A., Hasza, D. P. and Fuller, W. A. (1984). Testing for Unit Roots in Seasonal Time Series, *Journal of the American Statistical Association*, **79**, 355-367.

(5) Durbin, J. and Watson, G. S. (1950). Testing for serial correlation in least squares regression I, *Biometrika*, **37**, 409-428.

(6) Durbin, J. and Watson, G. S. (1951). Testing for serial correlation in least squares regression II, *Biometrika*, **38**, 159-178.

(7) Imhof, J. P. (1961). Computing the distribution of quadratic forms in normal variables, *Biometrika*, **48**, 419-426.

(8) Nabeya, S., and Tanaka, K. (1990). Limiting power of unit-root tests in time series regression, *Journal of Econometrics*, **46**, 247-271.

(9) Pantula, S. G., Gonzalez-Farias, G., and Fuller, W. A. (1994). A comparison of the unit-root test criteria, *Journal of Business and Economic Statistics*, **12**, 449-459.

(10) Sargan, J. D. and Bhargava, A. (1983). Testing residuals from least squares regression for being generated by a Gaussian random work, *Econometrica*, **51**, 153-174.

(11) Tanaka, K. (1996). *Time series analysis*, Wiley, New York.

(12) Vinod, H. D. (1973). Generalization of the Durbin-Watson statistic for higher order autoregressive processes, *Communication in Statistics*, **2**, 115-144.