콘크리트 슬래브의 미진동 제어

Micro-vibration Control in Concrete Slabs

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국문요약

본 연구는 허용진동환경을 만족하기 위한 콘크리트 슬래브의 미진동해석 및 제어기법에 관한 것이다. 본 연구에서 제시된 방법은 임의 두 점간의 전달함수와 가속도로부터 미지의 가진력을 산정하고 수치해석 및 구조실험의 검증을 통하여 콘크리트 슬래브의 미진동해석 및 제어를 실시하였다. 미진동제어 설계를 위한 구조물의 고유진동수는 25~30Hz 이상이 되는 것이 바람직하며, 콘크리트 구조물에서의 감쇠가 3%~5%인 경우 가진력 진동성분의 1.5배 이상 되는 것이 바람직함을 밝혔다.

주요어 : 미진동해석 및 제어기법, 실험적모드 해석법, 구조 동특성 변경, 하중응답해석

ABSTRACT

This study is to develop a technique for micro-vibration analysis and control of concrete slabs to fulfil the vibration criteria for working environments. The proposed technique is for determining the unknown forces from accelerance of two concerned points and the micro-vibration analysis and control of concrete slabs are then validated by numerical model and structural tests. And it is recommended that the natural frequency of structures for micro-vibration control design should be above 25 Hz~30 Hz, and 1.5 times forcing frequency in case of 3~5 % structural damping ratio of concrete structures.

Key words : *micro-vibration analysis and control technique, experimental modal analysis, structural dynamics modification, forced response simulation*

1. Introduction

Even though the initial study on micro-vibration was originated from high technology industries and high precision measurement fields, it is now extended into wide range of industrial and technology areas, and the demand for the micro-vibration control is increasing as the high technology industry is heading into high precision manufacturing.(1) Recently, more efforts are being put into the development of anti-vibration or vibration-control techniques for the vibration sensitive equipment which require stringent micro-vibration environments.

Generally, high precision equipment or vibration sensitive equipment are installed in the same general area with some equipment which induce vibration. It means that vibration control system is required, and in the present study concrete slabs are utilized in order to change system characteristics and to isolate one system from the other.

On the other hand, dynamic characteristics of the structural system need to be investigated clearly since it is an important element together with analysis of external excitation

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sources in establishing vibration control techniques. However, numerical analysis alone is not sufficient enough in investigating accurately the dynamic characteristics of the system and the resulting micro-vibration control technique tends to be less economical and less efficient. Until now, micro-vibration control technique was solely based on the impulse responses without considering the relationship between the input and output of the excited structure, which made it impossible to predict accurate responses by the actual excitation forces. Therefore, in most cases, the experienced experts were the basis for developing any vibration control technique. That is to say, no systematic and quantitative approaches were available either for the investigation of the dynamic characteristics in order to estimate the excitation force or for the development of efficient vibration control technique.[2]

In the present paper, a technique combining numerical and experimental modal analyses is proposed to control micro-vibration of concrete slabs in an economical and efficient manner.

2. Vibration criteria

In general, the allowable vibration criteria for each vibration sensitive equipment depends on the manufacturer and the installation condition. Therefore, it is reasonable to follow the allowable vibration range of the structure instead of that of the equipment. However, from the viewpoint of vibration control, more stringent vibration criteria make it more difficult and less economical to establish any vibration control scheme and it is important to investigate allowable responses accurately and to develop an efficient vibration control system.

2.1 Definition of micro-vibration

Unlike earthquake, wind or pollutive vibration, micro-vibration has low vibration level which can not be felt by human and is detectable only by measuring equipment. Its peak frequency is below 20Hz and it reduces the work efficiency of the workers who operate highly vibration sensitive equipment or high precision machines.[3]

2.2 Allowable vibration range

For specific equipment which requires highly precise working environments even within micro-vibration frequency range, allowable vibration range was selected within 50~58db (0.3~0.8gal) based on the manufacturer's standards as shown in Fig. 1.[4]

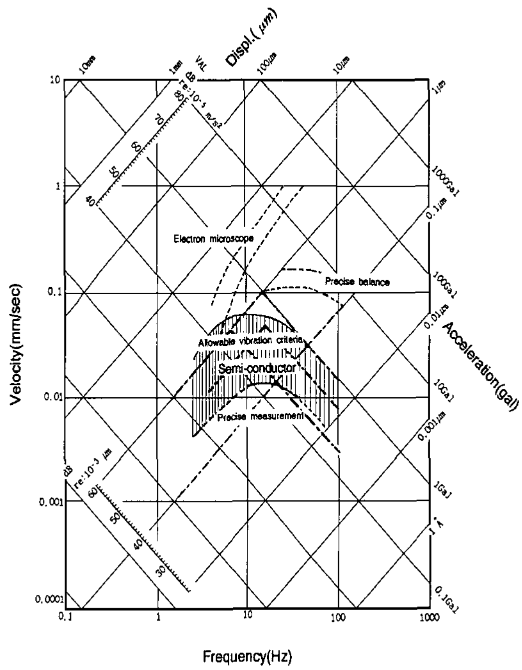


Fig. 1 Vibration criteria for high precose instruments

3. Modal analysis

3.1 Transfer function

The elastic motion of many types of complex structure such as micro-vibration of concrete slabs can be described by the second-order differential equation of (1), solution of which can be obtained through Laplace tansformation.

$$[[M]s^2+[C]s+[K]]\{X(s)\} = \{0\}$$
$$\text{or } [B]\{X(s)\} = \{0\} \quad (1)$$

where,

$[B(s)]=[M]s^2+[C]s+[K]$

s ; Laplace variable (complex)

The solution of equation (1) represents an eigenvalue problem where the non-trivial solutions can be obtained only when $Det\,[B]=0$. The complex eigenvalues(λ_k) (also called poles), which are the values of the Laplace variable(s), are the modal frequencies and modal damping values which cause the determinant of [B] to be zero. A corresponding eigenvector(also called the mode shape or mode vector) can be determined by solving equation (2) given a particular eigenvalue λ_k.

$$[B(\lambda_k)]\{\varphi\} = \{0\} \quad (2)$$

Since [B] represents the dynamic characteristics of the structure, it is called the system matrix whereas its inverse [H(s)] represents the Transfer Matrix (Function).

$$[H(s)] = [B(s)]^{-1} \quad (3)$$

Using equation (3), the system response can be rewritten as equation (4).

$$\{X(s)\} = [H(s)]\,\{F(s)\} \quad (4)$$

In other words, Transfer Matrix [H(s)] becomes the function of Laplace variable s and can be expressed as the ratio of two polynomial functions of the s-variable as shown in equation (5).

$$[H(s)]=\frac{Adj[B(s)]}{Det[B(s)]}=\sum_{k=1}^{2n}\frac{[a_k]}{s-p_k} \quad (5)$$

In equation (5), p's are the poles of Transfer Function [H(s)] which are also the roots of the characteristic polynomial Det[B(s)]. The numerator $[a_k]$ of the Transfer Function is called the residue and can be assembled into a residue matrix in terms of modal vectors or mode shapes, as given by equation (6).

$$[a_k] = A_k\{u_k\}\,\{u_k\}^T \quad (6)$$

where A_k ; scalar , u_k ; modal vector

Without any special assumption on damping characteristics, transfer matrix can be described in the parametric form as follows.

$$H(s) = \sum_{k=1}^{n}\left[\frac{\{u_k\}\{u_k\}^T}{s-p_k}+\frac{\{u_k^*\}\{u_k^*\}^T}{s-p_k^*}\right] \quad (7)$$

It is now noted that each mode of vibration is defined by a pair of complex conjugate poles (frequency and damping values) and a pair of complex mode shapes. Using equations (4), (5), (6), and (7), the responses in the Laplace domain can be rewritten as below in case of single-DOF.

$$X(s) = \frac{A}{[(s-p_k)+(s-p_k^*)]} \quad (8)$$

where, $p_k=\dfrac{-c+\sqrt{c^2-4mk}}{2m}=-\sigma+j\omega$

$$p_k^*=\frac{-c-\sqrt{c^2-4mk}}{2m}=-\sigma-j\omega$$

Fig. 2 represents the physical meaning of the equation (8), and fig. 3 shows the vibrational mode of single DOF of mass-spring-damper system using a pair of complex conjugate p_k and p_k^*.

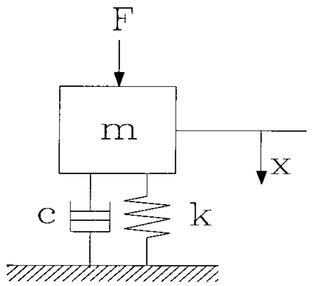


Fig. 2 Single-DOF

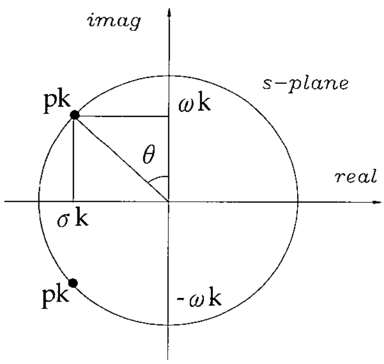


Fig. 3 Laplace domain

Impulse response in time domain can be obtained by using inverse-Laplace transform as shown in equation (9), which represents a damped single-DOF.

$$X = A\,e^{\sigma t}\sin(\omega t) \tag{9}$$

Thus, the real part of the pole is the decay rate and the imaginary part of the pole is the frequency of oscillation. Damping rate or Damping coefficient (σ) has the same physical characteristics as those of damped frequency (ω).

The coupled complex value is called complex frequency and becomes the real physical property of the system.(5) When a structure is excited using a broadband input force, many modes of vibration are excited simultaneously, and each mode has different damping ratio. The amplitudes of response are different according to the positions, and there is no need to have the same phase relation. In other words, the real vibration modes always have damping properties related to their modes, and their mode is somehow complex.

3.2 Structural dynamics modification

Structural stiffness modification is expressed as equation (10)

$$[\varDelta K] = [L][\alpha][L]^T \tag{10}$$

where $[\alpha]$ is eigenvalue matrix of the stiffness changes, [L] is eigenvector matrix. If stiffness modification is localized, the equation of motion would be modified as shown in equation (11).

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} + \alpha\{l\}\{l\}^T\{X\} = \{0\} \tag{11}$$

where $\{l\}$ is localized stiffness vector.

A new coordinate system can be obtained by using a modal coordinate $\{z\}$ and a transformation matrix made up of the mode shapes of the structure, i.e. $\{X\}=[\Phi]\{z\}$, and then equation (11) would be transformed into equation (12).

$$(\Omega^2[\Phi]^T[M][\Phi]+\Omega[\Phi]^T[C][\Phi] + [\Phi]^T[K][\Phi]+\alpha[\Phi]^T\{l\}\{l\}^T[\Phi]) \times\{z\}=\{0\} \quad (12)$$

The orthogonality properties of mode shape with respect to the mass and stiffness matrices would yield equation (13).

$$(\Omega^2[\,`I_\backslash]+\Omega[\,`2\sigma_\backslash] + [\,`\sigma^2+\omega^2{}_\backslash]+\alpha\{u\}\{u\}^T)\{z\}=\{0\}$$

where, $\{u\}=[\Phi]^T\{l\}$. (13)

Equation (13) is similar to eigenvalue problem whereas Ω and {z} become the eigenvalue and the eigenvector respectively. The rth equation in equation (13) now becomes as in equation (14).

$$\Omega^2 z_r+2\sigma_r\Omega z_r+(\sigma_r^2+\omega_r^2)z_r + \alpha\, u_r\sum_k^N u_k z_k=\{0\}$$

$$(\Omega^2+2\sigma_r\Omega+(\sigma_r^2+\omega_r^2))\, z_r\,/\,u_r = -\alpha\sum_k^N u_k z_k \quad (14)$$

The terms on the right side of equation (14) are independent of z_r, equation (14) becomes therefore equation (15), where z_r was arbitrary chosen.

$$(\Omega^2+2\sigma_1\Omega+(\sigma_1^2+\omega_1^2))\, z_1/\,u_1 = (\Omega^2+2\sigma_2\Omega+(\sigma_2^2+\omega_2^2))\, z_2/\,u_2 = \cdots\cdots = -\alpha\sum_k^N u_k z_k \quad (15)$$

From equation (15), the kth element of vector {z} can be obtained by equation (16).

$$z_k=(\Omega^2+2\sigma_r\Omega+(\sigma_r^2+\omega_r^2))z_r/u_r \times(u_k/(\Omega^2+2\sigma_k\Omega+(\sigma_k^2+\omega_k^2))) \quad (16)$$

Substituting z_k of equation (16) into equation (14) yields equation (17).

$$(\Omega^2+2\sigma_r\Omega+(\sigma_r^2+\omega_r^2))\, z_r\,/\,u_r = -\alpha\sum_k^N u_k u_k \frac{z_r(\Omega^2+2\sigma_r\Omega+(\sigma_r^2+\omega_r^2))}{u_r(\Omega^2+2\sigma_k\Omega+(\sigma_k^2+\omega_k^2))}$$

$$\frac{-1}{\alpha}=\sum_k^N\frac{u_k^2}{\Omega^2+2\sigma_k\Omega+(\sigma_k^2+\omega_k^2)} \quad (17)$$

Equation (17) represents the characteristic equation of the modified system in the new coordinate system and the roots of the polynomial becomes new eigenvalues. Once new eigenvalues are computed from equation (17), the corresponding modified eigenvectors can be determined from equation (15) to within an arbitrary constant. The kth element of the rth mode is given by equation (18).

$$z_k=C_r\frac{u_k}{(\Omega_r^2+2\sigma_k\Omega_r+(\sigma_k^2+\omega_k^2))} \quad (18)$$

Coefficient C_r is obtained by normalizing the modified mode shapes to the unit modal mass. The modified mode shapes {z} is related to the modal coordinates but not to the physical coordinates. Therefore the modified mode shapes needs to be transformed back to physical coordinates using $\{x\}=[\Phi]\{z\}$ as shown in equation (19).

$$\{x_r\} = [\Phi]\{z_r\} \quad (19)$$

4. Vibration test of concrete slabs

4.1 Discretization for numerical analysis and experimental modal analysis

The dimension of concrete slab for vibration test is 60cm×260cm×9cm, and the same discretization is used for the measurement of frequency response function at each point as that of numerical analysis. Fig. 4 shows the discretization of the structure with 80 points and 56 elements. As for the boundary conditions of numerical analysis, concrete slab is constrained in x and y-displacement and in z-rotation at points #1 and #65, and it is also constrained in z-rotation at points #16 and #80. Fig. 5 shows the vibration test setup of concrete slab.

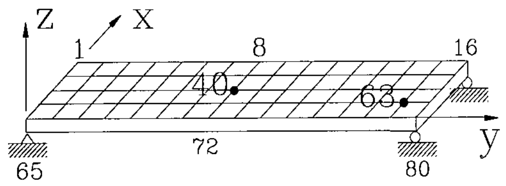


Fig. 4 Discretization of the structure

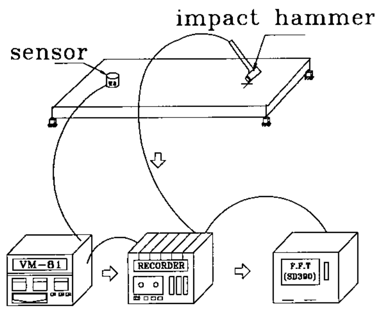


Fig. 5 Modal test setup

4.2 Determination of input force using transfer function

The main sources for vibration in semiconductor manufacturing facilities are refrigerators, air conditioners, power-transformers, pumps, elevators, and air conditioner-ducts as well as small air-compressors and pumps installed inside clean rooms.

In the present paper, accelerance, one of the frequency response functions, is used to determine the excitation force indirectly. In other words, excitation force (F') can be determined by using input force and input acceleration and the measured acceleration of the structure as shown in equation (20).

$$F' = \frac{F}{a - \Delta a} \times (a' - \Delta a) \tag{20}$$

where, F' = excitation force to be determined,
a' = measured acceleration,
F = input force for modal test,
Δa = acceleration by dark vibration,
a = acceleration by input force and dark vibration

It is noted that the technique to determine the excitation force indirectly by using response functions can be also applied to determine the excitation force originated by man induced load or vehicle load on the bridges. Figure 6 illustrates the calculated excitation force in the frequency domain.

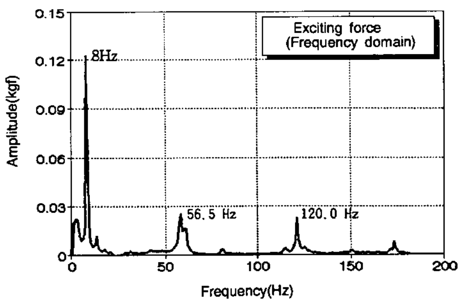


Fig. 6 Excitation force by the manufacturing facilities (frequency domain)

4.3 Results of numerical and experimental modal analyses

Experimental and numerical modal analyses of the structure were performed to establish test databases for structural dynamics modification and to validate the numerical analysis model, from which modal parameters were obtained as shown in Table 1. Fig. 7 and Fig. 8 show impulse responses at point #20 in time and frequency domains respectively when excitation force is applied at point #63. Numerical analysis yielded peak response of 144.0 gal in time domain whereas test results showed peak response of 185.0 gal and 22.2% of discrepancy exists between the experimental and numerical estimations. In the mean time, numerical and experimental analyses on natural frequencies in the frequency domain were also performed. The 1st, 2nd and 3rd natural frequencies were 21.9Hz, 78.0Hz, 148.8Hz in case of experimental analysis and 20.1Hz, 74.4Hz, 157.2Hz in case of numerical analysis with discrepancies of 8.2%, 4.6%, 5.6%. The results from the present analyses were comparable in accuracy with those of Kientzyand and Richardson[8] considering the fact that their analyses were on the rectangular aluminum plate with discrepancies of 1.8~7.4% for each mode. It is believed that the main reasons for the present discrepancies resulted from the inhomogeneous material properties of the concrete slab and the differences between the actual structural boundary conditions and the numerical ones. Poor coherence between the input and the output is also believed to contribute to the present discrepancies. Fig. 9 illustrates mode shapes obtained experimentally.

Table 1 Modal parameters

mode	experimental		numerical		
	frequency (Hz)	damping ratio(%)	frequency (Hz)	damping ratio(%)	MPF* (%)
1	21.9	1.61	20.1	1.61	82.35
2	78.0	5.25	74.4	5.25	0.16
3	148.8	9.46	157.2	9.46	9.90

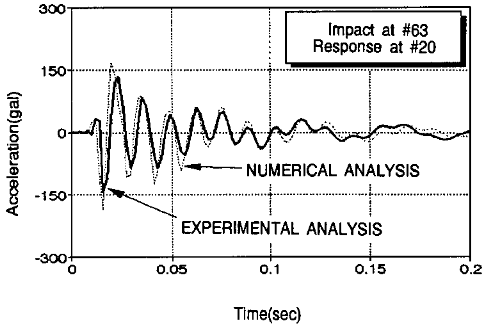


Fig. 7 Impulse response (time domain)

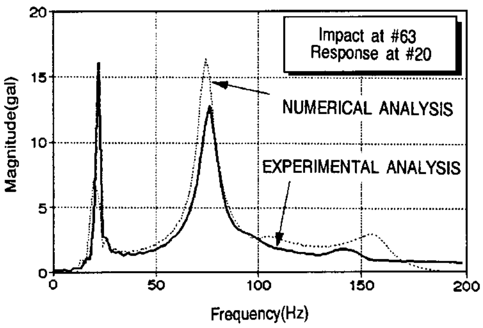
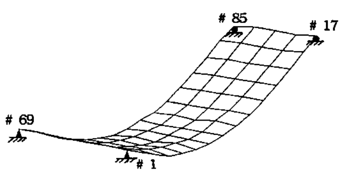
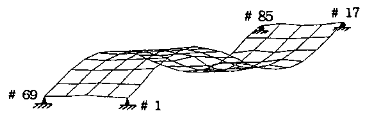


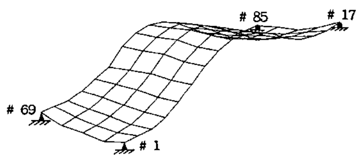
Fig. 8 Impulse response(frequency domain)



(a) 1st mode (Natural frequency : 21.9Hz, damping ratio :1.61%)



(b) 2nd mode (Natural frequency : 78.0Hz, damping ratio :5.25%)



(c) 3rd mode (Natural frequency : 148.8Hz, damping ratio :9.46%)

Fig. 9 Mode shapes by experiments

5. Vibration control through structural dynamics modification

5.1 Natural frequencies for structural design

High-precision manufacturing facilities in semi-conductor factories are usually concerned with the excitation force frequency below 10Hz. Since human is susceptive to the frequency range of 4~8Hz,[9] and keeping the natural frequency of the system below 10Hz may cause resonances during warmming up those facilities, it is decided to select the frequency range of 1~10Hz as frequency range to be avoided. In order to lower transmissibility within this frequency range, the natural frequency of the structural system is modified based upon damping ratio, forced frequency and the natural frequency of the system as shown in Table 2. In other words, considering the fact that the damping ratio of the concrete structure is, in general, 3~5% and the effect of vibration control table, it is recommended to keep the natural frequency of the system below 25~30Hz. In this case, the transmissibility of the system will be below 120%, whose relationship with natural frequency is shown in Fig. 10.

Table 2 Relationship of transmissibility with damping ratio and frequency ratio

ξ \ r	10/15	10/20	10/25	10/30
0.09	1.772	1.329	1.189	1.124
0.07	1.780	1.3315	1.190	1.125
0.05	1.791	1.332	1.190	1.125
0.03	1.797	1.333	1.190	1.125
0.01	1.799	1.333	1.190	1.125

5.2 Vibration control of concrete slab

From Table 3, it is clear that the 1st mode is dominant, and Fig. 11 shows impulse responses of the structure prior to the structural stiffness modification.

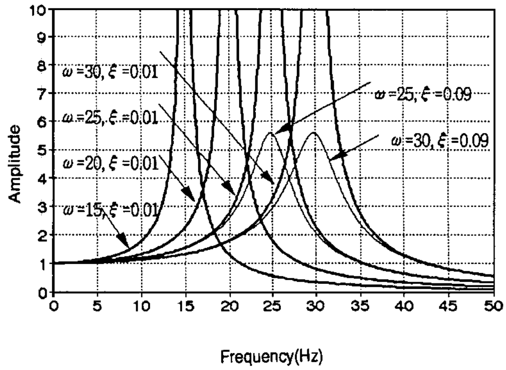


Fig. 10 Relationship of transmissibility with natural frequency

Table 3 Impulse responses before and after the structural dynamic modification

case	natural frequency (Hz)			damping ratio (%)			maximum response (gal)	DC~20Hz maximum response (gal)
	1st	2nd	3rd	1st	2nd	3rd		
1*	21.9	78.0	148.8	1.6	5.3	9.5	3.620 (120Hz)	1.000
2*	78.0	100.2	171.6	5.3	2.6	-	5.900 (120Hz)	0.115
3*	75.4	155.2	-	0.98	8.80	-	4.090 (120Hz)	0.136
4*	111.4	160.4	-	0.33	8.7	-	0.760 (56.5Hz)	0.057
5*	123.0	163.5	-	0.38	8.5	-	0.683 (56.5Hz)	0.053

1* before modification
2* additional support at point #43
3* additional supports at points #9, #77
4* additional supports at points #5, #9, #13, #73, #77, #81
5* additional supports at points #5, #9, #13, #43, #73, #77, #81

Since the damping ratio for the 1st mode is 1.6%, it is desirable to keep the new natural frequency above 8.6Hz considering the natural frequency suggested in Table 2. Then it becomes 1.07 times the excitation frequency to reduce the dynamic multification factor(D.M.F.) to 20% of that of resonance. In the mean time, since the damping ratio for the 2nd natural

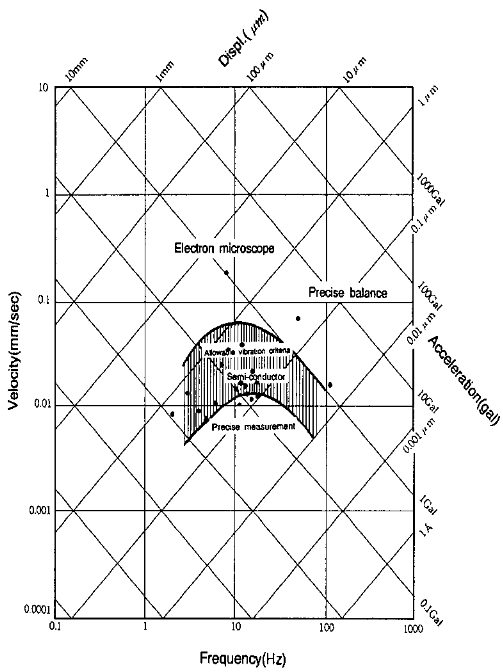


Fig. 11 Impulse responses prior to the structural dynamics modification

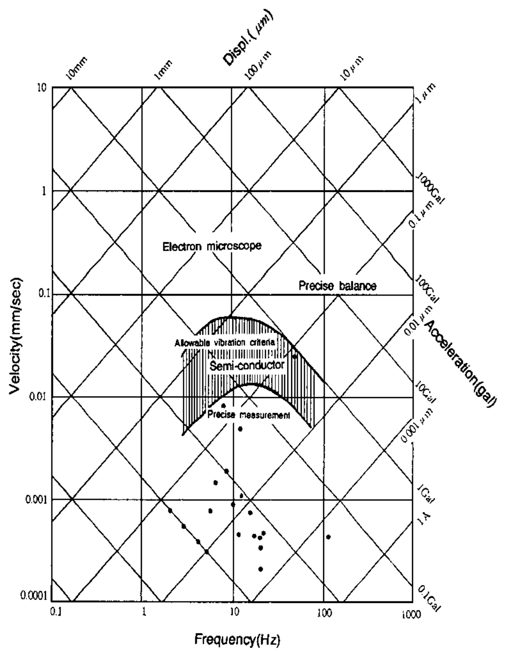


Fig. 12 Impulse responses after the structural dynamics modification

frequency is 5.3%, it is necessary to maintain the new natural frequency above 70.6Hz in order to make it 1.25 times the excitation frequency. However, it is preferable to raise the natural frequency for design purpose beyond 25~30Hz because the 1st natural frequency is rather low at 8.6Hz. Table 3 summarizes the maximum responses in the structural system under consideration before and after the structural dynamic modification subject to the similar modification conditions.

As shown in Fig. 12 the 1st and 2nd natural frequencies for case #5 are changed to 123.0Hz and 163.5Hz respectively, and the impulse responses in frequency domain is illustrated in Figure 12. It should be noted that the maximum responses for each mode is now reduced to 0.008~0.053gal which are within the allowable response range of 0.3~0.8gal, and it is 1/15~1/18 times the responses prior to the structural dynamic modification.

6. Conclusion

1. The dynamic characteristics of the system were obtained through combined experimental and numerical modal analyses. The predicted responses were obtained by using Structural Dynamics Modification(SDM) and Frequency Response System(FRS), and compared with the allowable micro-vibration range. Vibration control schemes were then introduced to make the structural system admissible for the micro-vibration criteria. The current technique enables to perform quantitative micro-vibration analyses and micro-vibration control in the concrete slabs for high-precision facilities.
2. The excitation forces for the machineries in operation could be accurately estimated

from the accelerances. These accelerances were obtained by considering dark vibration and the known input forces.

3. It is recommended in the design phase to maintain the natural frequencies of the structural system above 25~30Hz for the purpose of micro-vibration control.

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