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The growth rates and tune shifts due to the construction errors of RF cavity

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Abstract – The resonance frequencies, shunt impedances and Q-values for the higher-order modes in our designed cavity are calculated by the computer codes URMEL and MAFIA. A new computer code is developed to calculate the complex tune shifts for the randomness of the higher-order mode frequencies due to the construction errors of a cavity. The results with the construction errors are compared to those of without error cases for the dipole mode and quadrupole mode.

I. Introduction

Sacherer [1] developed an analysis method for longitudinal instabilities and the main result is an integral equation for the eigenmode and eigenfrequencies of the longitudinal bunch oscillations.

Our method of analysis is based on Zotter's formalism [2, 3], which is the eigenmode analysis of Sacherer's integral equation without mode coupling for a Gaussian beam, and based on Chin's method [4].

The instability analysis by the statistical methods has been carried out by many authors [4-6]. Especially, Chin took the impedance of a cavity, not a single cell, to take into account the actual fields seen by particles. The resonance frequencies, shunt impedances and Q-values for the higher-order modes in our designed cavity are calculated by the computer codes URMEL [7] and MAFIA [8].

Since the longitudinal growth rates are especially higher than the transverse ones, we calculate the longitudinal coherent tune shifts and growth rates as functions of the bunch length for the lower azimuthal modes.

A new computer program to calculate the complex tune shifts due to randomly distributed higher-order mode impedances is developed. The longitudinal instabilities for the higher-order modes are calculated by the eigenmode analysis of Sacherer's integral equation, and the growth rates and the tune shifts in the electron storage ring [9] are calculated by considering the randomness of the higher mode frequencies of a cavity due to the construction errors.

The results with the construction errors are compared to those of without error cases for the dipole mode and quadrupole mode.

II. The Eigenvalue Equation and Instabilities

Sacherer's integral equation can be reduced to the eigenvalue equation by expanding the radial mode function in orthogonal polynomial and the eigenvalue equation is written by

$$|M - \delta\nu I| = 0 \qquad (1)$$

where the matrix M is the interaction matrix which is no longer diagonal and $\delta\nu$ is the relative complex tune shift.

To calculate of the effects of the higher order modes of our model cavity, the matrix M can also be written as

$$M = M_{acc.} + M_{higher} \tag{2}$$

The effects of higher modes are statistically independent of the coupled-bunch mode number,

$$M_{kl} = \frac{m\xi R_s}{\sqrt{4Q^2-1}}[S(\nu_2)-S(\nu_1)] \tag{3}$$

where, m is the azimuthal mode number, ξ is the scaling factor, Q is the quality factor and

$$S(\nu) = \frac{E_{mk}E_{ml}}{2^{m+k+l}M}\sum_{p=-\infty}^{\infty}\frac{Z^{2(m+k+l)}e^{-z^2}}{p-\nu} \tag{4}$$

with $Z=Mp\sigma$,

$$E_{mk}E_{ml} = \frac{1}{2\pi\sigma^2\sqrt{(m+k)!\,k!(m+l)!\,l!}} \tag{5}$$

and

$$\nu_{1,2} = \frac{\omega_r}{2Q\,\omega_0}(j \pm \sqrt{4Q^2-1}) \tag{6}$$

with ω_r is the resonant angular frequency and ω_0 is the revolution angular frequency. Thus the matrix M can be written with the recurrence formula as

$$M_{kl} = \frac{m\xi R_s}{M\sqrt{4Q^2-1}}E_{mk}E_{ml}[B_g(\nu_2)-B_g(\nu_1)] \tag{7}$$

where,

$$B_g(nu) = \frac{1}{2^g}[Z^{2g}S_0(\nu)+\sum_{q=0}^{g-1}\Gamma(q+1/2)Z^{2g-2q-1}]$$
$$= \frac{Z^2}{2}B_{g-1}(\nu)+\frac{\Gamma(g-1/2)Z}{2^g} \tag{8}$$

with $B_0(\nu)=S_0(\nu)$ and $g=m+k+l$.

The infinite number of solutions for the eigenvalue is specified by the radial modes and azimuthal modes. The real part of the coherent frequency shift gives the real coherent mode frequency shift and the imaginary part gives the instability growth rates. The complex tune shifts of coherent oscillations for a single resonator impedance can be calculated by solving the eigenvalue equation. For the calculations of the growth rates and the tune shifts in our model storage ring, we have to consider the randomness of higher mode frequencies due to the construction errors of the cavities. Each impedance peak can contribute to both the growth rate and the damping rate of the Robinson instability according to the position of the impedance peak with respect to the harmonics of the revolution frequency. The Monte Carlo method is used to calculate the average values and rms spreads of the complex tune shifts due to the impedances for the accelerating mode and randomly distributed higher modes.

III. The Tune Shifts and Growth Rates with Statistical Analysis

The instability by the statistical analysis has been studied by many authors [4-6]. Especially, we took the impedance of a cavity, not a single cell, to take into account the actual fields seen by particles. The resonance frequencies, shunt impedances and Q-values for the higher order modes in our designed cavity with the length of 40 cm, diameter 46.03 cm, peak voltage 1.5 MV and frequency 501.96 MHz and R/Q=58.89 ohm are calculated by the computer codes URMEL [7] and MAFIA [8].

We need random numbers with a distribution that is different from a uniform one. The general approach to obtain a set of random numbers with a given distribution is to start with a uniform set and change it into the one required.

We assume that the spread in the higher mode frequencies due to the construction errors of cavities is a Gaussian with the standard deviation σ for the the cavity radius of b and the cavity length g. The magnitude of the standard deviation σ at resonant frequency f_r is estimated by the relative construction error ε as $\sigma\sim\varepsilon f_r$.

We assume that the relative construction error

Table 1. Main parameters for calculations

Machine parameters	ranges
Energy	2 GeV
Average machine radius	26.866 m
Synchrotron tune	0.0059
Revolution frequency	1.78 MHz
Momentum compaction factor	0.0024
Bunch length	0.01~0.1 cm
Harmonic number	282
RF frequency	501.96 MHz
Peak voltage	1.5 MV
Cavity length	40.00 cm
Cavity diameter	46.03 cm

for the cavity radius *b* and the cavity length *g* of the designed RF cavity is 0.5×10^{-3}, which is comparable to the relative construction error for the cavity diameter and the cavity length, for these calculations. The beam parameters [9] used in our calculations are the energy of 2 GeV, average machine radius of 26.866 *m*, momentum compaction factor 0.0024, betatron tune 6.29, revolution frequency 1.78 MHz, synchrotron tune 0.0059 and bunch length 0.01 to 0.1 cm as shown in Table 1.

The results for the tune shifts versus the bunch length are shown in Figs 1 and 2 for two azimuthal modes of the dipole where the bunches move rigidly as they execute longitudinal synchrotron oscillations and quadrupole where the bunch head and tail oscillate longitudinally out of phase and three radial modes. Fig. 1 shows the relationship between the tune shifts and the bunch length for the azimuthal mode m=1 (dipole mode) and the radial modes of k=0, 1 and 2.

Fig. 2 shows the relationship between the tune shifts and the bunch length for the azimuthal mode m=2 (quadrupole mode) and the radial modes k=0, 1 and 2.

In Figs 1 and 2, it is shown that tune shifts are not sensitive to the bunch length in both cases of the dipole and quadrupole modes.

Figs 3 and 4 show the comparison of the growth rates and the tune shifts for with and without errors in the case of m=1 and k=0. Figs 5 and 6 give the comparison of the growth rates and the tune shifts for with and without errors in the case of m=2 and k=0.

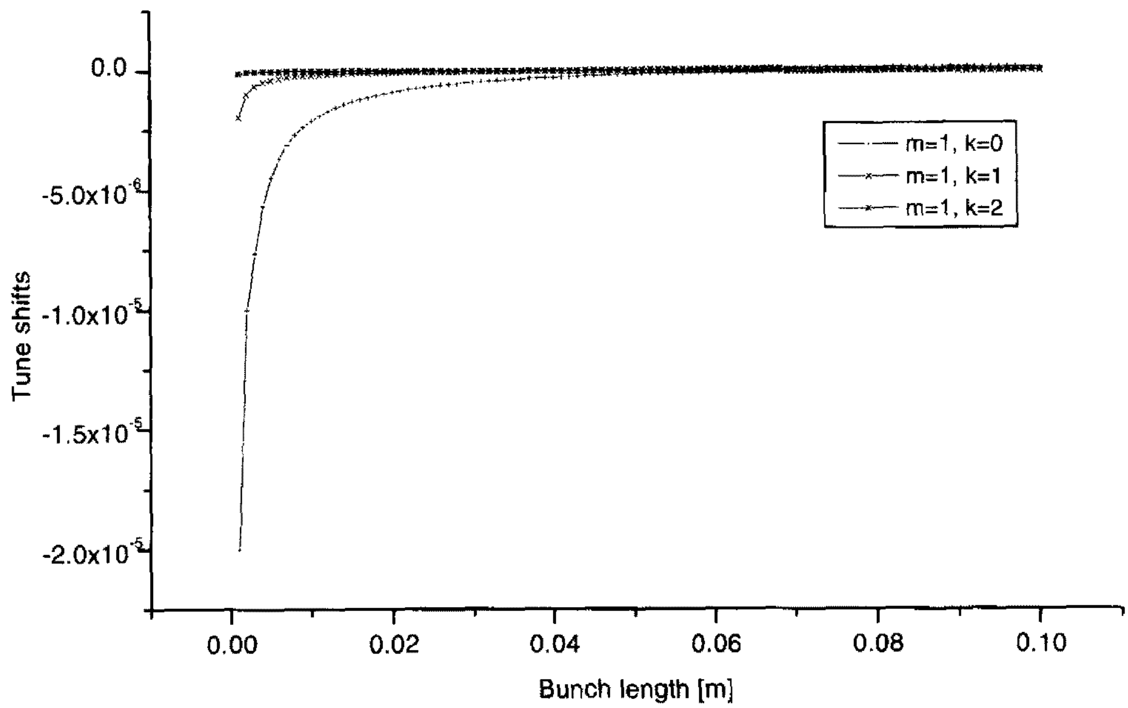

Fig. 1. The relationship between the tune shifts and the bunch length for m=1 and k=0, 1 and 2.

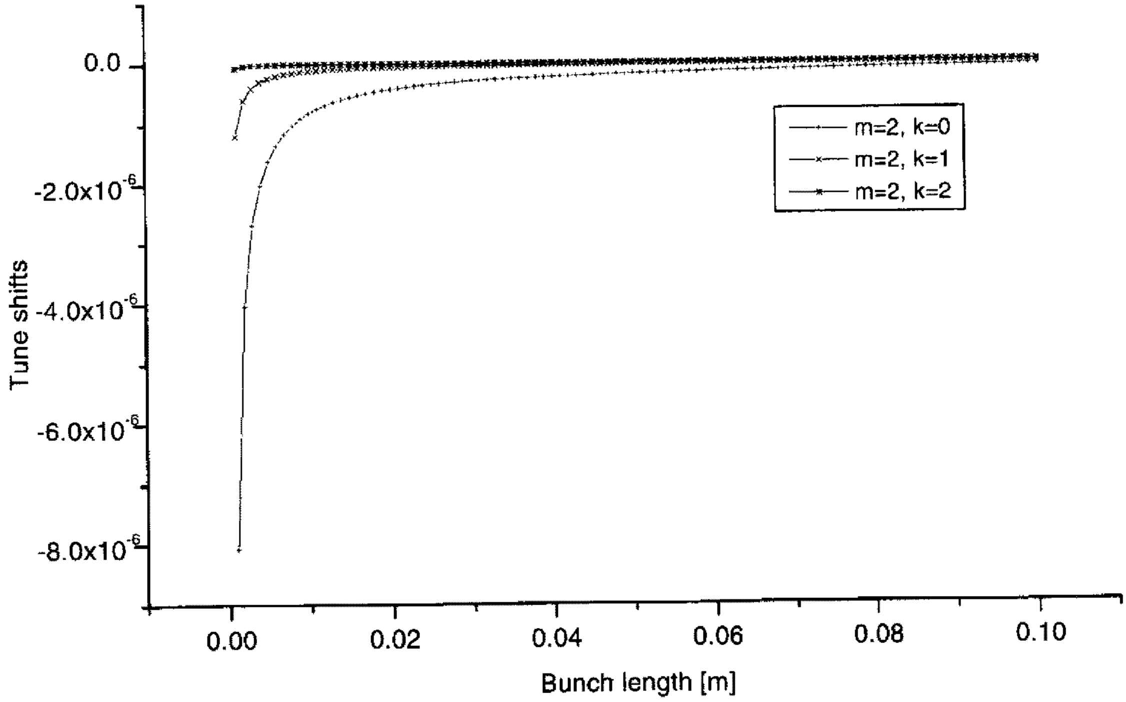

Fig. 2. The relationship between the tune shifts and the bunch length for m=2 and k=0, 1 and 2.

The growth rates with the construction errors are increased by the maximum error deviation of about a factor of ten compared to those obtained from without error cases by the statistical analysis.

For the tune shifts with and without errors, the maximum error deviations are increased by a factor of four.

We have calculated the growth rates and the tune shifts for the several hundred cases and considered several data to get the maximum devia-

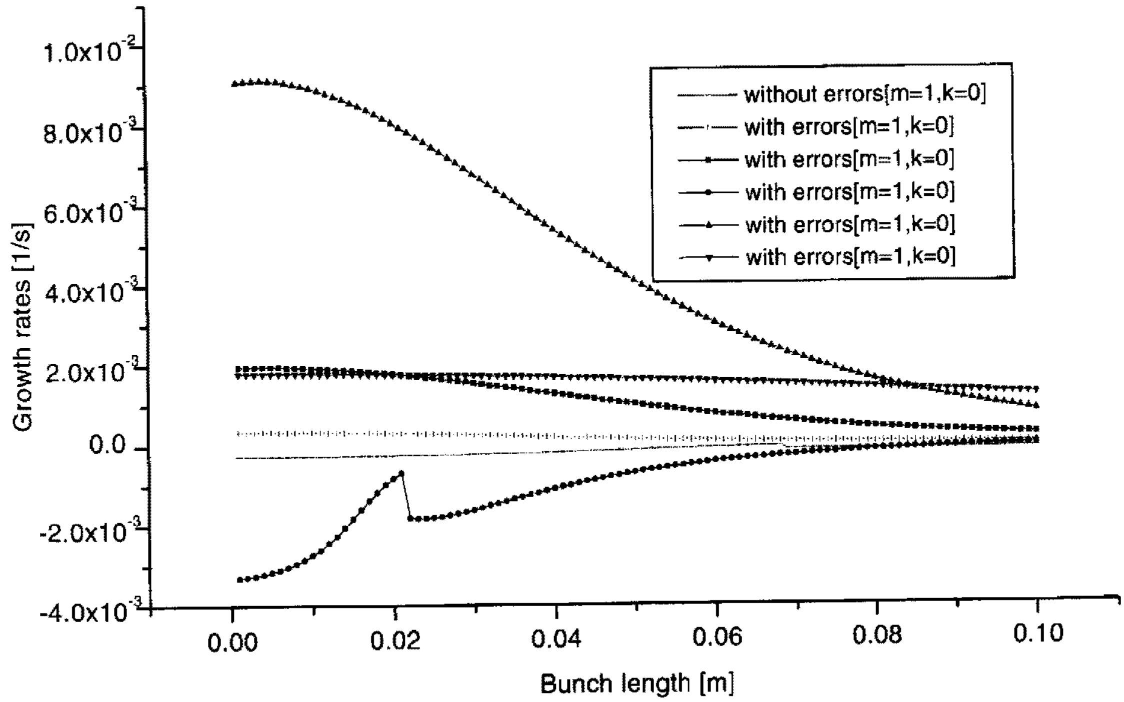

Fig. 3. The relationship between the growth rates and the bunch length with and without errors for m=1 and k=0.

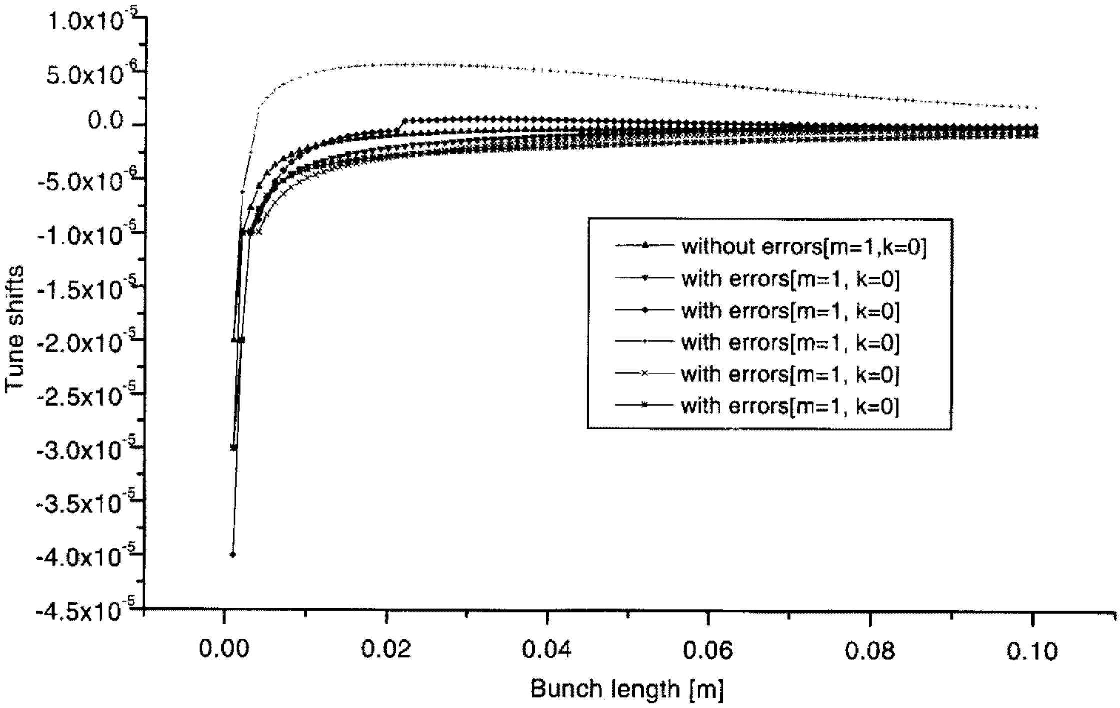

Fig. 4. The relationship between the tune shifts and the bunch length with and without errors for m=1 and k=0.

tion of the tune shifts and growth rates of coherent synchrotron oscillation due to randomly distributed RF cavity impedances.

The longitudinal radiation damping rate at 2 GeV storage ring is about 222 sec^{-1}, large enough to stabilize the dipole and quadrupole modes for various azimuthal modes. The contributions of higher modes in these calculations are not sensitive to the spread of resonances due to the construction errors of the designed cavity.

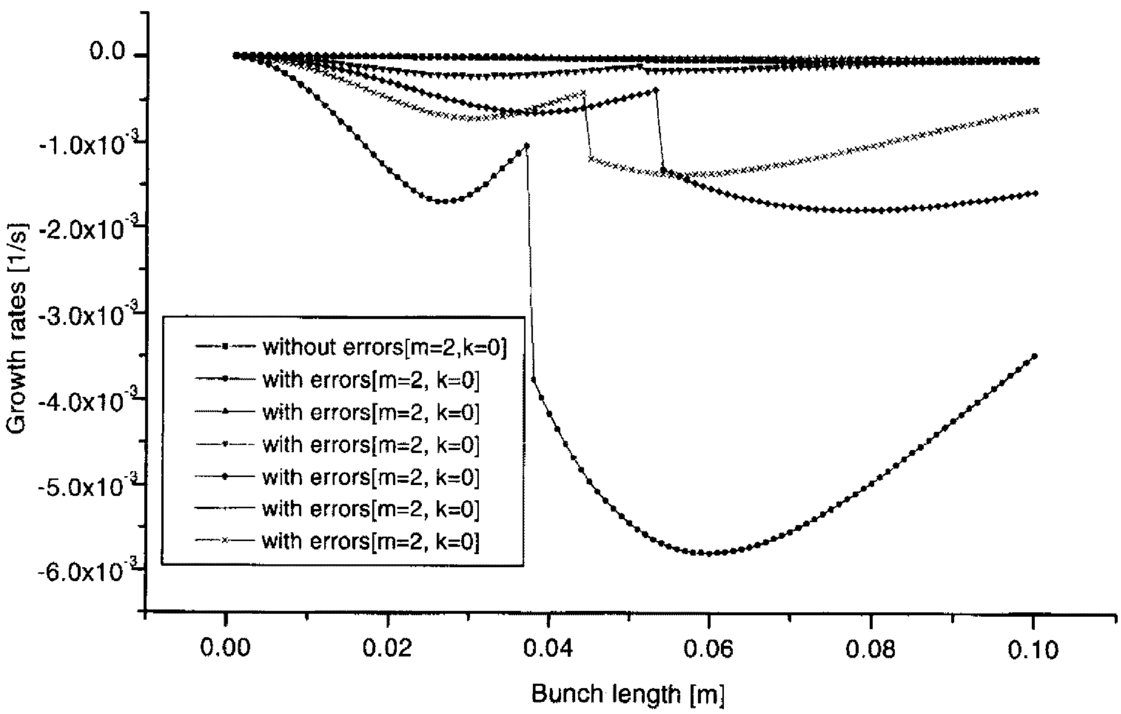

Fig. 5. The relationship between the growth rates and the bunch length with and without errors for m=2 and k=0.

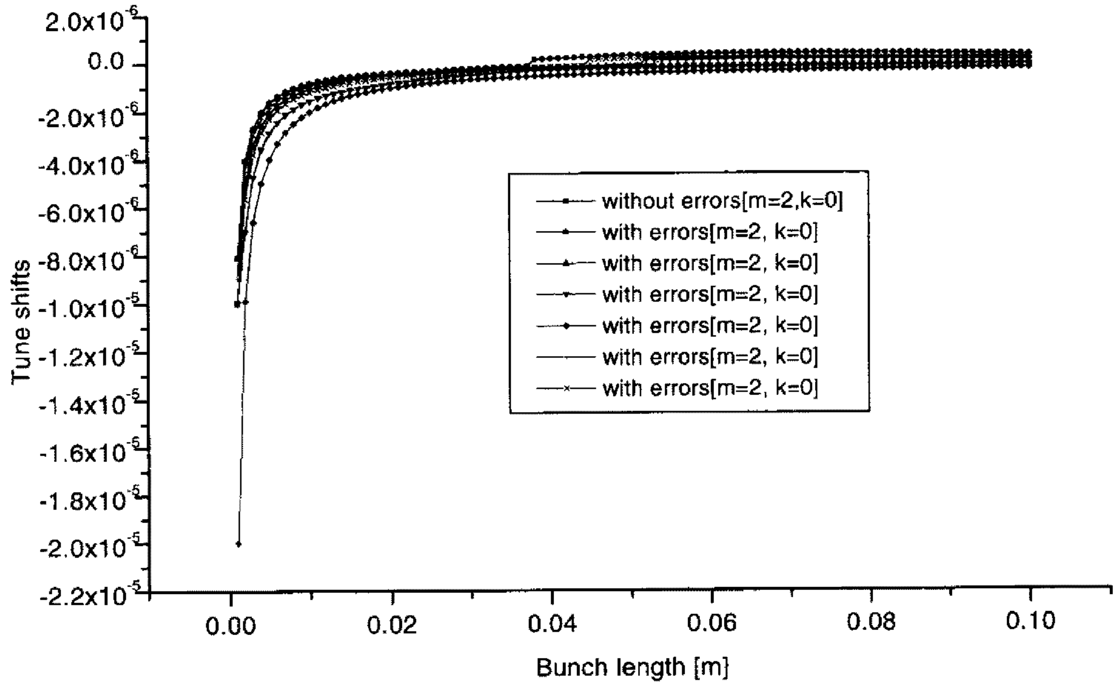

Fig. 6. The relationship between the tune shifts and the bunch length with and without errors for m=2 and k=0.

IV. Conclusion

The growth rates and the complex tune shifts due to randomly distributed higher-order mode impedances in the electron storage ring were calculated by a new computer code.

Tune shifts were not sensitive to the bunch length in both cases of the dipole and quadrupole modes.

The results with the construction errors were compared to those of without errors for the dipole mode and quadrupole mode. The growth rates with the construction errors are increased by the maximum error deviation of about a factor of ten compared to those obtained from without error cases by the statistical analysis.

For the tune shifts with and without errors, the maximum error deviations are increased by a factor of four. But the growth rates, in any cases, were small enough compared to the radiation damping time of 4.5 ms in our storage ring.

The results also indicated that the deviations of the growth rates and the tune shifts due to higher modes of cavities were not sensitive to the bunch length for the relative construction errors of cavities of about 5.0×10^{-4}.

Acknowledgments

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References

1. F. J. Sacherer, IEEE Trans. **NS-24**, 1393 (1977).
2. B. Zotter, CERN **SPS/81-18**, 1 (1981).
3. B. Zotter, CERN/ISR-TH/**80-03**, 1 (1981).
4. Y. Chin, KEK **81-20**, 3 (1982).
5. C. Pellegrini and M. Sands, PEP-**258**, 1 (1977).
6. C. Y. Yao and A. Chao, SLAC-PUB-**2680**, 1 (1981).
7. T. Weiland, Nucl. Instr. Methods, **NS-216**, 329 (1983).
8. The Mafia collaboration (1988).
9. S. K. Nam, KEK-Int., **88-17**, 1 (1988), S. K. Nam, Particle Accelerators, **33**(4), 69 (1990).