# Tracking Filter Design for a Maneuvering Target Using Jump Processes

Sang Seok Lim

## Abstract

This paper presents a maneuvering target model with the maneuver dynamics modeled as a jump process of Poisson-type. The jump process represents the deterministic maneuver(or pilot commands) and is described by a stochastic differential equation driven by a Poisson process taking values from a set of discrete states. Employing the new maneuver model along with the noisy observations described by linear difference equations, the author has developed a new linear, recursive, unbiased minimum variance filter, which is structurally simple, computationally efficient, and hence real-time implementable. Furthermore, the proposed filter does not involve a computationally burdensome technique to compute the filter gains and corresponding covariance matrices and still be able to track effectively a fast maneuvering target. The performance of the proposed filter is assessed through the numerical results generated from the Monte-Carlo simulation.

## I. Introduction

The Kalman filter has been accepted as one of the best methods for providing the estimate of the state of a moving target([1]-[4]). Although developed specifically for linear models, the Kalmen filter has found wide applications in the form of suboptimal filters for nonlinear models through linearization techniques. Since Singer[5] first published his paper on tracking manned-maneuvering target in 1970, considerable attention has been given in the literature[5]-[20] to the tracking of such targets using suboptimal Kalman filters.

As pointed out by McAulay and Denlinger[6], the basic problem posed by a maneuvering target is a mismatch existing between the modeled and the actual target dynamics. In this regards, a number of different approaches have been applied to the maneuvering target problem, ranging from the parametric description of the maneuver (i.e. white noise to colored noise and to a semi-Markov process) to the non-parametric model of the maneuver (i.e. modeled as an unknown input acceleration).

Thorp[7] suggested a binary random variable in the target equation and developed an estimator given by a weighted sum of two Kalman filter estimates with the weights depending on the likelihood ratio for the detection of a maneuver. However, this results in degradation in performance during non-maneuvering conditions. It also makes the estimator very prone to divergence, particularly if nonlinear equations have been approximated by a Taylor series. In the variable dimension(VD) algorithm proposed by Bar-Shalom and Birmiwal[9], the tracking filter operates in its normal mode in the absence of any maneuver and once a maneuver is detected the filter uses a different state model. The VD algorithm is simple, but can not be applied in real-time and has undesirable features of re-initialization as pointed out by Bogler[10]. Blom et al.[11] and Bar-Shalom et al.[12] proposed the interacting-multiple model algorithm, which consists of running a filter for each model, a model probability evaluator, and an estimate combiner at the output of the bank of filters. Each filter uses a mixed estimate at the beginning of each cycle.

Singer[5] modeled the target acceleration as a random process with known exponential autocorrelation. Though the resulting filter is capable of tracking target maneuvers, the quality of the estimate is degraded when a target is moving at a constant speed. McAulay and Denlinger[6] performed a hypothesis test for maneuver detection. They also proposed the use of rectilinear motion model and switching from one model to another and re-initializing at each turn.

Chan et al.[15] proposed an input estimation(IE) technique to compensate for a deterministic target maneuver. The maneuvering target input parameters, i.e. magnitude of the maneuver, are estimated in a mean square error sense to remove the filter bias caused by the target deviating from the

Manuscript received February 19, 1998; accepted April 4, 1998.
The author is with the Avionics Department, Hankuk Aviation University.

assumed constant velocity (straight line) course. Though this scheme does not assume any *a priori* knowledge of the maneuver, the detection scheme requires a significant amount of computation and memory, while assuming the maneuver is constant over the detection window. The IE algorithm is further developed by Bogler[10] for the case of a one-dimensional Kalman filter, thus reducing the computational burdens. By utilizing the alternate structure of the information type Kalman filter, Farooq et al.[17] further developed the concepts of Chan et al.[15] and obtained a decoupled (thus computationally more efficient) filter based on the analysis of correlation coefficients.

Moose et al.[16] and Gholson et al.[18] proposed a maneuver model based on a semi-Markov transition between states of time-invariant set of discrete values representing the maneuver commands. By incorporating the semi-Markovian concept into a Bayesian estimation scheme, an adaptive state estimator was developed. A somewhat similar method was proposed by Richer and Williams[19]. The approach by Moose et al.[16] has been proved successful in tracking a maneuvering submarine. However, the main drawback of the method is in its incapability of tracking *fast* maneuvering targets, such as aircraft, beyond a few transitions because of the enormous computational requirements[10]. Using the same target model as Moose, Demirbas[20] proposed a different approach that estimates the states by quantization, multiple hypotheses testing and a suboptimal decoding algorithm without linearizing the nonlinear observations. Abrupt changes in the Loran-C and Omega measurements arising from the marine integrated navigation system were modeled as a pure jump process of Poisson type by Ahmed and Lim[21] and Dabbous et al.[22].

Conventionally the target dynamics is modeled as a continuous random variable, statistically described by known and constant parameters, for example, the Singer model[5]. Provided this underlying target model is correct, then the Kalman filter yields optimum estimates. If the target initiates and sustains a sudden pilot-induced maneuver, then this underlying target model is no longer valid, because there is essentially a step discontinuity which is modeled as an input acceleration term. Unless this discontinuity is taken into consideration, the filter will accumulate errors and possibly lose the track. In this paper, keeping this problem of discontinuities in view, we propose a maneuvering target model with the maneuvers modeled as the jump process of Poisson type. The jump process represents the deterministic maneuver(or pilot commands) and is described by a stochastic differential equation driven by a Poisson process taking values from a set of discrete states. With the help of the new maneuver model based on the jump process, along with the observations which are described by a linear difference equation and driven by a sequence of white Gaussian noise, we propose a new recursive, unbiased minimum variance filter. The main contribution of the paper is the development of such an efficient filter for tracking a *fast* maneuvering target by employing the jump processes for pilot commands. This filter is structurally simple so that it does not require a time-consuming complex procedure to compute the filter gain or covariance matrices. The proposed filter is also computationally efficient so that it is real-time implementable. Futhermore, it is able to track effectively a fast maneuvering target.

The paper is organized as follows. In Section 2, a new target model using a jump process for the deterministic pilot command is presented. Based upon the target and observation models, an optimal (in the minimum variance sense), linear, recursive and discrete filter is developed in Section 3. In Section 4, the performance of the proposed filter is demonstrated through the Monte Carlo simulation. Finally the concluding remarks are presented in Section 5.

## II. System Model

In this section a system model for a maneuvering target is proposed and a discrete measurement model is presented.

### 1. Target Model

Let $w_x(t)$, $w_y(t)$ and $w_z(t)$ denote the components of a zero mean white Gaussian noise vector $w(t)$, and $u_x(t)$, $u_y(t)$ and $u_z(t)$ denote the components of a known deterministic pilot command vector $u(t)$ at time $t$ in $x$, $y$, $z$ directions respectively. Then the dynamics of maneuvering target with reference to cartesian coordinates can be characterized by the following differential equation:

$$\dot{x}(t) = -a\dot{x}(t) + w_x(t) + u_x(t), \tag{1}$$

$$\dot{y}(t) = -a\dot{y}(t) + w_y(t) + u_y(t), \tag{2}$$

$$\dot{z}(t) = -a\dot{z}(t) + w_z(t) + u_z(t), \tag{3}$$

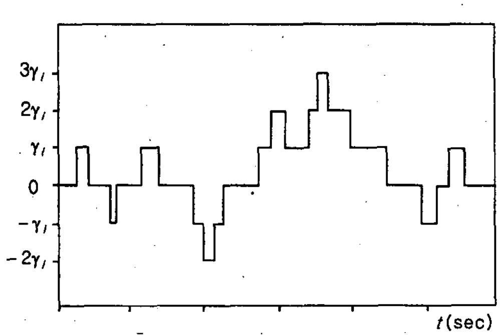


**Fig. 1.** The Pilot commands $u_i(t)$.

where $\alpha$ is the viscous drag coefficient, and the overdot(and double overdots) denotes the first(and second) derivatives with respect to time $t$. The pilot(maneuver) commands $u(t)$ are modeled as jump Markov processes with $(2n+1)$ discrete states. The behavior of the process $u_i(t), i=x,y,z,$ may be graphically represented by Fig. 1.

In Fig.1, $\gamma_i$ is a known positive parameter representing the size of jump between the two neighboring states, which corresponds to the mount of acceleration increment by the pilot and could be in general time-varying. However, without any lose of generality, we assume that it is time-invariant in the rest of the paper. Therefore, the process $u_i(t)$ is the solution of the following stochastic differential equation:

$$\begin{aligned} du_i(t) &= \gamma_i \{ 1_{[u_i(t_-)=-n\gamma_i]} N^i_{1,2}(dt) - 1_{[u_i(t_-)=+n\gamma_i]} N^i_{2n+1,2n}(dt) \\ &\quad + \sum_{j=2}^{2n} 1_{[u_i(t_-)=(j-n-1)\gamma_i]} (N^i_{j,j+1}(dt) - N^i_{j,j-1}(dt)) \Big\}, \qquad (4) \\ &\quad \text{for } i=x,y,z, \end{aligned}$$

where $1_{[\ldots]}$ is a characteristic function defined by

$$1_{[\ldots]} \equiv \begin{cases} 1 & \text{if } [\ldots] \text{ is true,} \\ 0 & \text{otherwise,} \end{cases} \qquad (5)$$

and $N^i_{j,j+1}$, $N^i_{j+1,j}$ ; $j=1,2,\ldots,2n$ ; $i=x,y,z$ are independent Poisson process with the transition rates $\lambda$'s defined by $E[N^i_{j+1,j}(\Delta)] \cong \lambda^i_{j+1,j}\Delta$ and $E[N^i_{j,j+1}(\Delta)] \cong \lambda^i_{j,j+1}\Delta$ for a sufficiently small sampling interval $\Delta$. Therefore, we note that the process $u_i(t), t \geq 0$ is a homogeneous jump process taking values from the set $\Gamma_i = \{-n\gamma_i, (1-n)\gamma_i, (2-n)\gamma_i, \ldots, 0, \gamma_i, 2\gamma_i, \ldots, n\gamma_i\}$, $i=x,y,z$ . This set can be represented by a so-called state transition diagram shown in Fig.2. This figure shows that the process $u_i(t)$ may stay in any one of the $(2n+1)$ discrete states and then randomly transfer to the one of the neighbouring states according to the properties outlined in Remark 1.

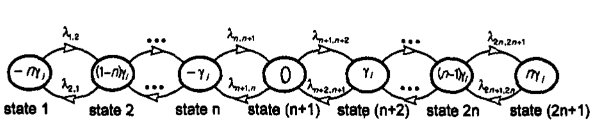


**Fig. 2.** The state transition diagram of $u_i(t)$.

**Remark 1.** (properties of the jump process $u(t)$)

(i) $u_i(t)$ can be characterized as being in one of the mutually exclusive, discrete states of the set $\Gamma_i$ at any time $t \geq 0$.

(ii) Changes of states between the two neighbouring states are possible at any time.

(iii) The probability of departure from one state to another state depends only on the current state and is independent of the time $t$.

(iv) The probability of more than one change of state during a sufficiently small time interval $\Delta$ is assumed to be negligibly small for the simplicity, though it is not essential.

**Remark 2.** (fitness of the jump process to maneuver commands)

Regarding the jump process in our target model, one may raise a natural question: Why should we model the maneuvers as a jump process governed by a Poisson-type counting measure? The arguments in favor of using such a model to represent maneuvers are as follows: i) the actual pilot induced commands in modern aircraft are often discrete and the profile of the commands very much resembles to the one shown in Fig.1, ii) any continuous process for a pilot command corresponding to a large acceleration can be closely approximated by a discrete process having several intermediate jump states with each jump corresponding to the velocity increment, and iii) the target model employing the jump process for maneuvers yields a computationally efficient tracking filter (see Remark 3 at the end of Section 3).

We now define the state vector $X(t)$ by $X=(x,y,z,\dot{x},\dot{y},\dot{z},u_x,u_y,u_z)'$, a Wiener process $W(t)$ by $W=(W_x,W_y,W_z)'$, and a Poisson process $N(t)$ by $N=(N_x,N_y,N_z)'$. Then the target dynamics (1)-(3) can be described by the following stochastic differential equation:

$$dX(t) = A\ X(t)dt + B\ dW(t) + C\ N(dt), \qquad (6)$$

where

$$A = \begin{pmatrix} 000 & 1 & 0 & 0 & 000 \\ 000 & 0 & 1 & 0 & 000 \\ 000 & 0 & 0 & 1 & 000 \\ 000 & -a & 0 & 0 & 000 \\ 000 & 0 & -a & 0 & 000 \\ 000 & 0 & 0 & -a & 000 \\ 000 & 0 & 0 & 0 & 000 \\ 000 & 0 & 0 & 0 & 000 \\ 000 & 0 & 0 & 0 & 000 \end{pmatrix}, \quad B = \begin{pmatrix} 000 \\ 000 \\ 000 \\ 100 \\ 010 \\ 001 \\ 000 \\ 000 \\ 000 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_x & 0 & 0 \\ 0 & \gamma_y & 0 \\ 0 & 0 & \gamma_z \end{pmatrix} \qquad (7)$$

The solution of Eq.(6) can be written as

$$X(t) = \Phi(t,t_0)X_0 + \int_{t_0}^{t} \Phi(t,s)\ B\ dW(s) + \int_{t_0}^{t} \Phi(t,s)\ C\ N(ds), \qquad (8)$$

where the transition matrix $\Phi(t,s)$ is

$$\Phi(t,s) \equiv \begin{pmatrix} 1 & 0 & 0 & a_1 & 0 & 0 & b_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & a_1 & 0 & 0 & b_1 & 0 \\ 0 & 0 & 1 & 0 & 0 & a_1 & 0 & 0 & b_1 \\ 0 & 0 & 0 & a_2 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_2 & 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (9)$$

with the parameters:

$$a_1 \equiv \frac{1}{\alpha}\{1-e^{-\alpha(t-s)}\},\quad a_2 \equiv e^{-\alpha(t-s)},$$
$$b_1 \equiv \frac{1}{\alpha^2}\{\alpha(t-s)e^{-\alpha(t-s)}-1\}. \tag{10}$$

Since, in practice, the observation process is usually of a discrete type, it is convenient to write the continuous dynamics of Eq.(6) in a discrete form. Let

$$t_0 \le t_1 \le t_2 \le \ldots \le t_k \le \ldots \le t_T \tag{11}$$

be the partitions of the period $[t_0, t_T]$ with the sampling interval $\Delta = |t_{k+1} - t_k|$. Then it is clear from Eq.(8) that

$$X(t_{k+1}) = \Phi(t_{k+1}, t_k)X(t_k) + \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, s)\, B\, dW(s) + \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, s)\, C\, N(ds) \tag{12}$$

In the following, we will use $k$ to denote $t_k$, $k=0,1,2,3,\ldots$, and $\Phi_k \equiv \Phi(k+1,k)$ for the simplicity of presentation. Now, a discrete model for the jump process in Eq. (4) can be written as

$$u_i(k+1) = u_i(k) + \gamma_i\{1_{[u_i(k)=-n\gamma_i]}N^i_{1,2}(\Delta) - 1_{[u_i(k)=+n\gamma_i]}N^i_{2n+1,2n}(\Delta) + \sum_{j=2}^{2n} 1_{[u_i(k)=(j-n-1)\gamma_i]}(N^i_{j,j+1}(\Delta) - N^i_{j,j-1}(\Delta)),\quad i=x,y,z \tag{13}$$

Defining the variables $\overline{B}(k) = \Phi_k B$, $\overline{C}(k) = \Phi_k C$, $\overline{W}(k+1) = W(k+1) - W(k)$ and $\overline{N}(k+1) = N(k+1) - N(k)$, Eq.(12) yields a discrete state model of the system in cartesian coordinates as

$$X(k+1) = \Phi_k X(k) + \overline{B}(k)\,\overline{W}(k+1) + \overline{C}(k)\,\overline{N}(k+1), \tag{14}$$

where $\overline{W}(k+1)$ satisfies $E\{\overline{W}(k)\,\overline{W}'(k)\} = \Delta\, Q(k)$. The initial state $X(0)$ is assumed to be Gaussian with mean $\hat{X}(0 \mid 0)$ and covariance $P(0 \mid 0)$. The two processes $\overline{N}$ and $\overline{W}$ are assumed to be independent of $X(0)$. Further, $\overline{N}(k+1)$ is independent of $\overline{W}(k+1)$ and it's components $\overline{N}_i(k+1)$, $i=x,y,z$ are defined by

$$\overline{N}_i(k+1) \equiv 1_{[u_i(k)=-n\gamma_i]}N^i_{1,2}(\Delta) - 1_{[u_i(k)=+n\gamma_i]}N^i_{2n+1,2n}(\Delta) + \sum_{j=2}^{2n} 1_{[u_i(k)=(j-n-1)\gamma_i]}(N^i_{j,j+1}(\Delta) - N^i_{j,j-1}(\Delta)),\quad i=x,y,z \tag{15}$$

### 2. *Measurement Model*

The measurements generated from the target are usually in terms of the range $r$, the bearing $\theta$ and the elevation $\varphi$ in spherical coordinates. The nonlinear relationship transforming the spherical observations to the cartesian form is as follows:

$$r = \sqrt{x^2+y^2+z^2},\quad \theta = \tan^{-1}\left(\frac{y}{x}\right),\quad \varphi = \tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right). \tag{16}$$

The standard Kalman filter can not be applied directly to the system described by Eqs.(14)-(16) because i) the radar observations are given in spherical coordinates while the target dynamics are described in cartesian coordinates, and ii) the representation of the system in one coordinate system results in a nonlinear estimation problem. Hence, in order to make the system amenable to linear estimation theory, the observation equations are usually approximated via a Taylor series around the estimated position $x_0, y_0, z_0$ (the position components of the state prediction $\hat{X}(k+1 \mid k)$ at time $k+1$ ). Thus, the approximate linear model of the observation is given by

$$Y(k+1) = H(k+1)X(k+1) + V(k+1), \tag{17}$$

where $V(k+1) \equiv (V_r(k+1), V_\theta(k+1), V_\varphi(k+1))'$ is a Gaussian white noise sequence satisfying $E\{V(k)V'(k)\} = R(k)$ and is independent of $\overline{N}(k+1)$, $\overline{W}(k+1)$ and $X(0)$. Further, $H(k+1)$ is the observation matrix given by

$$H(k+1) = \begin{pmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{21} & h_{22} & h_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{31} & h_{32} & h_{33} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

with

$$\begin{aligned} h_{11} &= x_0/a,\ h_{12} = y_0/a,\ h_{13} = z_0/a,\ \text{for } a = \sqrt{x_0^2+y_0^2+z_0^2}, \\ h_{21} &= -y_0/c^2,\ h_{22} = x_0/c^2,\ h_{23} = 0,\ \text{for } c = \sqrt{x_0^2+y_0^2}, \\ h_{31} &= -x_0 z_0/b,\ h_{32} = -y_0 z_0/b,\ h_{33} = c/a^2,\ \text{for } b = a^2 c, \end{aligned} \tag{19}$$

## III. The Minimum Variance Filter

In this section we develop a discrete, recursive, linear minimum variance filter for the system described by Eq.(14) and observation in Eq.(17). The results are summarized in Theorem 1.

### 1. *Derivation of the Filter Equation*

Define $F_k^Y \equiv \{Y(i),\ 0 \le i \le k\}$, as the *σ-algebra* (or the history of the observations) generated by the process $Y$ up to time $k$ and $F_k^{u_i} \equiv \{u_i(j),\ 0 \le j \le k\}$ as the history of the process $u_i$ up to time $k$. In the following, we compute the *a priori* estimate $\hat{X}(k+1 \mid k) \equiv E\{X(k+1) \mid F_k^Y\}$. For this, we take the conditional expectation, with respect to $F_k^Y$ on both sides of Eq.(14) to obtain

$$E\{X(k+1) \mid F_k^Y\} = \Phi_k E\{X(k) \mid F_k^Y\} + \overline{B}(k)E\{\overline{W}(k+1) \mid F_k^Y\} + \overline{C}(k)E\{\overline{N}(k+1) \mid F_k^Y\}. \tag{20}$$

Similarly, from Eq.(15) we have

$$\begin{aligned} E\{\overline{N}_i(k+1) \mid F_k^Y\} = {} & E\{1_{[u_i(k)=-n\gamma_i]}N^i_{1,2}(\Delta) \mid F_k^Y\} \\ & - E\{1_{[u_i(k)=+n\gamma_i]}N^i_{2n+1,2n}(\Delta) \mid F_k^Y\} \\ & + \sum_{j=2}^{2n} E\{1_{[u_i(k)=(j-n-1)\gamma_i]}(N^i_{j,j+1}(\Delta) \\ & - N^i_{j,j-1}(\Delta)) \mid F_k^Y\},\quad i=x,y,z. \end{aligned} \tag{21}$$

Using the properties of the conditional expectations [25], Eq.(21) becomes

$$\begin{aligned} E\{\overline{N}_i(k+1) \mid F_k^Y\} &= E\{1_{[u_i(k)=-n\gamma_i]}E\{N^i_{1,2}(\Delta) \mid F_k^Y \sqcup F_k^{u_i}\} \mid F_k^Y\} \\ &- E\{1_{[u_i(k)=+n\gamma_i]}E\{N^i_{2n+1,2n}(\Delta) \mid F_k^Y \sqcup F_k^{u_i}\} \mid F_k^Y\} \\ &+ \sum_{j=2}^{2n} E\{1_{[u_i(k)=(j-n-1)\gamma_i]}E\{(N^i_{j,j+1}(\Delta) - N^i_{j,j-1}(\Delta)) \\ &\mid F_k^Y \sqcup F_k^{u_i}\} \mid F_k^Y\}, \quad \text{for } i=x,y,z, \end{aligned} \tag{22}$$

where the symbol $\sqcup$ denotes the minimal *σ-algebra* generated by $F_k^Y$ and $F_k^{u_i}$. Employing the fact that

$$E\{N^i_{j,j+1}(\Delta) \mid F_k^Y \sqcup F_k^{u_i}\} = E\{N^i_{j,j+1}(\Delta)\} \cong \Delta\, \lambda^i_{j,j+1}, \tag{23}$$

$$E\{N^i_{j+1,j}(\Delta) \mid F_k^Y \sqcup F_k^{u_i}\} = E\{N^i_{j+1,j}(\Delta)\} \cong \Delta\, \lambda^i_{j+1,j} \quad \text{for } i=x,y,z;\ j=1,2,\ldots,2n, \tag{24}$$

we obtain from Eq.(22) that

$$\begin{aligned} E\{\overline{N}_i(k+1) \mid F_k^Y\} &= \Delta\lambda^i_{1,2}\, E\{1_{[u_i(k)=-n\gamma_i]} \mid F_k^Y\} \\ &- \Delta\lambda^i_{2n+1,2n}\, E\{1_{[u_i(k)=+n\gamma_i]} \mid F_k^Y\} \\ &+ \sum_{j=2}^{2n} E\{1_{[u_i(k)=(j-n-1)\gamma_i]} \mid F_k^Y\}\, (\lambda^i_{j,j+1} - \lambda^i_{j,j-1})\Delta \\ &= \Delta\lambda^i_{1,2}P^i_1(k) - \Delta\lambda^i_{2n+1,2n}P^i_{2n+1}(k) \\ &+ \sum_{j=2}^{2n} \Delta(\lambda^i_{j,j+1} - \lambda^i_{j,j-1})P^i_j(k) \quad \text{for } i=x,y,z \end{aligned} \tag{25}$$

where

$$P^i_l(k) \equiv \Pr[\, u_i(k) = (l-n-1)\gamma_i \mid F_k^Y] \quad \text{for } i=x,y,z;\ l=1,2,\ldots,2n+1. \tag{26}$$

Utilizing $E\{\overline{W}(k+1) \mid F_k^Y\}=0$, we obtain from Eqs.(20) and (25) that

$$\hat{X}(k+1 \mid k) = \Phi_k \hat{X}(k \mid k) + \overline{C}(k)\beta(k+1), \tag{27}$$

where $\beta(k+1) = (\beta_x(k+1), \beta_y(k+1), \beta_z(k+1))'$ and the component $\beta_i(k+1)$ is given by

$$\beta_i(k+1) \equiv \Delta\, \lambda^i_{1,2}\, P^i_1(k) - \Delta\, \lambda^i_{2n+1,2n}\, P^i_{2n+1}(k) + \sum_{j=2}^{2n} \Delta\, (\lambda^i_{j,j+1} - \lambda^i_{j,j-1})\, P^i_j(k) \quad \text{for } i=x,y,z. \tag{28}$$

The *a posteriori* estimate $\hat{X}(k+1 \mid k+1) \equiv E\{X(k+1) \mid F^Y_{k+1}\}$ is computed next. If the standard deviations of the elements of the noise processes $\overline{W}(k)$ and $V(k)$ are close to $\gamma_i$ one may use the standard Kalman filter without any significant degradation in the accuracy of estimates. In this paper, therefore we consider only the case when the jump size $\gamma_i$ is large enough compared with standard deviations of the elements of the noise processes $\overline{W}(k)$ and $V(k)$. In this case the standard Kalman filter yields unacceptable performance and the proposed filter becomes essential. Under the preceding assumption, one can determine the points the discontinuity (or the maneuver) of the state $X(k)$ from the observation $Y(k)$ by using a detection scheme (see Remark 4). It is clear from Eq.(14) that the process $X(k)$ is conditionally Gaussian given the observation history $\{Y(i),\ 0 \le i \le k\}$. Therefore, the estimate $\hat{X}(k+1 \mid k+1)$ can be assumed to have the following form

$$\hat{X}(k+1 \mid k+1) = \hat{X}(k+1 \mid k) + G(k+1)[\,Y(k+1) - H(k+1)\hat{X}(k+1 \mid k)], \tag{29}$$

where $G(k+1)$ is the filter gain yet to be determined. In the following, we use the minimum variance approach to obtain an explicit expression for the filter gain $G(k+1)$. Let

$$P(k \mid k-1) \equiv E\{(X(k) - \hat{X}(k \mid k-1))(X(k) - \hat{X}(k \mid k-1))'\}, \tag{30}$$

$$P(k \mid k) \equiv E\{(X(k) - \hat{X}(k \mid k))(X(k) - \hat{X}(k \mid k))'\}, \tag{31}$$

$$\Theta^i_j(k) \equiv \Pr\{u_i(k) = (j-n-1)\gamma_i\} \quad \text{for } i=x,y,z;\ j=1,2,\ldots,2n+1, \tag{32}$$

$$\overline{Q}(k+1) \equiv \Delta\, Q(k+1). \tag{33}$$

Employing Eqs.(14), (27) and (28), exploiting the independence of $\overline{W}(k)$ and $\overline{N}(k)$, and utilizing the fact that

$$E\{(X(k) - \hat{X}(k \mid k))\, \overline{W}'(k+1)\} = 0 \tag{34}$$

$$E\{(X(k) - \hat{X}(k \mid k))\, \overline{N}'(k+1)\} = 0 \tag{35}$$

$$E\{(X(k) - \hat{X}(k \mid k))\, V'(k+1)\} = 0 \tag{36}$$

$$E\{\overline{W}(k+1)(X(k) - \hat{X}(k \mid k))'\} = 0 \tag{37}$$

$$E\{\overline{N}(k+1)(X(k) - \hat{X}(k \mid k))'\} = 0 \tag{38}$$

$$E\{V(k+1)(X(k) - \hat{X}(k \mid k))'\} = 0 \tag{39}$$

we obtain from Eq.(30) that

$$P(k+1 \mid k) = \Phi_k P(k \mid k)\Phi_k' + \overline{B}(k)\overline{Q}(k+1)\overline{B}'(k) + \overline{C}(k)\Lambda(k+1)\overline{C}'(k), \tag{40}$$

where $\Lambda(k+1)$ is a diagonal matrix whose components are given by

$$\begin{aligned} \Lambda^{i,i}(k+1) &\equiv E\{\overline{N}_i(k+1)\, \overline{N}_i'(k+1)\} \\ &\cong \Theta^i_1(k+1)\lambda^i_{1,2}\Delta + \Theta^i_{2n+1}(k+1)\lambda^i_{2n+1,2n}\Delta \\ &+ \sum_{j=2}^{2n} \Theta^i_j(k+1)\,(\lambda^i_{j,j+1} + \lambda^i_{j,j-1})\Delta \end{aligned} \tag{41}$$

From the state transition diagram in Fig.2, it is obvious that for a sufficiently small $\Delta$, the probabilities $\{\Theta^i_j(k+1);\ j=1,2,\ldots,2n+1;\ i=x,y,z\}$ are recursively computed by the following relations:

$$\Theta^i_1(k+1) \cong (1-\lambda^i_{1,2}\Delta)\Theta^i_1(k) + (\lambda^i_{2,1}\Delta)\Theta^i_2(k), \tag{42}$$

$$\Theta^i_l(k+1) \cong (\lambda^i_{l-1,l}\,\Delta)\,\Theta^i_{l-1}(k) + (1-\lambda^i_{l,l+1}\Delta - \lambda^i_{l,l-1}\Delta)\Theta^i_l(k) + (\lambda^i_{l+1,l}\Delta)\Theta^i_{l+1}(k), \tag{43}$$

$$\Theta^i_{2n+1}(k+1) \cong (\lambda^i_{2n,2n+1}\Delta)\Theta^i_{2n}(k) + (1-\lambda^i_{2n+1,2n}\Delta)\Theta^i_{2n+1}(k), \quad \text{for } l=2,3,\ldots,2n;\ i=x,y,z. \tag{44}$$

Further it follows from Eqs.(29) and (31) that

$$\begin{aligned} P(k+1 \mid k+1) &= P(k+1 \mid k) \\ &- P(k+1 \mid k)H'(k+1)G'(k+1) \\ &- G(k+1)H(k+1)P(k+1 \mid k) \\ &+ G(k+1)[H(k+1)P(k+1 \mid k)H'(k+1) \\ &+ R(k+1)]G'(k+1). \end{aligned} \tag{45}$$

Utilizing the minimum variance approach, we obtain the optimal filter gain matrix $G(k+1)$ as

$$\begin{aligned} G(k+1) = & P(k+1 \mid k)H'(k+1)[H(k+1) \\ & P(k+1 \mid k)H'(k+1) + R(k+1)]^{-1}. \end{aligned} \tag{46}$$

Substituting Eq.(46) into Eq.(45), we obtain

$$P(k+1 \mid k+1) = [I - G(k+1)H(k+1)]P(k+1 \mid k). \tag{47}$$

*2. Linear Minimum Variance Filter*

The above results are summarized in the following theorem.

**Theorem 1.**(Linear Minimum Variance Filter)

*Let the process* $X(k)$ *and* $Y(k)$ *be governed by Equation* (14) *and Equation* (17), *respectively. Further, the increment* $\overline{N}(k)$ *of the Poisson process N is assumed to be independent of the increment* $\overline{W}(k)$ *of the Wiener process W. Then the corresponding linear, adaptive minimum variance filter consists of the following set of equations (Eqs.* (48)-(57)), *for the partitions defined by the expression* (11) *and a sufficiently small sampling interval* $\Delta$:

**Prediction phase:**

$$\hat{X}(k+1 \mid k) = \Phi_k \hat{X}(k \mid k) + \overline{C}(k)\beta(k+1), \tag{48}$$

$$\begin{aligned} P(k+1 \mid k) &= \Phi_k P(k \mid k)\Phi_k' + \overline{B}(k)\overline{Q}(k+1)\overline{B}'(k) \\ &+ \overline{C}(k)\Lambda(k+1)\overline{C}'(k), \end{aligned} \tag{49}$$

*where* $\beta(k+1) \equiv (\beta_x(k+1), \beta_y(k+1), \beta_z(k+1))'$ *and* $\beta_i(k+1)$ *are given by*

$$\begin{aligned} \beta_i(k+1) \equiv & \Delta\, \lambda^i_{1,2} P^i_1(k) - \Delta\, \lambda^i_{2n+1,2n} P^i_{2n+1}(k) \\ & + \sum_{j=2}^{2n} \Delta\, (\lambda^i_{j,j+1} - \lambda^i_{j,j-1}) P^i_j(k), \quad i = x, y, z. \end{aligned} \tag{50}$$

$\Lambda(k+1)$ *is the diagonal matrix and its elements are given by*

$$\begin{aligned} \Lambda^{i,i}(k+1) \cong & \Theta^i_1(k+1)\lambda^i_{1,2}\Delta + \Theta^i_{2n+1}(k+1)\lambda^i_{2n+1,2n}\Delta \\ & + \sum_{j=2}^{2n} \Theta^i_j(k+1)(\lambda^i_{j,j+1} + \lambda^i_{j,j-1})\Delta, \end{aligned} \tag{51}$$

*with*

$\Theta^i_j(k+1) \equiv \Pr\{u_i(k+1) = (j-n-1)\gamma_i\}$, $j = 1, 2, \ldots, 2n+1$,

*governed by*

$$\Theta^i_1(k+1) = (1 - \lambda^i_{1,2}\Delta)\Theta^i_1(k) + (\lambda^i_{2,1}\Delta)\Theta^i_2(k), \tag{52}$$

$$\begin{aligned} \Theta^i_l(k+1) = & (\lambda^i_{l-1,l}\,\Delta)\Theta^i_{l-1}(k) + (1 - \lambda^i_{l,l+1}\Delta \\ & - \lambda^i_{l,l-1}\Delta)\Theta^i_l(k) + (\lambda^i_{l+1,l}\Delta)\Theta^i_{l+1}(k), \end{aligned} \tag{53}$$

$$\begin{aligned} \Theta^i_{2n+1}(k+1) = & (\lambda^i_{2n,2n+1}\Delta)\Theta^i_{2n}(k) \\ & + (1 - \lambda^i_{2n+1,2n}\Delta)\Theta^i_{2n+1}(k), \\ & \text{for } i = x, y, z \text{ and } l = 2, 3, \ldots, 2n \end{aligned} \tag{54}$$

**Filtering phase:**

$$\begin{aligned} \hat{X}(k+1 \mid k+1) &= \hat{X}(k+1 \mid k) + G(k+1)[Y(k+1) \\ &- H(k+1)\hat{X}(k+1 \mid k)], \end{aligned} \tag{55}$$

$$\begin{aligned} G(k+1) = & P(k+1 \mid k)H'(k+1)[H(k+1) \\ & P(k+1 \mid k)H'(k+1) + R(k+1)]^{-1}, \end{aligned} \tag{56}$$

$$P(k+1 \mid k+1) = [I - G(k+1)H(k+1)]P(k+1 \mid k). \quad \square \tag{57}$$

**Remark 3.**

The proposed filter has a similar form as the standard Kalman filter, because for $\overline{C}(k) = 0$ (which represents the non-maneuvering target tracking situation), the proposed filter (Equations (48)-(57)) reduces to the standard Kalman filter. It is noted that the filter proposed by Gholson et al.[18] seems to be structurally similar to our proposed filter, however with one major difference with respect to computational requirements. Gholson's filter is based upon a semi-Markovian model to represent maneuvers and hence it is computationally intensive. The intense computational burden of the Gholson's algorithm has explicitly been pointed out by Gholson et.al[18] as well as by Bogler[10]. This is further explained in the next. If Gholson's scheme, i.e. the semi-Markovian maneuver model having $m$ discrete maneuver levels, is used, one has to perform about $m^6$computer instructions for calculating the state estimate $\hat{X}(k+1|k+1)$. On the contrary, if our proposed method, i.e. the maneuver model using the jump process having $m$ discrete jump levels, is used, one need to carry out only $2m$ computer instructions for obtaining the same quantity $\hat{X}(k+1|k+1)$, instead.

**Remark 4.** (Computation of the conditional probabilities $P^i_l(k)$)

An explicit expression for the conditional probabilities $P^i_l(k)$; $l = 1, 2, \ldots,\ 2n+1$; $i = x, y, z$, defined by Eq.(26) is, in general, difficult to obtain. Although a precise scheme for obtaining the conditional probabilities $P^i_l$ is highly desirable for an optimal filter performance, the excessive computational load may lead us to consider a sub-optimal solution as long as it warrant to yield a good filter performance, as discussed in the next. Since the standard deviations of the elements of the noise processes $\overline{W}(k)$ and $V(k)$ are, in practice, sufficiently small enough compared to the jump size $\gamma_i$, it is possible to approximate the value of $P^i_l$ when the presence or absence of a jump(or maneuver) is detected in the system. This can be accomplished by monitoring the velocity residuals $\tilde{v} \equiv v - \hat{v}$ where $v = \{\dot{x}, \dot{y}, \dot{z}\}'$ and $\hat{v} = \{\hat{\dot{x}}, \hat{\dot{y}}, \hat{\dot{z}}\}'$ and then comparing it against a threshold $v_{dth}$. If $\tilde{v} \geq v_{dth}$ then a jump (or maneuver) is detected and the corresponding probability $P_l^i$ is set to 0.9. Otherwise, it is set to 0.1. The performance of the proposed filter employing this sub-optimal scheme for $P_l^i(k)$ has been demonstrated in Section 4. In fact, the results of the simulation studies clearly show that this sub-optimal scheme for computation of $P_l^i(k)$ suffices to produce a fairly reliable state estimates for a fast maneuvering target.

## IV. The Monte Carlo Simulation

In this section, we consider a numerical example for the system model and the filter equation developed in the preceding two sections. In Section 4.1. we develop a numerical algorithm in order to compute the estimate $\hat{X}(k+1 \mid k+1)$ and the corresponding covariance matrix $P(k+1 \mid k+1)$ on the basis of theorem 1 in Section 3.2. Employing the proposed algorithm, the Monte Carlo simulation is carried out in Section 4.2. The simulation demonstrates the performance of the proposed filter for the target model proposed in Section 2 with known jump parameters. The concluding remarks are presented in Section 4.3.

### *1. Numerical Algorithm*

The major steps of the algorithm are summarized as follows:

Step 1: Set the time index $k$ to zero.

Step 2: Given $\Delta$, $n$, $\lambda^i_{j+1,j}$, $j=1,2,\dots,2n$, $i=x,y,z$, $Q$ and $R$, generate the random processes $\overline{W}(k+1)$, $V(k+1)$ and $\overline{N}(k+1)$

Step 3: Given $A$, $B$, $C$, $\Phi$ and $X(k)$, compute the state $X(k+1)$ using Eq.(14).

Step 4: Given $H(k+1)$, compute the observed process $Y(k+1)$ using Eq.(17).

Step 5: From the estimate $\hat{X}(k \mid k)$ compute $\hat{X}(k+1 \mid k)$ using Eqs. (48) and (50).

Step 6: From $\Theta^i_j(k)$, $j=1,2,\dots,2n+1$, $i=x,y,z$, compute the probabilities $\Theta^i_j(k+1)$, $j=1,2,\dots,2n+1$, $i=x,y,z$, using Eqs.(52) - (54).

Step 7: From $\Theta^i_j(k+1)$, $j=1,2,\dots,2n+1$, $i=x,y,z$ compute $\Lambda(k+1)$ using Eq.(51).

Step 8: From the covariance matrix $P(k \mid k)$ compute $P(k+1 \mid k)$ using Eq.(49).

Step 9: Calculate the gain matrix $G(k+1)$ using Eq.(56)

Step 10: Compute the updated estimate $\hat{X}(k+1 \mid k+1)$ using Eq.(55).

Step 11: Compute the updated covariance matrix $P(k+1 \mid k+1)$ using Eq.(57).

Step 12: Evaluate the R.M.S. errors (called **tRMS**) of $X(k+1)$ defined by

$$R.M.S.[X_l(k+1)] \equiv \sqrt{\frac{1}{k+1}\sum_{j=1}^{k+1}\{X_l(j) - \hat{X}_l(j \mid j)\}^2}, \; l=1,2,\dots,6, \quad \textbf{(58)}$$

where $X_l(k+1)$ is the $l$-th component of the state vector $X(k+1)$.

Step 13: Compute the R.M.S. ensemble errors(called **eRMS**) of the position and velocity.

Step 14: If the time $k \leq t_T$ ($t_T$ is the final time), set $k=k+1$ and go to *Step* 2. Otherwise go to *Step 15*.

Step 15: Print out the results and stop.

### *2. Monte Carlo Simulation and Analysis of the Results*

For the purpose of simulation we used the following system parameters: the drag coefficient $\alpha = 0.3$, the sampling interval $\Delta = 1.0$sec., the jump size $\gamma_i = 100$, $Q = diag.(25^2, 25^2, 25^2) ft^2$ and $R = diag.(35^2 ft^2,\ 0.002rad.,\ 0.002rad.)$. The initial position and velocity of the target were taken as $X(0) = (600, 850, 450, -150, 225, 120)'$. In order to demonstrate the performance of the filter, the Monte Carlo simulation was carried out in the following.

Suppose that the jump process $u_i(t)$ is given by seven discrete states with constant transition rates $\lambda$'s as follows.

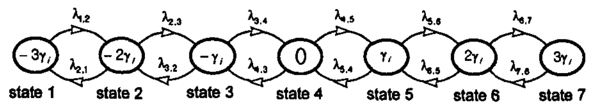


**Fig. 3.** The jump process $u_i(t)$.

In Fig.3 the parameters $\lambda$'s are assumed to be known and chosen as

$$\begin{array}{l} \lambda^i_{1,2}=0.90,\ \lambda^i_{2,3}=0.50,\ \lambda^i_{3,4}=0.30, \\ \lambda^i_{4,5}=0.15,\ \lambda^i_{5,6}=0.20,\ \lambda^i_{6,7}=0.35, \\ \lambda^i_{2,1}=0.25,\ \lambda^i_{3,2}=0.15,\ \lambda^i_{4,3}=0.15, \\ \lambda^i_{5,4}=0.30,\ \lambda^i_{6,5}=0.50,\ \lambda^i_{7,6}=0.80, \\ \quad \text{for } i=x,y, \end{array} \quad \textbf{(59)}$$

$$\begin{array}{l} \lambda^z_{1,2}=0.90,\ \lambda^z_{2,3}=0.80,\ \lambda^z_{3,4}=0.80,\ \lambda^z_{4,5}=0.07, \\ \lambda^z_{5,6}=0.01,\ \lambda^z_{6,7}=0.01, \lambda^z_{2,1}=0.10,\ \lambda^z_{3,2}=0.01, \\ \lambda^z_{4,3}=0.01,\ \lambda^z_{5,4}=0.80,\ \lambda^z_{6,5}=0.80,\ \lambda^z_{7,6}=0.90. \end{array} \quad \textbf{(60)}$$

According to the jump process(Fig.3) which has seven discrete states we have generated the pilot commands shown in Fig.4-Fig.6. Using the proposed filter we computed the corresponding estimate $\hat{X}(k \mid k)$ and the RMS errors in accordance with 200 sample paths for the state $X(k)$ and observation $Y(k)$. Detailed numerical results were obtain from the Monte Carlo simulation. Fig.7-Fig.9 present the observations, i.e. the range, the bearing angel and the elevation angle for one typical sample run. The velocity and position profiles(actual and estimated) for a typical sample run are shown in Fig.10-Fig.12 and Fig.13-Fig.15, respectively. The position and velocity tRMS errors are shown in Fig.16 and Fig.17, respectively and the eRMS errors are presented in Fig.18-Fig.20 and Fig.21-Fig.23. It is evident from Fig.10-Fig.23 that

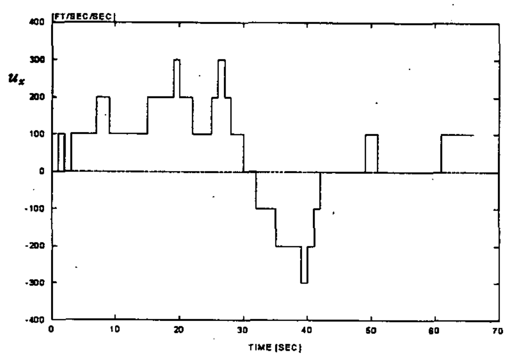


**Fig. 4.** The pilot command in $x$ direction.

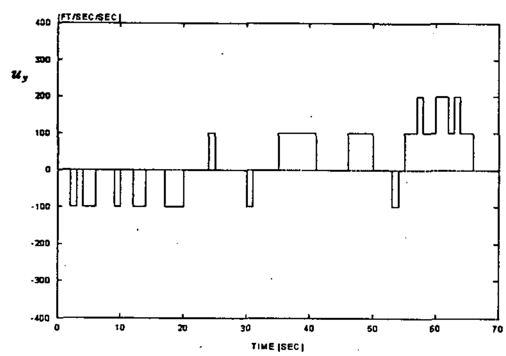


**Fig. 5.** The pilot command in $y$ direction.

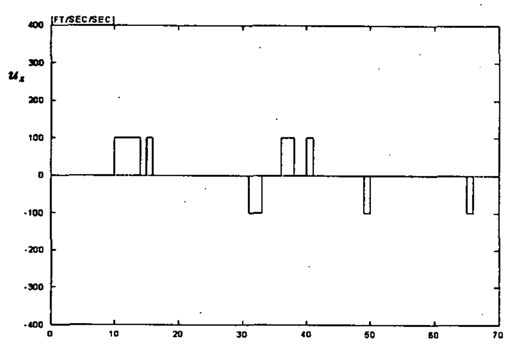


**Fig. 6.** The pilot command in $z$ direction.

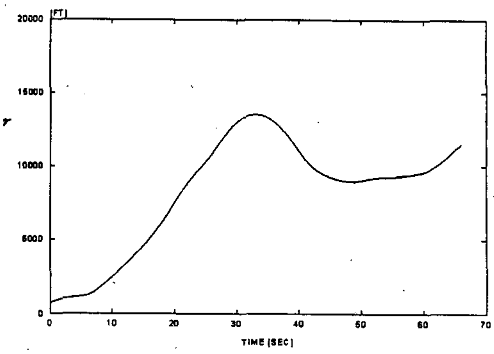


**Fig. 7.** The range.

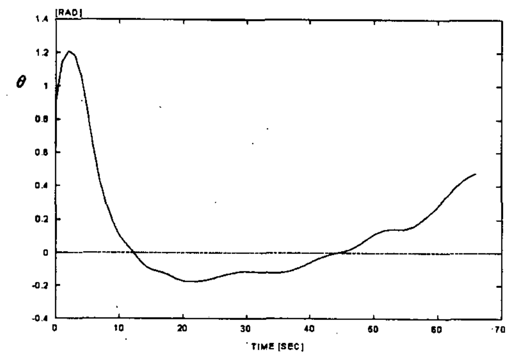


**Fig. 8.** The bearing angle.

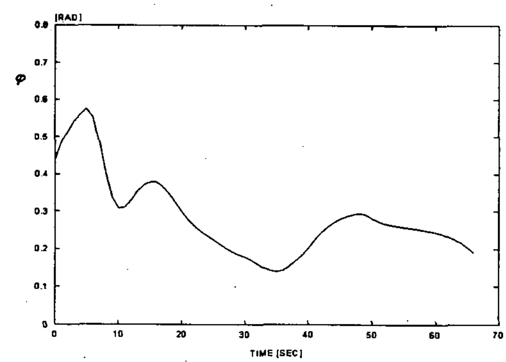


**Fig. 9.** The elevation angle.

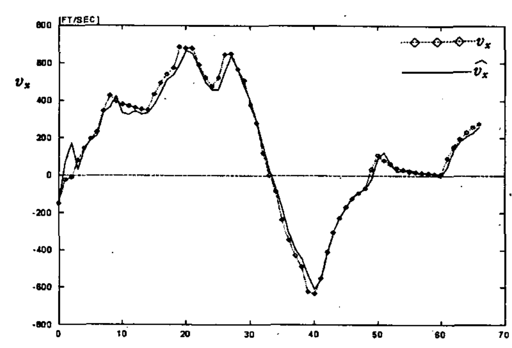


**Fig. 10.** The velocity in $x$ direction.

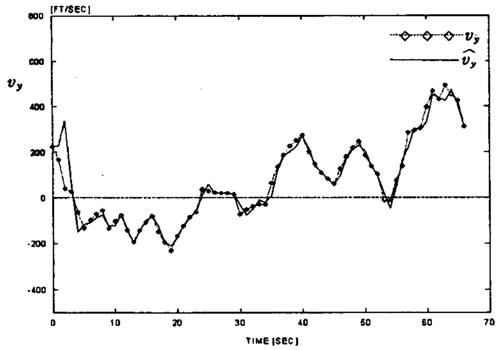


Fig. 11. The velocity in $y$ direction.

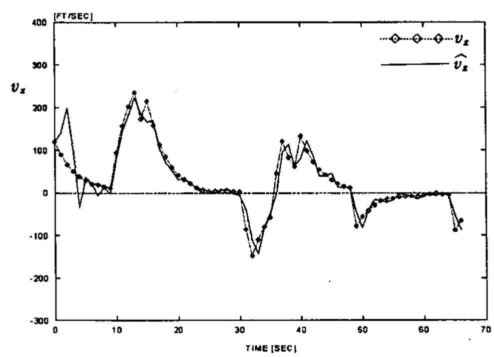


Fig. 12. The velocity in $z$ direction.

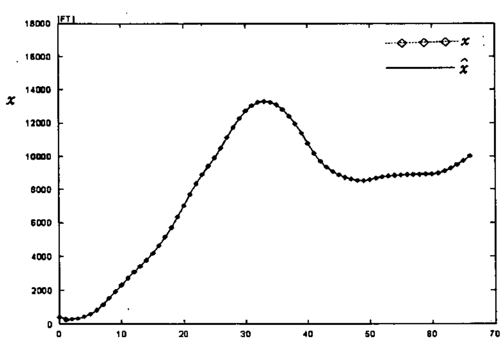


Fig. 13. The position in $x$ direction.

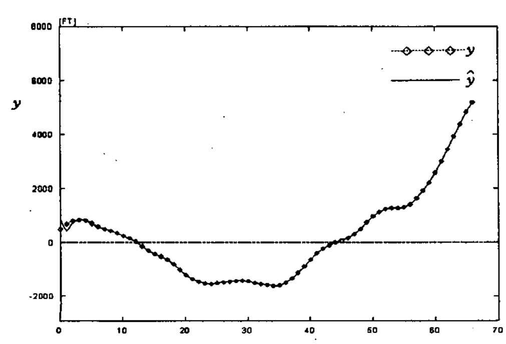


Fig. 14. The position in $y$ direction.

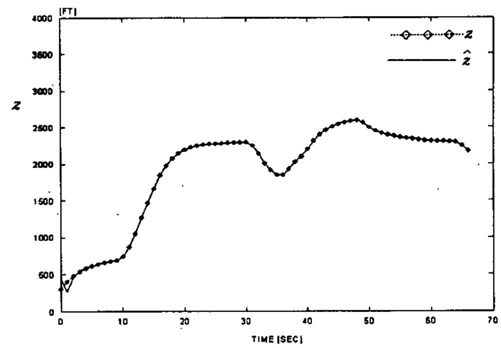


Fig. 15. The position in $z$ direction.

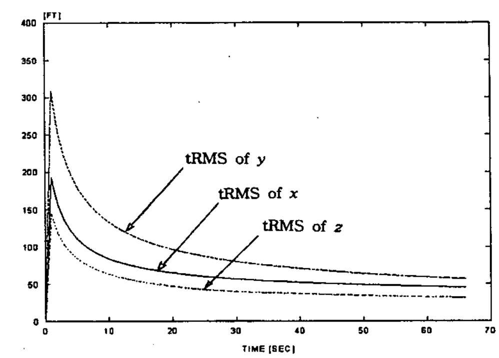


Fig. 16. The tRMS errors of the position.

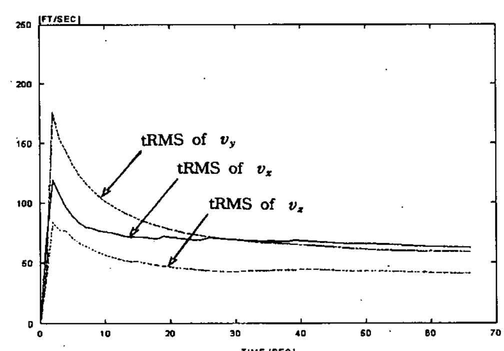


Fig. 17. The tRMS errors of the velocity.

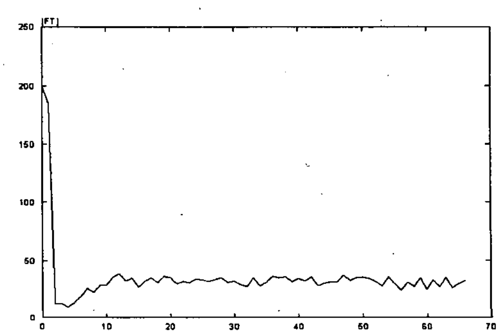


Fig. 18. The eRMS error of $x$.

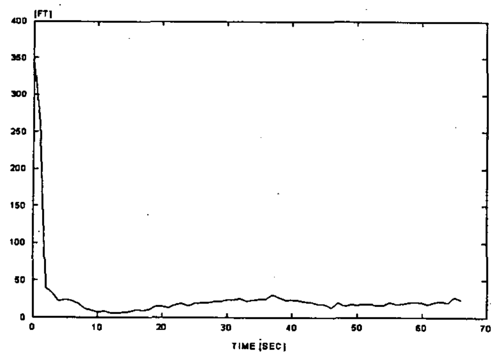


**Fig. 19.** The eRMS error of $y$.

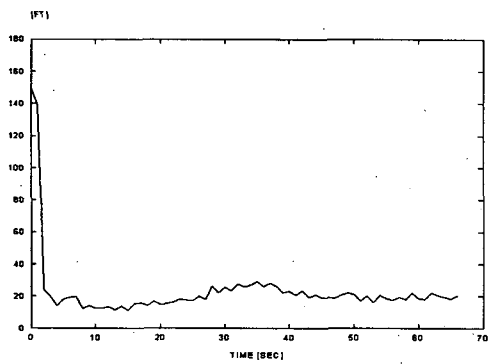


**Fig. 20.** The eRMS error of $z$.

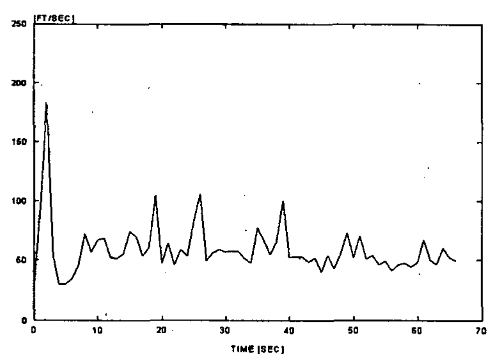


**Fig. 21.** The eRMS error of $v_x$.

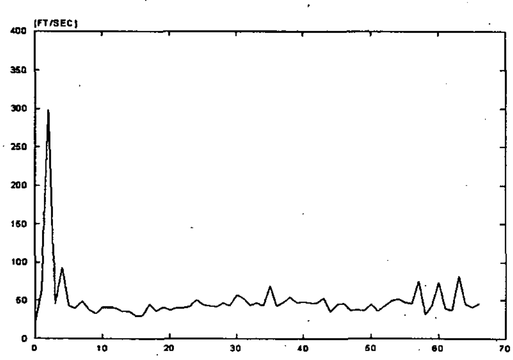


**Fig. 22.** The eRMS error of $v_y$.

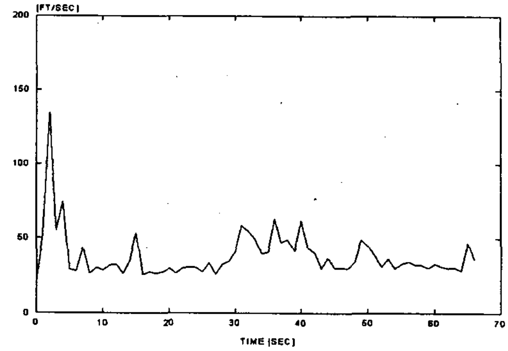


**Fig. 23.** The eRMS error of $v_z$.

(i) The estimated state $\hat{X}(k \mid k)$ is very close to the true state $X(k)$ throughout entire period of tracking and even during the rapid maneuvering under which the other tracking filters[10] usually suffer from significant degradation in performance.

(ii) In Fig.13-Fig.15, the estimated and the actual values are almost identical. This is, in fact, due to large scale of the plot compared to the real difference which is order of $30ft$. This can be more clearly observed from the position eRMS plot.

(iii) The eRMS errors presented in Fig.18 and Fig.19 clearly show that the proposed filter yields state estimates as close as to the level of the noise standard deviations. From the results for tRMS errors shown in Fig.16 and Fig.17, we note that the filter is convergent to the steady state value since the tRMS values are decreasing with time. This also implies that the proposed filter is stable.

(iv) On the whole, the response of the proposed filter to sudden maneuvering target is fairly good and hence one should expect an optimal estimate(in the sense of minimum variance) of fast maneuvering targets.

### 3. *Concluding Remarks*

In summary, the above simulation studies clearly demonstrate that the proposed filter can efficiently track a fast maneuvering target, where most of conventional adaptive filters fail to track properly. In the simulation discussed above, no attention was paid to optimize the computational time for the solution of the filter equation, yet the time was found to be reasonably small. A typical run for the 70 seconds tracking scenario of the system and filter equations given in Theorem 1 of Section 3.2 took 3.5sec (in CPU time) on a SUN Ultra-1 workstation computer. It is expected that the computational time could be further reduced by exploiting more efficient programming techniques.

## V. Conclusions

In this paper, a maneuvering target model with the maneuver dynamics modeled as a jump process of Poisson-type has been proposed. The jump process represents the deterministic maneuver(or pilot)commands and is described by a stochastic differential equation driven by a Poisson process taking values from a set of discrete states. Employing the new maneuver model and using the noisy observations described by a linear difference equation, the author has developed a new recursive, unbiased minimum variance filter, which is structurally simple, computationally efficient and hence real-time implementable. The main contribution of the paper is the development of such an efficient filter for tracking a fast maneuvering target on the basis of the jump processes to represent pilot commands. The proposed filter does not require a time-consuming complex procedure for computing the filter gain or covariance matrices unlike the other existing algorithms proposed in the literature. The performance of the proposed filter was assessed through the numerical results generated from the Monte Carlo simulation. It is clearly observed, from the numerical results for a fast maneuvering target, that the proposed filter provides estimates close to the true track. The sensitivity analysis of the filter with respect to the jump parameters is under study and the result will be reported in a forthcoming paper. Modifications of the proposed filter for applications to maneuvering targets in a cluttered environment present an interesting subject for a further development.

## References

[1] Y. Bar-Shalom and T. Fortmann, *Tracking and Data Association*, Academic Press, New York, 1988.

[2] A. V. Skorohod, *Studies in the Theory of Random processes*, Addison - Wesley, New York, 1965.

[3] A. H. Jazwinski, *Stochastic Processes and Filtering Theory*, Academic Press, New York, 1970.

[4] A. Gelb, *Applied Optimal Estimation*, MIT Press, Cambridge, Mass., 1974.

[5] R.A. Singer, "*Estimating Optimal Tracking Filter Performance for Manned Maneuvering Targets*", IEEE Trans. on Aerospace and Electronic Systems, AES-6, July 1970, pp.473-483.

[6] R.J. McAulay and E.J. Denlinger, "*A Decision - Directed Adaptive Tracker*", IEEE Trans. on Aerospace and Electronic Systems, AES-9, Mar. 1973, pp.229-236.

[7] J.S. Thorp, "*Optimal Tracking of Maneuvering Targets*", IEEE Trans. on Aerospace and Electronic Systems, AES-9, July 1973, pp.512-519.

[8] C.B. Chang, R.H. Whitning, and M. Athans, "*On the State and Parameter Estimation for maneuvering Reentry Vehicle*", IEEE Trans. on Automatic Control, AC-22, Feb. 1977, pp.99-105.

[9] Y. Bar-Shalom and K. Birmiwal, "*Variable Dimension Filter for Maneuvering Target tracking*", IEEE Trans. on Aerospace and Electronic Systems, AES-18, Sept. 1982, pp.621-629.

[10] P.L. Bogler, "*Tracking a Maneuvering Target using Input Estimation*", IEEE Trans. on aerospace and Electronic Systems, AES-23, May 1987, pp.298-310.

[11] H.A.P. Blom and Y. Bar-Shalom, "*The Interacting Multiple - Model Algorithm for Systems with Markovian Switching Coefficients*", IEEE Trans. on Automatic Control, AC-33, Aug. 1988, pp.780-783.

[12] Y. Bar-Shalom, K.C. Chang and H.A.P. Blom, "*Tracking a Maneuvering Target using Input Estimation Versus the Interacting Multiple Model Algorithm*", IEEE Trans. on Aerospace and Electronic systems, AES-25, Mar. 1989, pp.296-300.

[13] K. Spingarn, and H.L. Weidemann, "*Linear Regression Filtering and Prediction for Tracking Maneuvering Aircraft Vehicles*", IEEE Trans. on Aerospace and Electronic Systems, AES-8, Nov. 1972, pp.800-810.

[14] R.L.T. Hampton, and J.R. Cooke, "*Unsupervised Tracking of Maneuvering Vehicles*", IEEE Trans. on Aerospace and Electronic Systems, AES-9, Mar. 1973, pp.197-207.

[15] Y.T. Chan, A.G.C. Hu, and J.B. Plant, "*A Kalman Filter Based Tracking Scheme with Input Estimation*", IEEE Trans. on Aerospace and Electronic Systems, AES-15, Mar. 1979, pp.237-244.

[16] R.L. Moose, H.F. Vanlandingham, and D.H. McCabe, "*Modeling and Estimation for Tracking Maneuvering Target Problem*", IEEE Trans. on Aerospace and Electronic Systems, AES-15, May 1979, pp.448-445.

[17] M. Farooq, and S. Bruder, "*Information Type Filters for Tracking Maneuvering Target*", IEEE Trans. on Aerospace and Electronic Systems, AES-26, 1990, pp.441-454.

[18] N.H. Gholson and R.L. Moose, "*Maneuvering Target Tracking Using Adaptive State Estimation*", IEEE Trans.on Aerospace and Electronic Systems, AES-13, May 1977, pp.310-317.

[19] G.C. Ricker, and J.R. Williams, "*Adaptive Tracking Filter for Maneuvering Targets*", IEEE Trans. on Aerospace and Electronic Systems, AES-14, Jan. 1978, pp.185-193.

[20] K. Demirbas, "*Maneuvering Target Tracking with Hypothesis Testing*", IEEE Trans. on Aerospace and Electronic Systems, AES-23, Nov. 1987, pp.757-766.

[21] N.U. Ahmed and S.S. Lim, "*Continuation of the Study of Discontinuous Jump Processes in Omega and Loran-C Radio Signals*", Report for Defence Research

Establishment Ottawa, Department of National Defence, Canada, Aug. 1987.

[22] T.E. Dabbous, N. U. Ahmed, J. C. McMillan, and D. F. Liang, "*Filtering of Discontinuous Process Arising in Marine Integrated Navigation Systems*", IEEE Trans. on Aerospace and Electronic Systems, AES-24, 1988, pp.85-102.

[23] P. Boel, P. Varaiya, and E. Wong, *Martingales on Jump Processes II: Applications*, SIAM J. of Control, Vol.13, 1975, pp.1022-1060.

[24] M.V. Vaca and D. Snyder, "*Estimation and Decision for Observation Derived from Martingales: Part II*", IEEE Trans. on Information Theory, IT-24, No.1, 1978, pp.32-45.

[25] R.S. Lipster and A.N. Shiryayev, *Statistics of Random Processes,* Springer-Verlarg, New York, 1978.

**Sang Seok Lim** was born in 1955. He received the B.S. in Avionics in 1976 from the Hankuk Aviation University. He received the M.A.Sc in 1984 and Ph.D. in 1990 in Electrical Engineering from the University of Ottawa. From 1976 to 1982 he was employed by the ADD in Daejeon. He was at the Royal Military College of Canada from 1990 to 1992 as a research associate. From 1992 to 1995 he worked at DREO in Ottawa, Canada. Since 1995 he has been an Assistant Professor of Avionics at the Hankuk Aviation University. His current research interests include Radar target tracking, navigation systems, and avionics.