H_∞ Controller Design Based on NLCF Models: A Unified Approach for Continuous and Discrete Systems

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Abstract

In this paper, a unified approach to the H_∞ controller design is proposed under the δ-form for both continuous and discrete systems. Most of important basic concepts of H_∞ control,. such as inner, co-inner, GCARE and GFARE, are reformulated by the unified form. The NLCF(Normalized Left Comprime Factor) plant description has been reviewed in the δ-form, and some corresponding results are proposed. And the unified H_∞ controller is designed which is based on the McFarlane and Glover[1]. The state-space parameterization for all suboptimal controllers is given under the NLCF model which may not be strictly proper, and the central controller is derived by using the solution to Hankel norm approximation problem[2]. The unified controller is applied to the industrial boiler control problem to exemplify the performance of the controller.

I. Introduction

The H_∞ approach to optimal control problem which was originally formulated by Zames[3] has received a considerable amount of attention during the last decade, because it makes it possible to analytically approach to the area of robust stabilization of plants with unstructured uncertainties. However, the most approaches have a restriction that the plant must be reconstructed by a simplified form under some assumptions in order to be applied to general plants [4]. Moreover, solutions to H_∞ optimization problem are typically iterative in nature so that the maximum stability margin for this problem has been obtained from so-called ' γ -iteration' method[5,6], which requires a large computational burden. In recent paper, Englehart and Smith[7] have shown that an explicit formula for maximum stability margin can be derived without iteration in 4-block H_∞ optimal control problem.

McFarlane and Glover[1,6] have shown that, when the coprime factorization of plant is normalized, a surprisingly explicit solution to the robust stabilization problem can be derived and that the 4-block H_∞ control problem is reduced to 1-block problem saving the computational burden. In addition, they have proposed an open-loop shaping controller design technique with which the controller is systematically designed without choosing four frequency weighting functions independently. In particular, the design method above allows performance requirements to be specified within the normalized coprime factorization framework and the trade-offs between performance and robust stability objectives[8].

However, most of approaches to H_∞ control problems have been restricted to continuous-time systems. Although there are some discretization transformations, they have serious numerical errors. Therefore, in order to apply the continuous controllers to the practical problems, they have to be tuned manually by additional experiment because they can not be directly converted to discrete ones in the digital computer implementation of the controller.

The δ -form approach proposed by Middleton and Goodwin[9] is known to have numerical properties superior to those of usual shift form. Also, owing to the similar structure of the δ -operator with differential operator, it can generally use the continuous-time insights in the discrete-time problem and it directly represents the corresponding continuous form as the sampling interval approaches zero. That is, this approach makes possible to solve the unified solution to both continuous and discrete-time cases. These indicate that the δ -form approach may offer a powerful tool to solve the discrete-time control problem for continuous-time plants, that is, hybrid-time systems.

In this paper, a unified approach to the H_∞ control problem is proposed to overcome numerical problems which

Manuscript received February 25, 1997; accepted February 17, 1998.
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occur when the continuous H_∞ controller is discretized by the traditional shift operator as in the existing approaches. Firstly, the GCARE, GFARE, and some basic concepts such as inner, coinner, all-pass etc., are reformulated to supply theoretical background using the δ-transformation proposed by Middleton and Goodwin[9]. Secondly, NLCF(Normalized Left Comprime Factor) model description is reviewed under the δ-transformation, and some corresponding results are proposed in the unified form. Thirdly, the unified H_∞ controller is designed based on the NLCF model description, which corresponds to the control concept proposed by McFarlane and Glover[1,6]. Finally, the unified H_∞ controller is applied to an industrial boiler system to exemplify the performance of the controller proposed.

II. Definitions and Preliminary Results

Consider a continuous-time state-space system

$$\dot{x}(t) = A_c x(t) + B_c u(t) \qquad (2.1)$$
$$y(t) = C_c x(t) + D_c u(t)$$

where A_c, B_c, C_c and D_c are $n\times n$, $n\times m$, $m\times n$ and $m\times m$ matrices, respectively. System (2.1) can be converted to the discrete δ-form model by δ-operator[9] as follows:

$$\delta x(k) = Ax(k) + Bu(k) \qquad (2.2)$$
$$y(k) = Cx(k) + Du(k)$$

where $\delta = \frac{q-1}{\Delta}$, q denotes the usual forward shift operator and Δ is the sampling interval. And the system matrices are given as follows:

$$A = \Omega A_c \qquad (2.3)$$
$$B = \Omega B_c \qquad (2.4)$$
$$C = C_c \qquad (2.5)$$
$$D = D_c \qquad (2.6)$$
$$\Omega = \frac{1}{\Delta}\int_0^\Delta \exp(A_c\tau)d\tau$$

Taking the Laplace transformation in (2.1) and δ-transformation in (2.2) respectively, the complex variable γ of the δ-transformation is related to s, the complex variable of Laplace transformation, as follows :

$$\frac{\Delta}{2}|\gamma|^2 + Re(\gamma) < 0 \quad \longleftrightarrow \quad Re(s) < 0 \qquad (2.7)$$
$$\frac{\Delta}{2}|\gamma|^2 + Re(\gamma) = 0 \quad \longleftrightarrow \quad Re(s) = 0 \qquad (2.8)$$

It should be noted that, while the z-transformation for shift operator model might produce additional unstable zeros, but not the δ-transformation for (2.2) does[9].

RL_∞ denotes the space of proper, real rational functions with no poles on stability boundary contour with bounded norm denoted $\|\cdot\|_\infty$. RH_∞ denotes the subspace of RL_∞ with no poles outside the open stability boundary contour and RH_∞^- denotes the space of RL_∞ in RH_∞ with no poles on the stability boundary contour. And H_∞/L_∞ norm of a TFM, $G(\gamma)$, is denoted by

$$\|G(\gamma)\|_\infty = \sup_\omega \sigma_{\max}[\frac{(e^{j\omega\Delta}-1)}{\Delta}] \qquad (2.9)$$

where '$\sigma_{\max}$' denotes the maximum singular value.

State-space system is denoted

$$G(\gamma) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \qquad (2.10)$$

where $G(\gamma) = C(\gamma I - A)^{-1}B + D$, and the state-space representation of $G(\gamma)^*$ is then

$$G(\gamma)^* = \left[\begin{array}{c|c} -A^T\hat{A}^T & \hat{A}^T C^T \\ \hline -B^T\hat{A}^T & D^T - \Delta B^T\hat{A}^T C^T \end{array}\right] \qquad (2.11)$$

where $\hat{A} = (I+\Delta A)^{-1}$, A and $(I+\Delta A)$ must be invertible.

If $G(\gamma)$ is stable but not necessarily minimal with a state-space realization in (2.10), then the controllability and observability gramians, P and Q respectively, are defined as the solutions to the following unified Lyapunov equations :

$$AP + PA^T + BB^T + \Delta APA^T = 0 \qquad (2.12)$$
$$A^TQ + QA + C^TC + \Delta A^TPA = 0 \qquad (2.13)$$

With the notation in (2.12) and (2.13), the Hankel singular values of G with degree n are given by

$$\sigma_i \triangleq \lambda_i^{1/2}(PQ), \quad i = 1,\cdots,n \qquad (2.14)$$

ordered by convention, $\sigma_1 \geq \sigma_2 \cdots \geq \sigma_n \geq 0$. The Hankel norm, denoted $\|\cdot\|_H$, is defined to be σ_1.

III. Normalized Left Coprime Factorization in δ-domain

All-pass system (or lossless system) and coprime factorization are relevant to many aspects of control theory. In particular, such systems play important roles in H_∞ optimization and model reduction problems. Here, the all-pass system is defined in the δ-domain and the normalized left (respectively, right) coprime factorizations can be obtained in terms of the solution to the generalized control (respectively, filter) algebraic Riccati equation.

Lemma 3.1 Let $G(\gamma)$ be a stable, $m\times p$ transfer function matrix with minimal state-space realization

$$G(\gamma)=\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right],\quad \text{let } P=P^T \text{ be such that}$$

$$AP+PA^T+BB^T+\Delta APA^T=0 \tag{3.1}$$

then G is co-inner, i.e. $GG^*=I$, if and only if

$$BD^T+PC^T+\Delta APC^T=0 \tag{3.2}$$

$$DD^T+\Delta CPC^T=I \tag{3.3}$$

Proof: Refer to [11]. □□□

Lemma 3.2 Let $G(\gamma)$ be a stable, $m\times p$ transfer function matrix with minimal state-space realization

$$G(\gamma)=\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right],\quad \text{let } Q=Q^T \text{ be such that}$$

$$A^TQ+QA+C^TC+\Delta A^TQA=0 \tag{3.4}$$

then G is inner, i.e. $G^*G=I$, if and only if

$$D^TC+B^TQ+\Delta B^TQA=0 \tag{3.5}$$

$$D^TD+\Delta B^TQB=I \tag{3.6}$$

Proof: Refer to [11]. □□□

Finally, note that a square transfer function G is called all-pass if $GG^*=I$ (or equivalently if $G^*G=I$).

Theorem 3.1 Suppose $G(\gamma)=C(\gamma I-A)^{-1}B+D$ with A asymptotically stable. Then there exists $X=X^T$ satisfying the generalized control algebraic Riccati equation (GCARE) in δ-domain,

$$A^TX+XA+C^TC+\Delta A^TXA-L^T(S+\Delta B^TXB)L=0 \tag{3.7}$$

$$L=-(S+\Delta B^TXB)^{-1}[D^TC+B^TX(I+\Delta A)] \tag{3.8}$$

where $R=I+DD^T$ and $S=I+D^TD$.

Proof: The continuous Hamiltonian matrix for GCARE can be interpreted by the δ-operator as follows:

$$H_{\delta C}=\begin{bmatrix} I & \Delta BS^{-1}B^T\\ 0 & I+\Delta(A-BS^{-1}D^TC)^T\end{bmatrix}^{-1}$$

$$\begin{bmatrix}(A-BS^{-1}D^TC) & -BS^{-1}B^T\\ -C^TR^{-1}C & -(A-BS^{-1}D^TC)^T\end{bmatrix}$$

$$=\begin{bmatrix}\tilde{A}+\Delta BS^{-1}B^T(I+\Delta\tilde{A}^T)^{-1}\tilde{C} & -BS^{-1}B^T(I+\Delta\tilde{A}^T)^{-1}\\ -(I+\Delta\tilde{A}^T)^{-1}\tilde{C} & -(I+\Delta\tilde{A}^T)^{-1}\tilde{A}^T\end{bmatrix} \tag{3.9}$$

where $\tilde{A}=A-BS^{-1}D^TC$, $\tilde{C}=C^TR^{-1}C$. Then GCARE (Generalized Control Algebraic Riccati Equation) in δ-domain can be expressed using the Hamiltonian matrix $H_{\delta C}$ as follows:

$$0=[-X\ \ I]H_{\delta C}\begin{bmatrix}I\\X\end{bmatrix}$$

$$=(A-BS^{-1}D^TC)^TX+X(A-BS^{-1}D^TC)$$

$$+\Delta(A-BS^{-1}D^TC)^TX(A-BS^{-1}D^TC)$$

$$-[I+\Delta(A-BS^{-1}D^TC)^T]XB(S+\Delta B^TXB)^{-1}B^TX$$

$$[I+\Delta(A-BS^{-1}D^TC)]+C^T(I-DS^{-1}D^T)C \tag{3.10}$$

Using the matrix inversion lemma in (3.10), we have

$$0=A^TX+XA+\Delta A^TXA+C^TC$$

$$-C^TDS^{-1}[I-\Delta B^TXB(S+\Delta B^TXB)^{-1}]B^TX$$

$$\times(I+\Delta A)-(I+\Delta A)XB[I-\Delta(S+\Delta B^TXB)^{-1}B^TXB]S^{-1}D^TC$$

$$-C^TD[I-\Delta B^TXB-\Delta^2B^TXB(S+\Delta B^TXB)^{-1}B^TXB]S^{-1}D^TC$$

$$-(I+\Delta A^T)XB(S+\Delta B^TXB)^{-1}B^TX(I+\Delta A) \tag{3.11}$$

Applying the matrix inversion lemma again to (3.11), we can complete GCARE as

$$0=A^TX+XA+\Delta A^TXA+C^TC$$

$$-[C^TD+(I+\Delta A^T)XB](S+\Delta B^TXB)^{-1}$$

$$\times[B^TX(I+\Delta A)+D^TC]$$

Let the matrix formed of the generalized eigenvectors corresponding to the eigenvalues of $H_{\delta C}$ inside or on the stability boundary. Then

$$H_{\delta C}\begin{bmatrix}X_{11}\\X_{21}\end{bmatrix}=\begin{bmatrix}X_{11}\\X_{21}\end{bmatrix}\Lambda$$

where Λ is a diagonal matrix of eigenvalues, Jordan form matrix. From (3.9), we have

$$H_{\delta C}\begin{bmatrix}I\\X_{21}X_{11}^{-1}\end{bmatrix}=\begin{bmatrix}X_{11}\Lambda X_{11}^{-1}\\X_{21}\Lambda X_{11}^{-1}\end{bmatrix} \tag{3.12}$$

and the first row can be written by (3.9) as follows:

$$X_{11}\Lambda X_{11}^{-1}$$

$$=A-BS^{-1}[I-\Delta(I+\Delta XBS^{-1}B^T)^{-1}B^TXBS^{-1}]D^TC$$

$$-\Delta BS^{-1}(I+\Delta XBS^{-1}B^T)^{-1}B^TXA$$

$$=A-B(S+\Delta B^TXB)^{-1}(D^TC+B^TX(I+\Delta A)$$

$$=A+BL \tag{3.13}$$

Thus, we see that the matrix on the left side of (3.13) is the "A" matrix of the closed-loop system. Hence Λ represents the Jordan form of this matrix and X_{11} represents the corresponding matrix of eigenvectors. □□□

Theorem 3.2 Suppose $G(\gamma)$ with A asymptotically stable. Then there exists $Z = Z^T$ satisfying the generalized filter algebraic Riccati equation (GFARE) in δ-domain

$$AZ+ZA^T+BB^T+\Delta AZA^T-H(R+\Delta CZC^T)H^T=0 \quad (3.14)$$

$$H = -[(I+\Delta A)ZC^T+BD^T](R+\Delta CZC^T)^{-1} \quad (3.15)$$

Proof: From the continuous Hamiltonian matrix for GFARE, it can be also interpreted by δ-operator.

$$H_{\delta F}=\begin{bmatrix} I & \Delta C^TR^{-1}C \\ 0 & I+\Delta(A-BR^{-1}D^TC)^T\end{bmatrix}^{-1}$$
$$\begin{bmatrix} (A-BR^{-1}D^TC)^T & -C^TR^{-1}C \\ -BS^{-1}B^T & -(A-BR^{-1}D^TC)\end{bmatrix}$$
$$=\begin{bmatrix} \tilde{A}^T+\Delta C^TR^{-1}C(I+\Delta\tilde{A})^{-1}\tilde{B} & -C^TR^{-1}C(I+\Delta\tilde{A})^{-1} \\ -(I+\Delta\tilde{A})^{-1}\tilde{B} & -(I+\Delta\tilde{A})^{-1}\tilde{A}\end{bmatrix} \quad (3.16)$$

where $\tilde{A}=A-BD^TR^{-1}C$, $\tilde{B}=BS^{-1}B^T$ and it is multiplied as

$$0=[-Z \;\; I]H_{\delta F}\begin{bmatrix} I \\ Z\end{bmatrix} \quad (3.17)$$

$$=B(I-D^TR^{-1}D)B^T+Z(A-BD^TR^{-1}C)^T+(A-BD^TR^{-1}C)Z$$
$$+\Delta(A-BD^TR^{-1}C)Z(A-BD^TR^{-1}C)^T-[I+\Delta(A-BD^TR^{-1}C)]$$
$$\times ZC^T(R+\Delta CZC^T)^{-1}CZ[I+\Delta(A-BD^TR^{-1}C)^T] \quad (3.18)$$

Using the matrix inversion lemma in (3.18), we have

$$0=AZ+ZA^T+\Delta AZA^T+BB^T-BD^TR^{-1}[I-\Delta CZC^T(R+\Delta CZC^T)^{-1}]$$
$$\times CZ(I+\Delta A^T)-(I+\Delta A)ZC^T[I-(R+\Delta CZC^T)^{-1}CZC^T]R^{-1}DB^T$$
$$-BD^T[I-R^{-1}CZC^T-\Delta^2R^{-1}CZC^T(R+\Delta CZC^T)^{-1}CZC^T]R^{-1}DB^T \quad (3.19)$$

Now, applying the matrix inversion Lemma again in (3.19), we can complete GFARE as

$$0=AZ+ZA^T+\Delta AZA^T+BB^T-[(I+\Delta A)ZC^T+BD^T]$$
$$\times(R+\Delta CZC^T)^{-1}[DB^T+CZ(I+\Delta A^T)] \quad (3.20)$$

The strong solution of the GFARE in δ-domain is obtained by choosing $\begin{bmatrix} X_{11} \\ X_{21}\end{bmatrix}\in R^{2n\times n}$ to span the nth order stable invariant subspace of $H_{\delta F}$. The strong solution for (3.16) is then

$$X_{11}\Lambda X_{11}^{-1}=A-BD^TR^{-1}[I-\Delta CZC^T(R+\Delta CZC^T)^{-1}]C$$
$$+(I+\Delta A)ZC^T(R+\Delta CZC^T)^{-1}C$$
$$=A-[(I+\Delta A)ZC^T+BD^T](R+\Delta CZC^T)^{-1}C$$
$$=A+HC \quad (3.21)$$

Now, it is easy to know that the both of GCARE and GFARE in δ-domain also have dual property. □□□

Theorem 3.3 Let (A,B,C,D) be a minimal state-space realization with associated transfer function $G(\gamma) = C(\gamma I-A)^{-1}B+D$. If there is a normalized left coprime factorization $G(\gamma) = \tilde{M}^{-1}\tilde{N}$ such that

$$[\tilde{N} \;\; \tilde{M}] = \left[\begin{array}{c|cc} A+HC & B+HD & H \\ \hline YC & YD & Y\end{array}\right] \quad (3.22)$$

$\tilde{N}$, $\tilde{M}\in RH_\infty$ and $\tilde{N}\tilde{N}^*+\tilde{M}\tilde{M}^*=I$, then

(1) $H=-[\tilde{A}^{-1}ZC^T+BD^T](Y^TY)^{-1}$ (3.23)

(2) $Z=Z^T$ is unique positive definite and symmetric (3.24)

where $Y^TY = (R+\Delta CZC^T)^{-1}$.

Proof: Let $\tilde{G}(\gamma)=[\tilde{N} \;\; \tilde{M}]$. Then state-space realization of $\tilde{G}(\gamma)$ is

$$\tilde{G}(\gamma) = \tilde{C}(\gamma I-\tilde{A})^{-1}\tilde{B}+\tilde{D}$$

where $\tilde{A} = A+HC$, $\tilde{B} = [B+HD \;\; H]$, $\tilde{C} = YC$, $\tilde{D} = [YD \;\; Y]$.

Condition (1) : we can define the co-inner,

$$\tilde{G}(\gamma)\tilde{G}(\gamma)^* = I, \text{ if } \tilde{G}(\gamma) \text{ is stable and } \tilde{B}\tilde{D}^T+Z\tilde{C}^T+\Delta\tilde{A}Z\tilde{C}^T=0.$$

It can be written again as

$$\tilde{B}\tilde{D}^T = -(Z\tilde{C}^T+\Delta\tilde{A}Z\tilde{C}^T)$$
$$[B+HD \;\; H]\begin{bmatrix} D^TY \\ Y\end{bmatrix} = -[ZC^TY+\Delta(A+HC)ZC^TY]$$
$$H(I+DD^T+\Delta CZC^T)=-[(I+\Delta A)ZC^T+BD^T]$$

$$\therefore \; H = -(\tilde{A}^{-1}ZC^T+BD^T)(Y^TY)^{-1} \quad (3.25)$$

Condition (2) : $\tilde{G}(\gamma)$ also satisfies the Lyapunov equation (2.12) as

$$\tilde{A}Z+Z\tilde{A}^T+\tilde{B}\tilde{D}^T+\Delta\tilde{A}Z\tilde{A}^T=0 \quad (3.26)$$

$$(A+HC)Z+Z(A+HC)^T+[B+HD \;\; H]\begin{bmatrix} D^TY \\ Y\end{bmatrix}$$
$$+\Delta(A+HC)Z(A+HC)^T$$
$$=AZ+ZA^T+BB^T+\Delta AZA^T-H(R+\Delta CZC^T)H^T$$
$$=AZ+ZA^T+HH^T+HDD^TH+\Delta AZA^T+\Delta HCZC^TH^T$$
$$-(ZC^T+BD^T+\Delta AZC^T)H^T-H(DB^T+\Delta CZA^T+HCZ)=0 \quad (3.27)$$

This can arrive finally at the Generalized Filter Algebraic Riccati Equation (GFARE) in δ-domain for Z. The only remaining point is to establish that $\tilde{A}$ is indeed a stable matrix, i.e., that $A+HC$ is Hurwitz, which can be easily shown by (3.21). Hence Z is the solution of GFARE. □□□

IV. H_∞ Suboptimization Problem with NLCF Model

The following theorem given by McFarlane and Glover[1,6] shows the optimization of 4-block H_∞ problem solving 1-block Hankel norm approximation problem in δ-domain while the NLCF model is normalized.

Theorem 4.1 For $\varepsilon < \varepsilon_{max}$ NLCF model, followings are equivalent[1,6].

(1) (G, K) is internally stable and satisfies

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I-GK)^{-1} [G \quad I] \right\|_\infty \le \varepsilon^{-1} \tag{4.1}$$

(2) All optimal controllers are given by

$$\left\| \begin{bmatrix} -\widetilde{N}^* \\ \widetilde{M}^* \end{bmatrix} + \begin{bmatrix} U \\ V \end{bmatrix} \right\|_\infty \le (1-\alpha^{-2})^{1/2} \tag{4.2}$$

where, $\alpha = \varepsilon^{-1}$, $K = UV^{-1}$ for $U, V \in RH_\infty$.

On the theorem 3.3 and (2.12), the state-space model of $[-\widetilde{N} \ \widetilde{M}]^*$ is

$$\begin{bmatrix} -\widetilde{N}^* \\ \widetilde{M}^* \end{bmatrix} = \left[\begin{array}{c|c} -(A+HC)^T \hat{A}_H{}^T & \hat{A}_H{}^T C^T Y^T \\ \hline (B+HD)^T \hat{A}_H{}^T & -D^T Y^T - \Delta (B+HD)^T \hat{A}_H{}^T C^T Y^T \\ -H^T \hat{A}_H{}^T & Y^T - \Delta H^T \hat{A}_H{}^T C^T Y^T \end{array}\right] \tag{4.3}$$

where, $\hat{A}_H{}^T = [I + \Delta (A+HC)^T]^{-1}$.

We first apply the result of the theorem 4.1 to obtain all controllers using (4.3). Then all Q_K satisfying the result of Hankel norm approximation problem[2] are given as follows:

$$Q_K = F_U(S(\gamma), \phi), \tag{4.4}$$

where $\phi \in RH_\infty{}^{m\times p}$, $\|\phi\|_\infty \le 1$, $\begin{bmatrix} -\widetilde{N}^* \\ \widetilde{M}^* \end{bmatrix} = \left[\begin{array}{c|c} A_U & B_U \\ \hline C_U & D_U \end{array}\right]$

$$S(\gamma) = \left[\begin{array}{c|cc} S_{11} & -\alpha^2 S_{11} & S_{12} \\ \hline S_{21} & 1-\alpha^2 D_{21}^{-1} & S_{22} \end{array}\right]$$

with state-space form $\left[\begin{array}{c|c} S_{11} & S_{12} \\ \hline S_{21} & S_{22} \end{array}\right] = \left[\begin{array}{c|c} S_a & S_b \\ \hline S_c & S_d \end{array}\right]$

$$S_a = -A_U^T \hat{A}_H{}^T$$

$$S_b = [R_U^{T^{-1}} (\hat{A}_{U^T} C_U{}^T D_{U11} + Q_U B_U) D_{U21}^{-1} \quad R_U^{T^{-1}} \hat{A}_{U^T} C_U^T D_{U12} D_\perp]$$

$$S_c = \begin{bmatrix} -C_U P_U - D_U B_U^T \hat{A}_U{}^T \\ B_U^T \hat{A}_U{}^T \end{bmatrix}$$

$$S_d = \begin{bmatrix} (D_{U11} - D_U) D_{U21}^{-1} & D_{U12} D_\perp \\ -D_{U21}^{-1} & 0 \end{bmatrix}$$

and P_U, Q_U are Gramians for $\begin{bmatrix} -\widetilde{N}^* \\ \widetilde{M}^* \end{bmatrix}$.

$R_U = P_U Q_U - \rho^2 I$ and $D_\perp$ is a arbitrary matrix such that $[\rho^2 D_{U12}^{-1} (D_U - D_{U11}) D_{U21}^{-1} \quad D_\perp]$ is a unitary matrix.

The next theorem gives a simplified solution to Hankel norm approximation problem in this case.

Theorem 4.2 For $\|\widetilde{N} \quad \widetilde{M}\|_\infty < \beta < 1$, the parameterization of all controllers satisfying theorem 4.1 is given by

$$K = (S_{11U} + \tilde{S}_{12U} \Phi)(S_{11V} + \tilde{S}_{12V} \Phi)^{-1} \tag{4.5}$$

where $S_{11} = \begin{bmatrix} S_{11U} \\ S_{11V} \end{bmatrix}$, $\beta S_{12} = \begin{bmatrix} \tilde{S}_{12U} \\ \tilde{S}_{12V} \end{bmatrix}$, $\beta = \varepsilon^{-1}(1-\varepsilon^2)^{\frac{1}{2}}$

and $\Phi \in RH_\infty{}^{m\times p}$, $\|\Phi\|_\infty \le 1$.

Proof: Let $\alpha = (1-\varepsilon^2)^{\frac{1}{2}}$, then from (4.4) $\begin{bmatrix} U \\ V \end{bmatrix}$ satisfying theorem 4.1 and

$$\begin{bmatrix} U \\ V \end{bmatrix} = (S_{11} - \alpha^2 S_{11}\phi_1 + S_{12}\phi_2)[S_{21} - (\alpha^2 S_{21} + D_{21}^{-1})\phi_1 + S_{22}\phi_2]^{-1} \tag{4.6}$$

where $\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \in RH_\infty^{p\times m}$, and $\phi_1 \in RH_\infty^{p\times m}$, $\phi_2 \in RH_\infty^{m\times m}$.

Let $U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$, then $K = UV^{-1} = U_1 V (U_2 V)^{-1} = U_1 U_2^{-1}$

since V is a unit in RH_∞. Now, all controller K can be constructed with only U, and U can be represented as

$$U = (S_{11} + S_{12}\beta)(I - \alpha^2 \phi_1)$$

where $\beta = \phi_2 (I - \alpha^2 \phi_1)^{-1} \in RH_\infty^{m\times p}$

and $\|\beta\|_\infty \le \varepsilon^{-1}(I-\varepsilon^2)^{-\frac{1}{2}}$ [6]. And similarly as above, $(I-\alpha^2\phi_1)$ is a unit in RH_∞. All controller can be given by a coprime factorization of

$$U = \begin{bmatrix} S_{11U} + \tilde{S}_{12U}\Phi \\ S_{11V} + \tilde{S}_{12V}\Phi \end{bmatrix}$$

Therefore the resulting controller is parameterized by

$$K = (S_{11U} + \tilde{S}_{12U}\Phi)(S_{11V} + \tilde{S}_{12V}\Phi)^{-1}$$ □□□

Now the state-space parametrization for all suboptimal controllers, $K = UV^{-1}$, is given in the next theorem in combination with the state-space realization of $[-\widetilde{N} \quad \widetilde{M}]^*$.

Theorem 4.3 All $\delta-H_\infty$ controllers for the NLCF robust stabilization problem satisfying theorem 4.2 are given by

$$K=F_U(L,\ \Phi) \tag{4.7}$$

where the state-space form of L is $L=\left[\begin{array}{c|c} L_a & L_b \\ \hline L_c & L_d \end{array}\right]$

$$L_a=A+HC \tag{4.8}$$

$$\begin{aligned} L_b= R^{T^{-1}}[\,(B+HD)D_{U11}-HD_{U112} \\ +Q_U\widehat{A}_H{}^T C^T Y^T\,]D_{21}^{-1} \;\vdots\; \beta R_U^{T^{-1}} \\ \times[\,(B+HD)D_{\perp1}-HD_{\perp2}\,] \end{aligned} \tag{4.9}$$

$$L_c=\begin{bmatrix} -(B+HD)^T\,\widehat{A}_H{}^T[P_U+\Delta\,\widehat{A}_H{}^T C^T Y^T YC]-D^T Y^T YC \\ H^T\,\widehat{A}_H{}^T[P_U+\Delta C^T Y^T YC]-Y^T YC \end{bmatrix} \tag{4.10}$$

$$L_d=\begin{bmatrix} [D_{U11}+D^T Y^T-\Delta(B+HD)^T\,\widehat{A}_H{}^T C^T Y]D_{U21}^{-1} & \beta D_{\perp1} \\ (D_{U112}-Y^T+\Delta H^T\,\widehat{A}_H{}^T C^T Y^T)D_{U21}^{-1} & \beta D_{\perp2} \end{bmatrix} \tag{4.11}$$

and

$$D_{11}=\begin{bmatrix} D_{111} \\ D_{112} \end{bmatrix}=\begin{bmatrix} -\Delta\alpha^2(B+HD)^T\,\widehat{A}_H{}^T R_U^{-1}C^T Y^T(I+\Delta Y(Q_U R_U^{-1}C^T Y^T)^{-1} \\ \Delta\alpha^2 H^T\,\widehat{A}_H{}^T R_U^{-1}C^T Y^T(I+\Delta YCQ_U R_U^{-1}C^T Y^T)^{-1} \end{bmatrix}$$

$$D_{U12}^T D_{U12}=\alpha^2[I+\Delta\begin{bmatrix} B+HD \\ -H^T \end{bmatrix}R_U^{-1}P_U[\,B+HD \quad -H\,]]^{-1}$$

$$D_{U12}D_\perp=\begin{bmatrix} D_{\perp1} \\ D_{\perp2} \end{bmatrix}$$

$$D_{U21}^T D_{U21}=\alpha^2(I+\Delta YCQ_U R_U^{-1}C^T Y^T)^{-1}$$

$\alpha=(1-\varepsilon^2)^{\frac{1}{2}}$ and $\Phi\in RH_\infty^{m\times p}$, $\|\Phi\|_\infty\le 1$.
and P_U and Q_U are the solutions to P_U, Q_U -Lyapunov equations of $[-\widetilde{N} \quad \widetilde{M}\,]^*$, respectively.

Proof: From the (4.4) and the theorem 4.2, we have figure of

$$K=(S_{U11}+\widetilde{S}_{U12U}\phi)(S_{U11V}+\widetilde{S}_{U11V}\phi)^{-1} \tag{4.12}$$

therefore $L(\gamma)=[\,S_{11} \quad \widetilde{S}_{12}\,]=$

$$\left[\begin{array}{c|cc} -A_U^T\,\widehat{A}_{H^T} & R_U^{T^{-1}}(\,\widehat{A}_{U^T}C_U{}^T D_{U11}+Q_U B_U)D_{U21}^{-1} & \beta R_U^{T^{-1}}\,\widehat{A}_{U^T}C_U^T D_{U12}D_\perp \\ \hline -C_U P_U-D_U B_U^T\,\widehat{A}_U{}^T & (D_{U11}-D_U)D_{U21}^{-1} & \beta D_{U12}D_\perp \end{array}\right]$$

□□□

A particular controller, central controller , in the theorem 4.3 is given by $K_0=L_{11}L_{21}^{-1}$ corresponding to $\Phi=0$, because it is written as a unity feedback system[1]. Notice, at the moment when the sampling time approaches zero, the controller K is the same with continuous-time ones developed by McFarlane and Glover[1]. Therefore, it is possible to design the unified H_∞ controller of continuous and discrete H_∞ controllers by solving H_∞ optimization problem in δ-domain.

V. Application to Industrial Boiler Control

To exemplify the performance of the proposed controller, we will design a continuous and a discrete H_∞ suboptimal controller for a gas- or oil-fired boiler using the proposed algorithm and the well-known loop-shaping design technique. The design example deals with the heating-cogeneration boiler model which exhibits nonlinearities, instability, time delays, non-minimum phase behaviour, and coloured noise disturbances with sensor noise in the frequency range of the significant plant dynamics[10]. A properly functioning boiler must satisfy the following requirements; i) a desired steam pressure must be maintained at the outlet of the drum; ii) the water in the drum must be maintained at the desired level to prevent overheating of drum components or flooding of steam lines; iii) the mixture of fuel and air in the combustion chamber must meet standards for safety, efficiency, and protection of the environment, which is usually accomplished by maintaining a desired percentage of oxygen in the stack in excess of that required for a perfect, or stoichiometric combustion, usually referred to as excess oxygen. The control variables and measurements are as below:

u_1, u_2, u_3 - fuel, air flow, and feedwater flow rate,
y_1, y_2, y_3 - steam pressure(p.s.i), excess oxygen(%), and water level(in).

The linear and nonlinear model is given in [10] which is controllable and observable. We firstly find discrete-time system matrices in δ-domain with sampling time 0.1(sec), which is selected with consideration of the closed-loop bandwidth, as follows:

$$A=\begin{bmatrix} -5.507e-3 & 0 & 0 & -1.584e-1 \\ 0 & -2.041e-1 & 0 & 0 \\ -1.216e-2 & 0 & 0 & -5.660e-1 \\ 0 & 0 & 0 & -3.992e-2 \end{bmatrix}$$

$$B=\begin{bmatrix} 2.797e-1 & 0 & -1.348e-2 \\ -9.279e+0 & 7.580 & 0 \\ -1.020e-3 & 0 & 7.317e-1 \\ 2.993e-2 & 0 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} -1.421e+1 & 0 & 0 & 0 \\ 0 & 1.000e+1 & 0 & 0 \\ 3.221e-1 & 0 & 1.434e-1 & 1.116e+1 \end{bmatrix}$$

$$D=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.272e+0 & 0 & -2.080e-1 \end{bmatrix}$$

Note that the discrete-time model doesn't have any additional nonminimum phase, since δ-transformation always maintains the minimum characteristics even in discretization procedure like this case.

The loop shaping weighting function we decided on is

$$W = \frac{10s^2 + 10s + 0.5s}{s(100s + 5.5)} I_{3\times 3}$$

in which the gain, and pole and zeros locations were arrived at the required loop shape at high-and low-frequency. Fig. 1 shows the unshaped open loop singular values plot. The corresponding shaped plant and weighting function singular value plots are given in Fig. 2 and Fig. 3. From Fig. 1 to Fig. 8 we use solid and dashed curves to distinguish continuous and discrete time results.

Then the allowable maximum stability margin for shaped plant is $\varepsilon_{\max} = 0.370$. In presented design, the stabilizing H_∞ controller, obtained in the design procedure of the previous Section, is chosen to be a suboptimal controller. In general, the suboptimal stability margin, ε, is chosen such that $0.95\varepsilon_{max} \le \varepsilon \le \varepsilon_{max}$, and it clearly reflects the maximum achievable stability margin for the particular problem[6]. In this case ε is selected as $0.98\varepsilon_{max}$.

The compensated open loop is given in Fig. 4. Fig. 5 shows the sensitivity function of $\sigma_{\max}[I - GK)^{-1}]$. In Fig. 6, the plot of $\sigma_{max}[(I-GK)]^{-1}G]$ highlights the effect of including integral action in the shaping function, giving zero steady-state transmission of input disturbance signal. In Fig. 7 the bound of additive uncertainty, $1/\{\sigma_{\max}[K(I-GK)^{-1}]\}$, can be allowed for in excess the magnitude of plant at high frequency. Fig. 8 shows $1/\{\sigma_{\max}[GK(I-GK)^{-1}]\}$ and indicates that at low frequency, admissible bound of output multiplicative uncertainties with 100% of plant magnitude can be tolerated and at high frequency, the uncertainty well in excess of the plant magnitude can be tolerated.

It is noted that the plots of the closed-loop TFMs in δ-domain coincide with ones of s-domain within the ranges of smaller frequencies than sampling frequency. This indicate that the design of discrete-time controllers with fast sampling frequency has good numerical properties. Fig. 9 - Fig. 16 shows the time-responses with respect to step input to both nominal and nonlinear plant. In Fig. 9 - Fig. 10, the time-responses with linear continuous and discrete H_∞ controller are shown. Fig. 11 - Fig 16 shows the nonlinear behaviors with continuous and discrete H_∞ controller. All of performances specified, that of maximum overshoot and steady-state error, are satisfactory. Also it is true that the responses with discrete-time H_∞ controller are very close to that of continuous-time H_∞ controller.

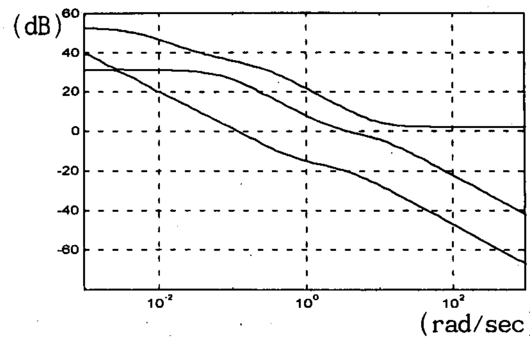

Fig. 1. Nominal plant

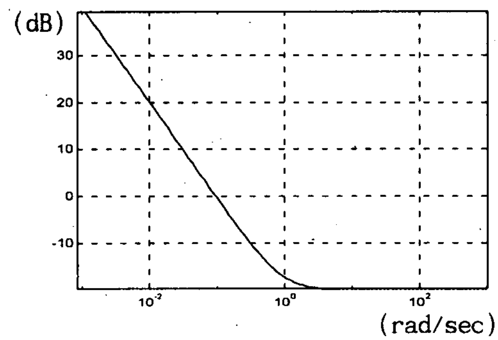

Fig. 2. Weighting function

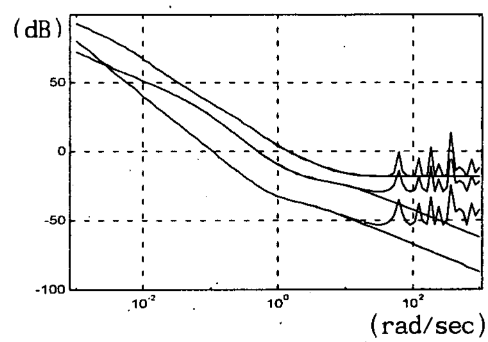

Fig. 3. Shaped plant

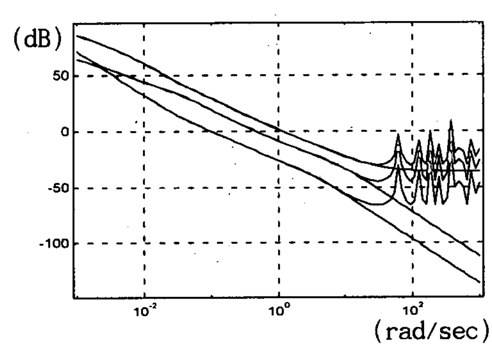

Fig. 4. Compensated open loop

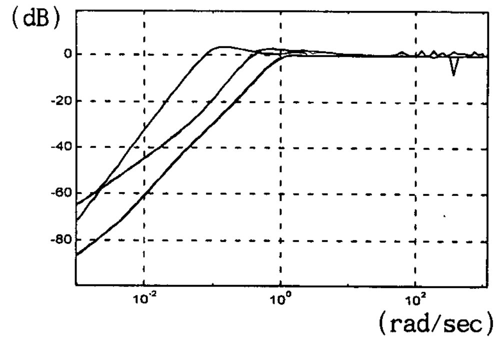

Fig. 5. Sensitivity function

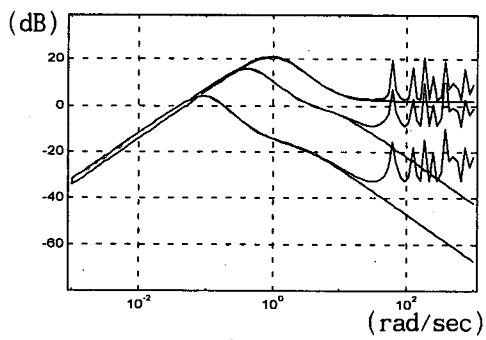

Fig. 6. Input disturbance

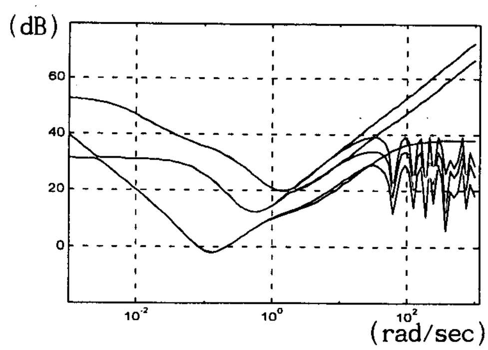

Fig. 7. Additive uncertainty

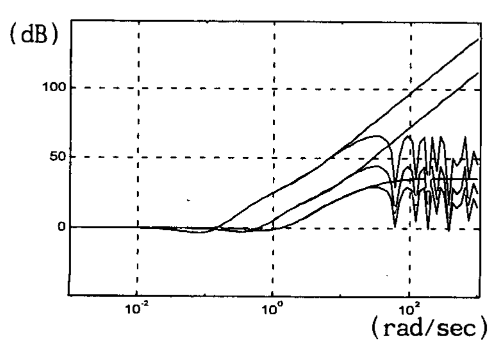

Fig. 8. Multiplicative uncertainty

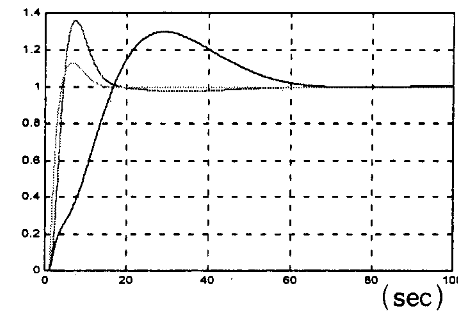

Fig. 9. Step response by continuous H_∞ controller

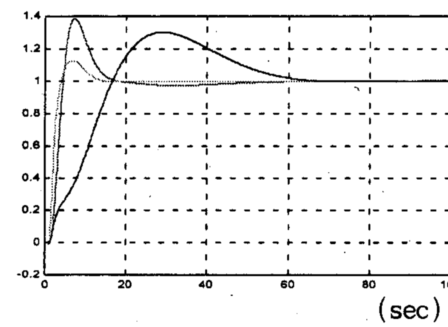

Fig. 10. Step response by discrete H_∞ controller

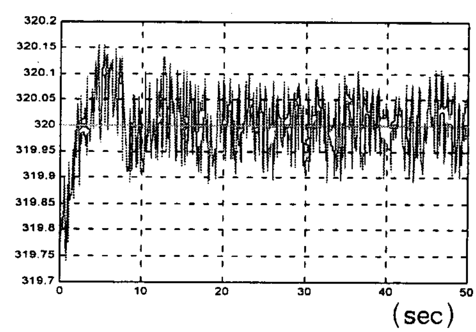

Fig. 11. Steam pressure(p.s.i) by continuous H_∞ controller

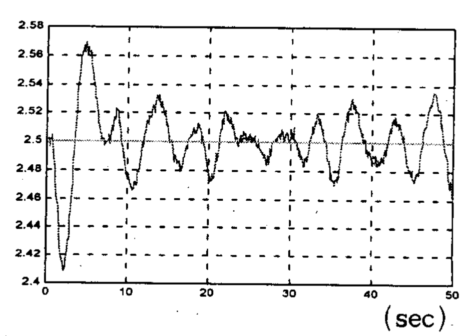

Fig. 12. Excess oxygen(%) by continuous H_∞ controller

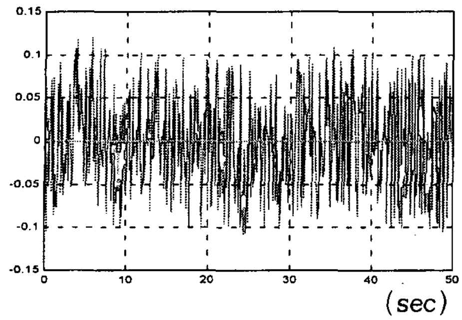

Fig. 13. Water level(in) by continuous H_∞ controller

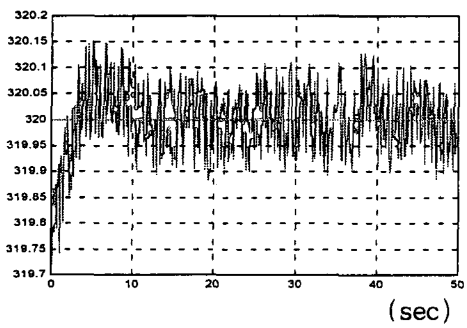

Fig. 14. Steam pressure(%) by discrete H_∞ controller

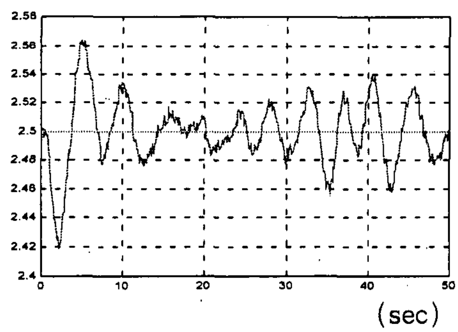

Fig. 15. Excess oxygen(%) by discrete H_∞ controller

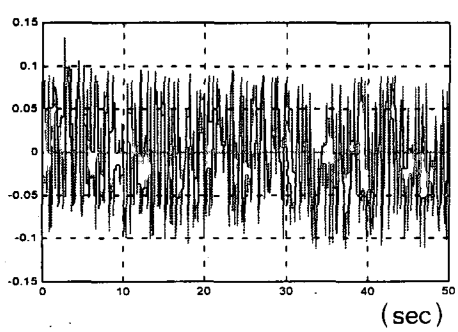

Fig. 16. Water level(in) by discrete H_∞ controller

VI. Conclusions

In this paper, most of important concepts of H_∞ control theory have been rebuilt in δ-domain. Particularly, we have derived new generalized control/filter Riccati equations using δ operator to provide theoretical background. The NLCF(Normalized Left Coprime Factorization) description with maximum stability margin has been achieved as a unified model to approach continuous and discrete ones and demonstrated a connection between robust stabilization using H_∞ optimization and Hankel norm approximation problem in δ-domain. Particularly, the unified H_∞ control method of continuous and discrete ones is developed by solving δ -domain problem. We can now start from δ-model and analysis continuous and discrete H_∞ controller by adjusting sampling time. It also minimize the errors through discretization.

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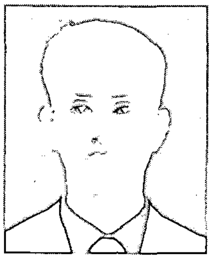

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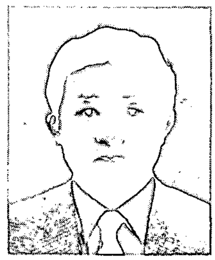

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