# Adaptive Data Association for Multi-Target Tracking using Relaxation

## Yang-Weon Lee and Hong Jeong

#### **Abstract**

This paper introduces an adaptive algorithm determining the measurement-track association problem in multi-target tracking (MTT). We model the target and measurement relationships with mean field theory and then define a MAP estimate for the optimal association. Based on this model, we introduce an energy function defined over the measurement space, that incorporates the natural constraints for target tracking. To find the minimizer of the energy function, we derived a new adaptive algorithm by introducing the Lagrange multipliers and local dual theory. Through the experiments, we show that this algorithm is stable and works well in general environments. Also the advantages of the new algorithm over other algorithms are discussed.

#### I. Introduction

The primary purpose of a multi-target tracking(MTT) system is to provide an accurate estimate of the target position and velocity from the measurement data in a field of view. Naturally, the performance of this system is inherently limited by the measurement inaccuracy and source uncertainty which arises from the presence of missed detection, false alarm, emergence of new targets into the surveillance region and disappearance of old targets from the surveillance region. Therefore, it is difficult to determine precisely which target corresponds to each of the closely spaced measurements. Although trajectory estimation problems have been well studied in the past, much of this previous work assumes that the particular target corresponding to each observation is known. Recently, with the proliferation of surveillance systems and their increased sophistication, the tools for designing algorithms for data association have been announced.

Generally, there are three approaches in data association for MTT: non-Bayesian approach based on likelihood function[11], Bayesian approach[6,10], and neural network approach[15,19]. The major difference of the first two approaches is how to treat the false alarms. The non-Bayesian approach calculates all the likelihood functions of all the possible tracks with given measurements and selects the track

which gives the maximum value of the likelihood function. Meanwhile, the tracking filter using Bayesian approach predicts the location of interest using a posteriori probability.

The two approaches are inadequate for real time applications because the computational complexity is overwhelming even for relatively large targets and measurements and yet a computationally efficient substitute based on a careful understanding of its properties is lacking.

As an alternative approach, Sengupta and Iltis [15] suggested a Hopfield neural probabilities data association (HNPDA) network to approximately compute a posteriori probability,  $\beta_j^l$  for the joint probabilities data association filter(JPDAF)[17] as a constrained minimization problem.

However, the neural network developed in [15] has been shown to have improper energy functions. Since the value of  $\beta_j^t$ 's in the original JPDAF are not consistent with  $X_j^t$  of [15], these dual assumptions of no two returns from the same target and no single return from two targets should be used only in the generation of the feasible data association hypotheses, as pointed out in [5]. This resulted from misinterpretations of the properties of the JPDAF which the network was supposed to emulate.

Furthermore, heuristic choices of the constant coefficients in the energy function in [15] didn't guarantee the optimal data association. In this paper, we derive the new scheme for data association which reflects the natural constraints of the MTT problem and convert the derived model into the minimization problem of energy function by MAP estimator [18]. The coefficients of energy function is calculated by Lagrange multiplier [2] and local dual theory [1].

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This paper is organized as follows. § II gives a detailed description of the MTT problem and explains that data association problem can be formulated as a constrained optimization problem. In § III, as an optimal method for solving this problem, we propose the use of differential of the Lagrange multiplier. § IV, some simulation results of the proposed algorithm are given.

## II. Problem Formulation and Energy Function

This section derives an energy function whose minimizer is a MAP estimator for optimal data association. At first the data association is studied thoroughly in terms of natural constraints. These constraints are then interpreted as an energy function.

#### 1. Representing Measurement-Target Relationship

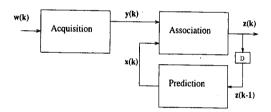


Fig. 1. An overall scheme for target tracking.

Fig1. shows the overall scheme. This system consists of three blocks: acquisition, association, and prediction. The purpose of the acquisition is to determine the initial starting position of the tracking. After this stage, the association and prediction interactively determine the tracks. Our primary concern is the association part that must determine the actual measurement and target pair, given the measurements and the predicted gate center.

Let m and n be the number of measurements and targets respectively, in a surveillance region. Then, the relationships between the targets and measurements are efficiently represented by the validation matrix  $\Omega$  [17]:

$$Q = \{ \omega_{it} | j \in [1, m], t \in [1, n] \}$$
 (1)

where the first column denotes clutter and always  $\omega_{j0} = 1 \, (j \in [1, m])$ . For the other columns,  $\omega_{jt} = 1 \, (j \in [1, m], t \in [1, n])$ , if the validation gate of target t contains the measurement j and  $\omega_{jt} = 0$ , otherwise. Based on the validation matrix, we must find hypothesis matrix [17]  $\widehat{Q}(=\{\widehat{\omega}_{jt} | j \in [1, m], t \in [1, n]\})$  that must obey the data association hypothesis(or feasible events [17]):

$$\begin{cases} \sum_{j=1}^{m} \widehat{\omega}_{jt} = 1 \text{ for } (t \in [1, n]), \\ \sum_{t=0}^{n} \widehat{\omega}_{jt} = 1 \text{ for } (j \in [1, m]), \end{cases}$$
 (2)

Here,  $\hat{\omega}_{jt} = 1$  only if the measurement j is associated with clutter t = 0 or target  $t(t \neq 0)$ . Generating hypothesis matrices leads to a combinatorial problem, where the number of data association hypothesis increases exponentially with the number of targets and measurements.

For example, let's consider a case in Fig. 2,

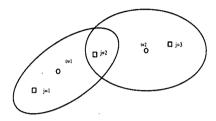


Fig. 2. An example with two targets and three measurements

where there are only two targets. Then, as can be seen in this figure, there are two validation gates whose centers are denoted by the filled disks. This radar scan contains three measurements, denoted by the filled squares, and one of them falls inside the intersection of the two validation gates. By definition, this situation can be represented by the validation matrix:

$$\mathcal{Q} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Applying the restrictions given (2) leads eight hypothesis matrices:

$$\mathcal{D}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \ \mathcal{D}_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \ \mathcal{D}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

It is easy that the computational cost of data association increases exponentially with n and m. In fact, the size of search space is  $O(2^{mn})$ . Naturally, the efficient generation and selection of hypothesis matrices for any number of targets and measurements are the great interest in the implementation of MTT.

#### 2. Constraining Target Trajectories

Let's consider a particular situation of radar as in Fig. 3.

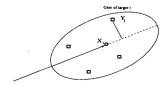


Fig. 3. Target trajectory and the measurements

In this figure, the position of the gate center of target t at

time k is represented by  $x_t(k)$ . Also,  $y_j$  means the coordinate of the measurement j at time k. Among the measurements included in this gate, at most one must be chosen as an actual target site. Note that the gate center is simply an estimate of this actual target position obtained by a prediction filter[16].

Since the target must change its direction smoothly, a possible candidate must be positioned on the site which is close to the trajectory as possible. As a measure of this distance, one can define the minimum distance between  $\mathbf{x}_t = (\mathbf{x}_t, \mathbf{y}_t)$  and the measurement  $\mathbf{y}_t = (\mathbf{x}_i, \mathbf{y}_i)$  as

$$\|\mathbf{x}_{t} - \mathbf{y}_{j}\|^{2} \equiv \frac{(\mathbf{x}_{j}y_{t} - \mathbf{y}_{j}x_{t})^{2}}{(x_{t}^{2} + y_{t}^{2})}$$
 (3)

where  $x_t$  and  $y_t$  are the position differences between  $\mathbf{x}_t(\mathbf{k})$  and  $\mathbf{x}_t(\mathbf{k}-1)$  in x and y axis.

Fig. 4 shows the distribution of the distance measure depending on the target direction within the target validation gate.

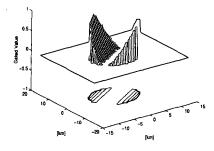


Fig. 4. Profile of the distance measure in (3)

#### 3. MAP Estimates for Data Association

The ultimate goal of this problem is to find the hypothesis matrix  $\widehat{Q}(=\{\widehat{\omega}_{jt} | j \in [1, m], t \in [1, n]\})$ , given the observation  $Y = \{Y_k | k \in [1, m]\}$ , which must satisfy (2).

Let's consider that  $\widehat{\mathcal{Q}}$  is a parameter space and  $(\mathcal{Q}, Y, X)$  is an observation space. Then, a posteriori can be derived by the Bayes rule:

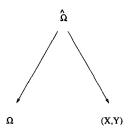
$$P(\widehat{Q}|\mathcal{Q}, \mathbf{y}, \mathbf{x}) = \frac{P(\mathcal{Q}, \mathbf{y}, \mathbf{x}|\widehat{Q})P(\widehat{Q})}{P(\mathcal{Q}, \mathbf{y}, \mathbf{x})}$$

$$= \frac{P(\mathcal{Q}|\widehat{Q}), P(\mathbf{y}, \mathbf{x}|\widehat{Q})P(\widehat{Q})}{P(\mathcal{Q}, \mathbf{y}, \mathbf{x})}$$
(4)

Here, we assumed that  $P(\mathcal{Q}, \mathbf{y}, \mathbf{x} \mid \widehat{\mathcal{Q}}) = P(\mathcal{Q} \mid \widehat{\mathcal{Q}})$   $P(\mathbf{y}, \mathbf{x} \mid \widehat{\mathcal{Q}})$ , since the two variables  $\mathcal{Q}$  and (X, Y) are separately observed. This assumption makes the problem more tractable as we shall see later.

This relationship is illustrated in Fig. 5.

Fig. 5. The parameter space and the observation space



Given the parameter  $\widehat{\Omega}$ ,  $\Omega$  and (X, Y) are observed. If the conditional probabilities describing the relationships between the parameter space and the observation spaces are available, one can obtain the MAP estimator:

$$\widehat{\mathcal{Q}}^* = \underset{\widehat{\mathcal{Q}}}{\operatorname{argmax}} P(\widehat{\mathcal{Q}}|\mathcal{Q}, \mathbf{y}, \mathbf{x})$$
 (5)

#### 4. Representing Constraints by Energy Function

Here we assume that clutter distribution around the targets are Gaussian since it make harder to keep the tracks comparing to the uniform distribution. To determine the solution of 5 efficiently, the mean field theory [12] from statistical mechanics is used. The mean field theory is based on the fact that the mean value is the minimum variance Bayes estimator and becomes the MAP estimator when the distribution is Gaussian. As the states are the Markov process, so we assume that the conditional probabilities are all Gibbs distributions:

$$P(\widehat{\Omega}|\Omega, \mathbf{y}, \mathbf{x}) \equiv \frac{1}{Z} \exp\{-E(\widehat{\Omega}|\Omega, \mathbf{y}, \mathbf{x},)\}$$

$$P(\mathbf{y}, \mathbf{x}|\widehat{\Omega}) \equiv \frac{1}{Z_{1}} \exp\{-E(\mathbf{y}, \mathbf{x}|\widehat{\Omega})\},$$

$$P(\Omega|\widehat{\Omega}) \equiv \frac{1}{Z_{2}} \exp\{-E(\Omega|\widehat{\Omega})\},$$

$$P(\widehat{\Omega}) \equiv \frac{1}{Z_{3}} \exp\{-E(\widehat{\Omega})\},$$

$$P(\Omega, \mathbf{y}, \mathbf{x}) \equiv \frac{1}{Z_{4}} \exp\{-E(\Omega, \mathbf{y}, \mathbf{x})\},$$
(6)

where  $Z_s$  ( $s \in [1, 2, 3, 4]$ ) called a partition function:

$$Z_{s} = \int_{\widehat{\Omega} = \varepsilon} \exp\{-E(\widehat{\Omega})\} d\widehat{\Omega}$$
 (7)

Here, E denotes the energy function. Substituting (6) into (4) yields

$$E(\widehat{\Omega}|\Omega, \mathbf{y}, \mathbf{x}) = E(\mathbf{y}, \mathbf{x}|\widehat{\Omega}) + E(\Omega|\widehat{\Omega}) - E(\Omega, \mathbf{y}, \mathbf{x})$$
 (8)

$$\widehat{\mathcal{Q}}^{*} = \underset{\widehat{\mathcal{Q}}}{\operatorname{argmin}} E(\widehat{\mathcal{Q}}|\mathcal{Q}, \mathbf{y}, \mathbf{x}),$$

$$= \underset{\widehat{\widehat{\mathcal{Q}}}}{\operatorname{argmin}} [E(\mathbf{y}, \mathbf{x}|\widehat{\mathcal{Q}} + E(\mathcal{Q}|\widehat{\mathcal{Q}}) + E(\widehat{\mathcal{Q}})].$$
(9)

The energy functions are realizations of the constraints both for the target trajectories and measurement-target relationships. For instance, the first term in (9) represents the distance between measurement and target and must be minimized using the constraints in (3). The second term intend to suppress the measurements which are uncorrelated with the validated measurements. The third term denotes constraints of the validation matrix and it can be designed to represent the two restrictions as shown in (2). The energy equations of each terms are defined respectively:

$$\begin{cases}
E(\mathbf{y}, \mathbf{x} | \widehat{\Omega}) \equiv \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(\mathbf{x}_{j} y_{t} - \mathbf{y}_{j} \mathbf{x}_{t})^{2}}{\mathbf{x}_{t}^{2} + \mathbf{y}_{t}^{2}} \widehat{\omega}_{jt}, \\
E(\Omega | \widehat{\Omega}) \equiv \sum_{i=1}^{n} \sum_{j=1}^{m} (\widehat{\omega}_{jt} - \omega_{jt})^{2}, \\
E(\widehat{\Omega}) \equiv \sum_{i=1}^{n} (\sum_{j=1}^{m} \widehat{\omega}_{jt} - 1) + \sum_{j=1}^{m} (\sum_{i=0}^{n} \widehat{\omega}_{jt} - 1).
\end{cases} (10)$$

Putting (10) into (9), one gets

$$\widehat{Q}^{\bullet} = \operatorname{argmin} A \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(\mathbf{x}_{1} y_{i} - \mathbf{y}_{1} x_{i})^{2}}{(x_{i}^{2} + y_{i}^{2})^{2}} \widehat{\omega}_{ii} + \frac{B}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} (\widehat{\omega}_{ii} - \omega_{ii})^{2} + \sum_{i=1}^{n} (\sum_{j=1}^{m} \widehat{\omega}_{ii} - 1) + \sum_{j=1}^{m} (\sum_{i=1}^{m} \widehat{\omega}_{ij} - 1),$$
(11)

In (11), the first term favors associations which locates near the velocity line by weighted validation matrix. The second term tends to discourage unrealistic association by comparing the generated matched events with the validation matrix. The third and fourth terms represent the constraints as explained in (2).

## III. Relaxation Scheme

The optimal solution for (11) is hard to find by any deterministic method. So, we convert the present constrained optimization problem to an unconstrained problem by introducing the Lagrange multipliers and local dual theory[1,2]. In this case, the problem is to find  $\hat{\omega}^*$  such that  $\hat{\omega}^* = \operatorname{argmin}_{\widehat{\omega}} L(\hat{\omega}, \lambda, \varepsilon)$  where

$$L(\widehat{\omega},\lambda,\varepsilon) = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(x_{j}y_{i} - y_{j}x_{i})^{2}}{x_{i}^{2} + y_{i}^{2}} \widehat{\omega}_{ji} + \frac{\alpha}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} (\widehat{\omega}_{ji} - \omega_{ji})^{2} + \sum_{i=1}^{n} \lambda_{i} (\sum_{j=1}^{m} \widehat{\omega}_{ji} - 1) + \sum_{i=1}^{m} \varepsilon_{j} (\sum_{j=1}^{n} \widehat{\omega}_{ji} - 1),$$
(12)

 $\alpha$  is coefficient of matching term and  $\lambda_i$  and  $\varepsilon_j$  are the Lagrange multiplier. Here we modify (12) to include the effect of the first column which represents the clutter or newly appearing target. In this paper, we also assume that m > n, since most of the multitarget problem is caused by many confusing measurements more than number of original targets, and that the summation of probability either in a row or column is one. Therefore, the Lagrangian  $L(\hat{\omega}, \lambda, \varepsilon)$  can be changed by

$$L(\hat{\omega}, \lambda, \varepsilon) = \sum_{t=1}^{m} \sum_{j=1}^{m} \frac{(x_{j}y_{t} - y_{j}x_{t})^{2}}{x_{t}^{2} + y_{t}^{2}} \hat{\omega}_{H} (1 - \delta_{t}) + \frac{\alpha}{2} \sum_{t=1}^{m} \sum_{j=1}^{m} (\hat{\omega}_{H} - \omega_{H})^{2} + \sum_{t=0}^{m} \lambda_{t} \left\{ \sum_{j=1}^{m} \hat{\omega}_{H} - 1 - d_{mn} \delta_{t} \right\} + \sum_{j=1}^{m} \varepsilon_{j} (\sum_{t=0}^{n} \hat{\omega}_{H} - 1),$$
where  $d_{m} = m - n - 1.$  (13)

We now look for a dynamical system of ordinary differential equations. The state of this system is defined by

 $\hat{\omega} = \{ \hat{\omega}_{jt} \}$  and the energy equation L is continuously differentiable with respect to  $\hat{\omega}_{jt}$ ,  $(j = 1, \dots, n)$  and

Since we are dealing with a continuous state problem, it is logical to use the Lagrange multipliers in the differential approach:

$$\begin{cases}
\frac{d\hat{\omega}}{dt} = -\eta(\hat{\omega}) \frac{dL(\hat{\omega}, \lambda, \varepsilon)}{d\hat{\omega}} \\
\frac{d\lambda_t}{dt} = \frac{dL(\hat{\omega}, \lambda, \varepsilon)}{d\lambda_t} \\
\frac{d\varepsilon_i}{dt} = \frac{dL(\hat{\omega}, \lambda, \varepsilon)}{d\varepsilon_t},
\end{cases} (14)$$

where  $\eta(\widehat{\omega})$  is a modulation function that ensures that the trajectories of (14) are in a state space contained in Euclidean mn-space. Performing gradient ascent on  $\lambda$  and  $\varepsilon$  have been shown [20] to be very effective in the resolution of constrained optimization problems.

To find a minimum of this equation by iterative calculations, we can use the gradient descent method:

$$\begin{cases} \widehat{\omega}^{n+1} = \widehat{\omega}^{n} - \eta(\widehat{\omega} \nabla_{\widehat{\omega}} L(\widehat{\omega}, \lambda, \varepsilon) \triangle t, \\ \lambda^{n+1} = \lambda^{n} + \nabla_{\lambda} L(\widehat{\omega}, \lambda, \varepsilon) \triangle t, \\ \varepsilon^{n+1} = \varepsilon^{n} + \nabla_{\varepsilon} L(\widehat{\omega}, \lambda, \varepsilon) \triangle t, \end{cases}$$
(15)

where  $\widehat{\omega}^0$ ,  $\lambda^0$ , and  $\varepsilon^0$  are initial states,  $\triangle t$  is the unit step size for each iteration, and  $\nabla_{\widehat{\omega}}$ ,  $\nabla_{\lambda}$ ,  $\nabla_{\varepsilon}$  are gradients. The trajectory of this dynamical equation is chosen in such a way that the energy  $L(\widehat{\omega},\lambda,\varepsilon)$  decreases steadily along the path. Hence,  $L(\widehat{\omega},\lambda,\varepsilon)$  is a Lyapunov function for this dynamical equation. Note that this algorithm converges to a minimum point nearest to the initial state. In general, the gradient search method has the property of converging to one of the local minima depending the initial states.

We assume that the energy is analytic and also that the energy is bounded below, i.e.,  $L \ge 0$ . A complete form of the relaxation equations are given by

$$\widehat{\omega_{jt}^{n+1}} = \widehat{\omega_{jt}^{n}} - \triangle t \left[ \frac{\left( \mathbf{x}_{j} \mathbf{y}_{t} - \mathbf{y}_{j} \mathbf{x}_{t} \right)^{2}}{\left( \mathbf{x}_{t}^{2} + \mathbf{y}_{t}^{2} \right)} (1 - \delta_{t}) + \alpha \left( \widehat{\omega_{jt}^{n}} - \omega_{jt} \right) + \lambda_{t}^{n} + \lambda_{t}^{n} + \lambda_{t}^{n+1} = \lambda_{t}^{n} + \triangle t \left[ \sum_{j=1}^{m} \widehat{\omega_{jt}^{n}} - 1 - d_{mn} \delta_{t} \right], \tag{16}$$

$$\varepsilon_{j}^{n+1} = \varepsilon_{j}^{n} + \triangle t \left[ \sum_{l=0}^{n} \widehat{\omega_{jt}^{n}} - 1 \right].$$

This equation can be computed by an array processor. A processing element in this array stores and updates the states by using information coming form nearby processors, together with their previous states. To terminate the iteration, we can define in advance either the maximum number of iterations or a lower bound of the change of  $\hat{\omega}$ ,  $\varepsilon$  and  $\lambda$  in successive steps.

## IV. Experimental Results

In this section, we present some results of the experiments comparing the performance of the proposed MAP estimate adaptive data association(MAPADA) with that of the Hopfield Neural PDA(HNPDA) of Sengupta and Iltis [15]. Though the MAPADA has a good structure for a parallel hardware, currently the algorithm is simulated by a serial computer. The performance of the MAPADA is tested in two separate cases in the simulation. In the first case, we consider two crossing and parallel targets for testing the track maintenance and accuracy in view of clutter density. In the second case, all the targets as listed in Table 1 are used for testing the multi-target tracking performance.

The dynamic models for the targets have been digitized using sampling period T normalized to 1 s and the state vectors have been represented in 2-dimensional Cartesian coordinates. Furthermore, only position measurements have been assumed to be available. The surveillance region used in the simulation is a 20 km by 20 km square and the initial positions and velocities of 10 targets in 2-dimensional plane are given in Table 1. Since every targets except target 8 and 9 are non maneuvering, the generic target dynamic model has the following form:

$$(k+1) = Fx(k) + Gw(k),$$
 (17)

$$z(k) = Hx(k) + v(k), (18)$$

where

$$x(k) = (x(k)(k)y(k)(k))',$$
  

$$w(k) = (w_1(k) w_2(k))',$$
  

$$E(w(k)) = 0,$$
  

$$E(w(k)w'(j)) = Q\delta(k-j),$$

$$F = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$G = \begin{pmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{pmatrix},$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

and

$$E(v(k)) = 0;$$
  $E(v(k)v'(j)) = R\delta(k-j).$ 

The correct returns from the targets are generated by adding noise to the computed true position of the target. The standard deviation of the measurement noise has been selected as  $\sqrt{R_i} = 0.15$  km for both the X and Y components. The correct return would pass a detector with probability of detection  $P_D = 0.7$ . The clutter's generated to get normal distribution around the targets.

The total umber of clutter returns observed in the region is a Poisson random umber. The density of clutter,  $\mathcal{C}$  is selected from 0.2 to 0.8/ km². For filter initiation, clutter was introduced after the time k=5. For the performance comparing, we have done the Monte Carlo simulation of N=40 runs. The crossing and parallel targets whose initial parameters are taken from target 1,2,3,and 4, respectively in Table 1 are tested.

In Fig. 6 and 8 sample of track estimation errors between HNPDA and MAPADA are shown, and the rms error in position and velocity in clutter density,  $C\!=\!0.2$ , are given in Fig. 7 and 9. The rms estimation errors and track maintenance capability from the filtering based on the crossing and parallel targets are listed in Table 3. We note that an obvious trend in the results is making harder to maintain tracks by increasing the clutter density. We note also that, although we have simulated just two scenarios, the performance of the MAPADA is quite steady comparing with that of the HNPDA in view of both tracking accuracy and maintenance.

Table 1. Initial Positions and Velocities of 10 targets.

Target	Position (km)		Velocity (km/s)		
i	x_	у	×	ý	
1	-4.0	1.0	0.2	-0.05	
2	-4.0	1.0	0.2	0.05	
3	-6.0	-5.0	0.0	0.3	
4	-5.5	-5.0	0.0	0.3	
5	8.0	-7.0	-4.0	0.0	
6	-8.0	-8.0	4.0	0.0	
7	-5.0	9.0	0.25	0.0	
88	-5.0	8.9	0.25	0.0	
9	0.5	-3.0	0.1	0.2	
10	9.0	-9.0	0.01	0.2	

Table 2. Maneuver Parameter of target 8 and 9.

Target i	Maneuvering type	Acceleration $(m/s^2)$	Turn period (sec)	Turn angle (deg)
8	Dog-leg	20	10	30
9	Constant Acceleration	10	1-35	0

Table 3. The track performance based on the crossing and parallel targets.

Clutter density	Position error (km)		Velocity error (km/s)		Track maintenance (%)	
(/km²)	HNPDA	MAPADA	HNPDA	MAPADA	HNPDA	MAPADA
0.2	0.47	0.13	0.46	0.06	84	84
0.4	0.62	0.16	0.69	0.04	52 .	76
0.6	0.79	0.17	1.17	0.05	48	74
0.8	1.17	0.29	1.45	0.02	36	76
Average	€0.76	0.19	0.94	0.04	55	80

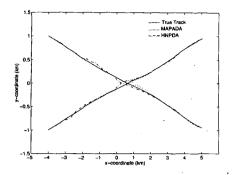


Fig. 6. Tracking results for the crossing targets

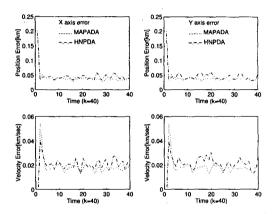


Fig. 7. RMS errors in position velocity for crossing targets.

In the second test as shown in Fig. 8., target's model which are maneuvering is simulated by the Singer model developed in [9]. Table 4 summarizes the rms position and velocity errors for each targets. The performance of the MAPADA is superior to that of HNPDA. The rms error of HNPDA for the target 8 has not been included since it loses track during the simulation.

Table 4. RMS Errors in the case of then targets

<b>(T)</b>	Position error		Velocity error		Track maintenance	
Target .	(km)		(km/s)		(%)	
1	HNPDA	MAPADA	HNPDA	MAPADA	HNPDA	MAPADA
1	0.64	0.42	0.69	0.18	95	100
2	0.64	0.42	0.42	0.17	95	100
3	0.78	0.42	0.22	0.18	100	100
4	0.60	0.43	0.15	0.18	93	100
5	0.59	0.45	0.67	0.18	85	100
6	0.57	0.45	0.20	0.18	100	100
7	0.57	0.42	0.31	0.49	90	100
8	-	2.95	1	1.18	0	53
9	0.62	0.44	0.27	0.21	80	98
10	0.59	0.45	0.21	0.18	100	98

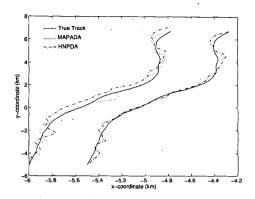


Fig. 8. Tracking results for the parallel targets.

## V. Conclusion

The purpose of this paper was to explore adaptive data association method as a tool for applying the multi-target tracking. It was shown that it always yields consistent data association, in contrast to the Hopfield Neural PDA, and that these associated data measurements are very effective for multi-target filter. Although the MAPADA find the convergence recursively, the MAPADA is a general method about the solving the data association problems in multi-target tracking. A feature of our algorithm is that it requires only storage, where m is the number of candidate measurement associations and n is the number of trajectories, compared to some branch and bound techniques, where the memory requirements grow exponentially with the number of targets. The experimental results show that the MAPADA is superior to the HNPDA in terms of both rms errors and track maintenance rate. This algorithm has several applications and can be effectively used in radar target tracking system.

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