Nonlinear Feedback Linearization-H_∞/Sliding Mode Controller Design for Improving Transient Stability in a Power System

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Abstract

In this paper, the standard Dole, Glover, Khargoneker, and Francis (abbr. : DGKF 1989) H_∞ controller (H_∞C) is extended to the nonlinear feedback linearization- H_∞/sliding mode controller (NFL- H_∞/SMC), to tackle the problem of the unmeasurable state variables as in the conventional SMC, to obtain smooth control as the linearized controller in a linear system, and to improve the time-domain performance under a worst scenario. The proposed controller is obtained by combining the H_∞ estimator with the nonlinear feedback linearization-sliding mode controller (NFL-SMC) and it does not need to measure all the state variables as in the traditional SMC. The proposed controller is applied as a nonlinear power system stabilizer (PSS) for the improvement of the power system damping characteristics of an single machine infinite bus system (SMIBS) connected through a double circuit line. The effectiveness of the proposed controller is verified by nonlinear time-domain simulation in case of a 3-cycle line-to-ground fault and in case of the parameter variations for the AVR gain K_A and for the inertia moment M.

I. Introduction

Sliding mode controller (SMC) and nonlinear feedback linearization-sliding mode controller (NFL-SMC) have applied as an effective way of the design of a power system stabilizer (PSS) for damping oscillations in a power system [1-14].

However, these NFL-SMCs applied to the PSS are based on the assumption that the complete state is avaliable for implementation of the control law [13,14].

In this paper, to cope with the problem of the unmeasurable state variables as in the conventional SMC, to obtain smooth control as the linearized controller in a linear system(or to cancel the nonlinearity in a nonlinear system), and to improve the time-domain performance under a worst scenario, the standard Dole, Glover, Khargoneker, and Francis (abbr. : DGKF 1989) H_∞ controller(H_∞C) [15] is extended to the nonlinear feedback linearization- H_∞ /sliding mode controller (NFL- H_∞/SMC).

Manuscript received July 14, 1997; accepted May 18, 1998.
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The proposed controller is obtained by combining the nonlinear feedback linearization-sliding mode controller (NFL-SMC) [13-14] with the H_∞ estimator [15], and eliminates the need to measure all the state variables as in the traditional SMC. The estimated control input derived by Lyapunov's second method, keeps the system stable.

The proposed controller is applied as the nonlinear power system stabilizer (PSS) for the improvement of the power system damping characteristics of an single machine infinite bus (SMIB) connected through a double circuit line [20-21].

The effectiveness of the proposed controller is verified by nonlinear time-domain simulation in case of a 3-cycle line-to-ground fault, and in case of the parameter variations for the AVR gain K_A and for the inertia moment M.

The organization of this paper is as follows: In section II the preliminary for the NFLC is represented. In section III the proposed NFL- H_∞/SMC is presented. In section IV we briefly review the nonlinear power system model. In section V the feedback linearization of a nonlinear power system is represented. In section VI the data analysis is presented. In section VII the nonlinear time-domain simulation is shown.

II. The Preliminary for the NFLC

In this section, the preliminary for a nonlinear feedback linearization controller (NFCL) is presented [16-19].

Let us consider the general nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \tag{1}$$

$$y(t) = h(x(t)) \tag{2}$$

in which $f(x)$ and $g(x)$ are smooth vector fields, and $h(x)$ is a smooth function, defined on R^n.

The linearizing diffeomorphism using Lie derivative [13-16] is

$$\begin{aligned} z(t) = T(x(t)) &:= [h\ L_f h\ L_f^2 h\ L_f^3 h \ldots\ldots\ldots]^T \\ &= [z_1(t)\ z_2(t)\ z_3(t)\ z_4(t)\ldots\ldots]^T \end{aligned} \tag{3}$$

where the Lie derivative $L_f h$ is simply the directional derivative of h along the direction of the vector f.

Remark 1: $L_f h = \frac{\partial h}{\partial x} f, \ldots\ldots, L_f^r h = \frac{\partial (L_f^{r-1} h)}{\partial x} f.$ (4)

The state space form based on NFL can be expressed as

$$\dot{z}(t) = Az(t) + Bu(t) \tag{5}$$

$$y(t) = Cz(t) \tag{6}$$

The derivatives of the output are

$$y(t) = L_f^0 h(x(t))$$

$$\frac{dy(t)}{dt} = L_f h(x(t)) + L_g h(x(t))u(t)$$

$$\frac{d^2 y(t)}{dt^2} = L_f^2 h(x(t)) + L_g L_f h(x(t))u(t)$$

....

$$\frac{d^r y(t)}{dt^r} = L_f^r h(x(t)) + L_g L_f^{r-1} h(x(t))u(t) \tag{7}$$

Remark 2: The eq. (1) and eq. (2) are said to have a relative degree r at a point x^o, if (i) $L_g L_f^k h(x) = 0$ for all x in a neighborhood of x^o, and, for all $k < r-1$ and (ii) if $L_g L_f^{r-1} h(x^o) \neq 0$.

The control input vector based on NFL is

$$u(t) = g(x(t)), v(t)) := -\frac{L_f^r h}{L_g L_f^{r-1} h} + \frac{1}{L_g L_f^{r-1} h} v(t) \tag{8}$$

where $v(t) = \frac{d^r y(t)}{dt^r}$ has a linear relation.

III. The Proposed NFL- H_∞/SMC Design

In this section, the standard Dole, Glover, Khargoneker, and Francis (abbr. : DGKF 1989) H_∞ controller (H_∞C) [12] is extended to the nonlinear feedback linearization-H_∞/sliding mode controller (NFL- H_∞/SMC).

The state equations based on NFL under worst case can be expressed as

$$z(t) = T(x(t)) \tag{9}$$

$$\dot{z}(t) = Az(t) + B_1 w_{worst}(t) + B_2 u(t) \tag{10}$$

$$p(t) = C_1 z(t) + D_{11} w_{worst}(t) + D_{12} u(t) \tag{11}$$

$$y(t) = C_2 z(t) + D_{21} w_{worst}(t) + D_{22} u(t) \tag{12}$$

where $x \in R^n$, $z \in R^n$, $w_{worst} \in R^{m1}$, $u \in R^{m2}$, $p \in R^{p1}$, $y \in R^{p2}$, A is the $n \times n$ system matrix, B_1 is the $n \times m_1$ exogenous input matrix, B_2 is the $n \times m_2$ control matrix, C_1 is the $p_1 \times n$ regulated output matrix, C_2 is the $p_2 \times n$ output or measurement matrix, D_{11} is the $p_1 \times m_1$ regulated direct feed-forward matrix, D_{12} is the $p_1 \times m_2$ regulated direct feed-forward matrix, D_{21} is the $p_2 \times m_1$ output direct feed-forward matrix, and D_{22} is the $p_2 \times m_2$ output direct feed-forward matrix.

The standard H_∞ estimator state equation based on NFL under worst case [15] can be expressed as

$$\dot{\hat{z}}(t) = A\hat{z}(t) + B_2 u(t) + B_1 \hat{w}_{worst}(t) + Z_\infty K_e (y(t) - \hat{y}(t)) \tag{13}$$

where $\hat{w}_{worst}(t) = \gamma^{-2} B_1^T X_\infty \hat{z}(t)$ (14)

$$\begin{aligned} \hat{y}(t) &= C_2 \hat{z}(t) + \gamma^{-2} D_{21} B_1^T X_\infty \hat{z}(t) \\ &= [C_2 + \gamma^{-2} D_{21} B_1^T X_\infty] \hat{z}(t) \end{aligned} \tag{15}$$

Remark 3: The γ in eq. (14) is a positive scalar value and is iterated until the desired specification is obtained.

The controller gain K_c is given by

$$K_c = \tilde{D}_{12} (B_2^T X_\infty + D_{12}^T C_1) \tag{16}$$

where $\tilde{D}_{12} = (D_{12}^T D_{12})^{-1}$ (17)

The estimator gain K_e is given by

$$K_e = (Y_\infty C_2^T + B_1 D_{21}^T)\, \tilde{D}_{21} \tag{18}$$

where $\tilde{D}_{21} = (D_{21} D_{21}^T)^{-1}$ (19)

The term Z_∞ is given by

$$Z_\infty = (I - \gamma^{-2} Y_\infty X_\infty)^{-1} \tag{20}$$

The controller Riccati equation term X_∞ is given by

$$X_\infty = Ric\begin{bmatrix} A - B_2 \tilde{D}_{12} D_{12}^T C_1 & \gamma^{-2} B_1 B_1^T - B_2 \tilde{D}_{12} B_2^T \\ -\tilde{C}_1^T \tilde{C}_1 & -(A - B_2 \tilde{D}_{12} D_{12}^T C_1)^T \end{bmatrix} \quad (21)$$

where $\tilde{C}_1 = (I - D_{12} \tilde{D}_{12} D_{12}^T) C_1$ (22)

The estimator Riccati equation term Y_∞ is given by

$$Y_\infty = Ric\begin{bmatrix} (A - B_1 \tilde{D}_{21} D_{21}^T C_2)^T & \gamma^{-2} C_1 C_1^T - C_2^T \tilde{D}_{21} C_2 \\ -\tilde{B}_1 \tilde{B}_1^T & -(A - B_1 \tilde{D}_{21}^T \tilde{D}_{21} C_2) \end{bmatrix} \quad (23)$$

where $\tilde{B}_1 = B_1 (I - D_{21}^T \tilde{D}_{21} D_{21})$ (24)

The estimated control input vector based on NFL is

$$u(t) = -K_c \hat{z}(t) \quad (25)$$

where $\hat{z} \in R^n$ is the estimated state variables, and K_c is the input gain of the worst case-control.

The internally stabilizing control gain using the packed matrix notation is

$$K(s) = \begin{bmatrix} A_1 & Z_\infty K_e \\ -K_c & 0 \end{bmatrix} \quad (26)$$

where
$$A_1 := A - B_2 K_c - Z_\infty K_e C_2 + \gamma^{-2}(B_1 B_1^T - Z_\infty K_e D_{21} B_1^T) X_\infty \quad (27)$$

The closed loop system can be expressed as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & -B_2 K_c \\ Z_\infty K_e C_2 & A_2 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ Z_\infty K_e D_{21} \end{bmatrix} w_{worst}(t) \quad (28)$$

$$\begin{bmatrix} p(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_1 & -D_{12} K_c \\ C_2 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} w_{worst}(t) \quad (29)$$

where
$$A_2 := A - B_2 K_c + \gamma^{-2} B_1 B_1^T X_\infty - Z_\infty K_e (C_2 + \gamma^{-2} D_{21} B_1^T X_\infty) \quad (30)$$

Remark 4: A stabilizing compensator can be obtained if and only if there exist a positive semi-definite solution to the two Riccati equations, $\rho(X_\infty Y_\infty) < \gamma^2$, where $\rho(A)$ is a spectral radius of $A =$ largest eigenvalue of $A = \lambda_{\max}(A)$.

From eq. (10) and eq. (14), the state equation based on NFL can be expressed as

$$\begin{aligned} \dot{z}(t) &= Az(t) + B_1 w_{worst}(t) + B_2 u(t) \\ &= Az(t) + B_1(\gamma^{-2} B_1^T X_\infty) z(t) + B_2 u(t) \\ &= (A + B_1(\gamma^{-2} B_1^T X_\infty)) z(t) + B_2 u(t) \end{aligned} \quad (31)$$

Suppose the sliding mode exists on all hyperplanes, then, the switching surface vector and the differential switching surface vector can be expressed as

$$\sigma(z(t)) = G^T z(t) \quad (32)$$

$$\dot{\sigma}(z(t)) = G^T \dot{z}(t) \quad (33)$$

where $z \in R^n$ is the NFL-based state variables, G^T the sliding surface gain, and the design procedure for obtaining G^T in the eq. (32) is found in references[1-14].

To determine a control law that keeps the system on $\sigma(z(t)) = 0$, we introduce the Lyapunov's second method

$$V(z(t)) = \sigma^2(z(t))/2 \quad (34)$$

The time derivative of $V(z(t))$ can be expressed as

$$\dot{V}(z(t)) = \sigma(z(t))\dot{\sigma}(z(t)) \quad (35)$$

$$\begin{aligned} &= G^T z(t) G^T \dot{z}(t) \\ &= G^T z(t) G^T [(A + B_1(\gamma^{-2} B_1^T X_\infty)) \\ &\quad + B_2 u_{NFL-W-SMC}(t)] \le 0 \end{aligned} \quad (36)$$

where $u_{NFL-W-SMC}(t)$ is the input vector of a nonlinear feedback linearization-worst case-sliding mode control.

The eq. (36) can be reduced as the control input with switching function

$$u^{+}_{NFL-W-SMC}(t) \ge -(G^T B_2)^{-1} [G^T(A + B_1(\gamma^{-2} B_1^T X_\infty))] z(t) \quad \text{for } G^T z(t) > 0 \quad (37)$$

$$u^{-}_{NFL-W-SMC}(t) \le -(G^T B_2)^{-1} [G^T(A + B_1(\gamma^{-2} B_1^T X_\infty))] z(t) \quad \text{for } G^T z(t) < 0 \quad (38)$$

The eq. (37) and eq. (38) can be formed as the control input of the NFL-W-SMC with sign function

$$u^{sign}_{NFL-W-SMC}(t) \ge -(G^T B_2)^{-1} [G^T(A + B_1(\gamma^{-2} B_1^T X_\infty))] z(t) \, sign(\sigma(z(t))) \quad (39)$$

The equation (39) can be reformed as follows:

$$u^{sign}_{NFL-W-SMC}(t) = -K_{W-SMC} z(t) \, sign(\sigma(z(t))) \quad (40)$$

where $K_{W-SMC} := (G^T B_2)^{-1} [G^T(A + B_1(\gamma^{-2} B_1^T X_\infty))]$ (41)

Finally, the estimated control input vector of the proposed NFL-H_∞/SMC is expressed as

$$u^{sign}_{NFL-H_\infty/SMC}(t) = -K_{W-SMC} \hat{z}(t) \, sign(\sigma(\hat{z}(t))) \quad (42)$$

where $\hat{z}(t) \in R^n$ is the estimated state variables, and the K_{W-SMC} is the input gain of the worst case-sliding mode control.

Remark 5: The estimated state $\hat{z}(t)$ in eq. (42) is obtained from eq. (13).

The block diagram of the proposed NFL-H_∞/SMC under worst case is shown in Fig. 1.

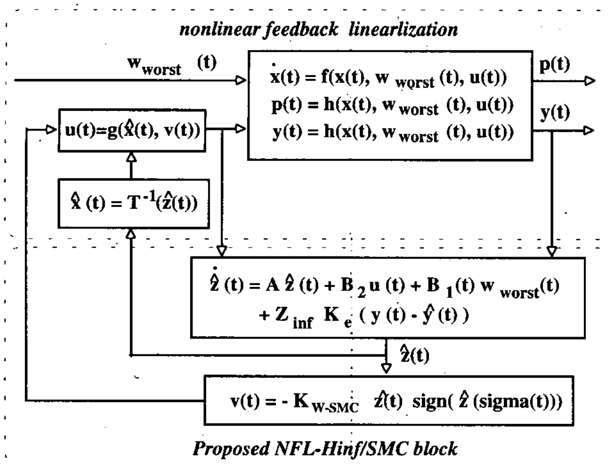


Fig. 1. Block diagram of the proposed NFL- H_∞/SMC.

The internally stabilizing control gain using the packed matrix notation is

$$K_{H_\infty/SMC}(s) = \begin{bmatrix} A_1 & Z_\infty K_e \\ -K_{W-SMC}sign(\sigma(\hat{z}(t))) & 0 \end{bmatrix} \tag{43}$$

where $A_1 := A - B_2K_{W-SMC}sign(\sigma(\hat{z}(t))) - Z_\infty K_e C_2$

$$+ r^{-2}(B_1B_1^T - Z_\infty K_e D_{21}B_1^T)X_\infty \tag{44}$$

The closed loop system can be expressed as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & -B_2K_{W-SMC}sign(\sigma(\hat{z}(t))) \\ Z_\infty K_e C_2 & A_2 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ Z_\infty K_e D_{21} \end{bmatrix} w_{worst}(t) \tag{45}$$

$$\begin{bmatrix} p(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_1 & -D_{12}K_{W-SMC}sign(\sigma(\hat{z}(t))) \\ C_2 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} w_{worst}(t) \tag{46}$$

where $A_2 := A - B_2K_{W-SMC}sign(\sigma(\hat{z}(t))) + r^{-2}B_1B_1^TX_\infty$

$$- Z_\infty K_e(C_2 + r^{-2}D_{21}B_1^TX_\infty) \tag{47}$$

The algorithm for the proposed NFL- H_∞/SMC under a worst scenario is summarized as follows:

(i) Set the nonlinear system representation.
(ii) Set the H_∞ state equation under a worst case.
(iii) Set the H_∞ estimator equation under a worst case.
(iv) Differentiate with respect to the output equation only until the input term appears.
(v) Set the linearized system representation.
(vi) Check if the assumptions (the rank conditions) are satisfied. If they are not, reformulate the problem by adding (fictitious) inputs or outputs.
(vii) Select a large positive value of γ.
(viii) Solve the two Riccati equations. Determine if the solutions are positive semi-definite; also, verify that the spectral radius condition is met.
(ix) If all the above conditions are satisfied, lower the value of γ. Otherwise, increase it. Respeat steps (viii) and (ix) until either an optimal or satisfying solution is obtained.
(x) Choose the equation of hyperplane $\sigma(z(t)) = G^Tz(t)$.
(xi) Compute the estimated control input with the sign function.
(xii) Apply the estimated control input to the plant.

IV. Nonlinear Power System Model

In this section, we briefly review the nonlinear power system equations.

1. Nonlinear power system model [21]

The d-axis current and the q-axis current are

$$i_d(t) = con_1 e_q'(t) - con_2(R_2 \sin\delta(t) + X_1\cos\delta(t)) \tag{48}$$

$$i_q(t) = con_3 e_q'(t) - con_4(-X_2 \sin\delta(t) + R_1\cos\delta(t)) \tag{49}$$

where

$$con_1 := \frac{(C_1X_1 - C_2R_2)}{(R_1R_2 + X_1X_2)}, \quad con_2 := \frac{V_\infty}{(R_1R_2 + X_1X_2)}$$

$$con_3 := \frac{(C_1R_1 + C_2X_2)}{(R_1R_2 + X_1X_2)}, \quad con_4 := \frac{V_\infty}{(R_1R_2 + X_1X_2)}$$

$$Z_1 := R_1 + jX_1, \quad Z_2 := R_2 + jX_2, \quad Y := G + jB$$

$$Z_T := \frac{Z_1Z_2}{Z_1 + Z_2}, \quad 1 + Z_TY := C_1 + jC_2$$

$$C_1 := RG - XB, \quad C_2 := XG + RB$$

$$R_1 := R - C_2x_a', \quad R_2 := R - C_2x_q$$

$$X_1 := X + C_1x_q, \quad X_2 := X + C_1x_d' \tag{50}$$

The expressions for $v_d(t)$, $v_q(t)$, $v_T(t)$, and $T_e(t)$ are

$$v_d(t) = x_qi_q(t) \tag{51}$$

$$v_q(t) = e_q'(t) - x_d'i_d(t) \tag{52}$$

$$v_T^2(t) = v_d^2(t) + v_q^2(t) \tag{53}$$

$$T_e(t) \cong P_e(t) = i_d(t)v_d(t) + i_q(t)v_q(t)$$
$$= e_q'(t)i_q(t) + (x_q - x_d')i_d(t)i_q(t) \tag{54}$$

where $i_d(t)$ is the d-axis current, $i_q(t)$ is the q-axis current, $T_e(t)$ is the electric torque, $P_e(t)$ is the electric power, $e_q'(t)$ is the q-axis transient voltage, $\delta(t)$ is the torque angle, V_∞ is the infinite bus voltage, x_d is the d-axis reactance, x_q is the q-axis reactance, and x_d' is the d-axis transient reactance.

Remark 6: In eq. 54, the electric torque T_e of a synchronous machine near the synchronous speed can be approximated by the electric power P_e.

The nonlinear 4-th order state equations including the limits imposed on AVR output, i.e. field voltage e_{fd}, and, on the stabilizing signal u_E are represented as

$$\dot{\omega}(t)=\frac{1}{M}T_m-\frac{1}{M}T_e(t) \tag{55}$$

$$\dot{\delta}(t)=\omega_o(\omega(t)-1) \tag{56}$$

$$\dot{e}_q'(t)=-\frac{1}{T_{do}'}e_q'(t)+\frac{(x_d-x_d')}{T_{do}'}i_d(t)+\frac{1}{T_{do}'}e_{fd}(t) \tag{57}$$

$$\dot{e}_{fd}(t)=-\frac{1}{T_A}e_{fd}(t)+\frac{K_A}{T_A}(V_{ref}-v_T(t)+u_E(t)) \tag{58}$$

$$e_{fd\,\min}\le e_{fd}\le e_{fd\,\max}\quad\text{and}\quad u_{E\,\min}\le u_E\le u_{E\,\max} \tag{59}$$

$e_{fd\,\max}=6.0$ $e_{fd\,\min}=-6.0$ and $u_{E\,\max}=2.0$ $u_{E\,\min}=-0.2$

where $\omega(t)$ is the angular velocity, $e_{fd}(t)$ is the exciter output voltage, T_m is the mechanical torque, T_A is the voltage regulator gain, T_{do}' is the d-axis transient open circuit time constant, M is the inertia coefficient, ω_o is the synchronous angular velocity, V_{ref} is the reference voltage, v_T is the terminal voltage, and u_E is the supplementary excitation control input.

The 4-th order state variables can be represented by

$$\begin{aligned}x(t)&=[x_1(t)\ x_2(t)\ x_2(t)\ x_4(t)]^T\\&:=[\omega(t)\ \delta(t)\ e_q'(t)\ e_{fd}(t)]^T\end{aligned} \tag{60}$$

The nonlinear 4-th order state equations from eq. (55)-(58) are written in the state space form as

$$\dot{x}(t)=f+gu_E(t) \tag{61}$$

$$y(t)=h \tag{62}$$

where $f:=(f_1\ f_2\ f_3\ f_4)^T$

$$=\begin{pmatrix}\frac{1}{M}T_m-\frac{1}{M}T_e(t)\\ \omega_o(\omega(t)-1)\\ -\frac{1}{T_{do}'}e_q'(t)+\frac{(x_d-x_d')}{T_{do}'}i_d(t)+\frac{1}{T_{do}'}e_{fd}(t)\\ -\frac{1}{T_A}e_{fd}(t)+\frac{K_A}{T_A}(V_{ref}-v_T(t))\end{pmatrix}$$

$$g:=\left(0\ 0\ 0\ \frac{K_A}{T_A}\right)^T$$

$$h:=\omega \tag{63}$$

The linearized differential equations of a single-machine, infinite bus system are found in Reference [21].

V. Nonlinear Feedback Linearization in a Power System

In this section, the nonlinear feedback linearization to cancel the nonlinearities in a power system is presented.

The nonlinear feedback linearization in a power system from eq. (48)-(58) are obtained by differentiating the angular velocity until the input term appears

$$z_1:=L_f^o=h=\omega \tag{64}$$

$$z_2:=L_fh=\frac{\partial h}{\partial x}f=\frac{\partial\omega}{\partial x}f=\frac{1}{M}(T_m-T_e) \tag{65}$$

$$L_gh=\frac{\partial h}{\partial x}g=\frac{\partial\omega}{\partial x}g=0 \tag{66}$$

$$\begin{aligned}z_3&:=L_f^2h=\frac{\partial(L_fh)}{\partial x}f=\frac{\partial}{\partial x}\left(\frac{1}{M}(T_m-T_e)\right)\\&=-\frac{1}{M}(pd_9f_2+pd_{10}f_3)\end{aligned} \tag{67}$$

$$L_gL_fh=\frac{\partial(L_fh)}{\partial x}g=\frac{\partial}{\partial x}\left(\frac{1}{M}(T_m-T_e)\right)g=0 \tag{68}$$

$$\begin{aligned}z_4&:=L_f^3h=\frac{\partial}{\partial x}(L_f^2h)f=\frac{\partial}{\partial x}\left(-\frac{1}{M}(pd_9f_2+pd_{10}f_3)\right)f\\&=[pd_{11}\ pd_{12}\ pd_{13}\ pd_{14}]f\end{aligned} \tag{69}$$

$$\begin{aligned}L_gL_f^2h&=\frac{\partial}{\partial x}(L_f^2h)g=\frac{\partial}{\partial x}\left(-\frac{1}{M}(pd_9f_2pd_{10}f_3)\right)g\\&=[pd_{11}\ pd_{12}\ pd_{13}\ pd_{14}]g\end{aligned} \tag{70}$$

where

$$pd_1:=-\frac{V_\infty}{Z_e^2}(R_2\cos(\delta)-X_1\sin(\delta)) \tag{71}$$

$$pd_2:=\frac{V_\infty}{Z_e^2}(X_2\cos(\delta)+R_1\sin(\delta)) \tag{72}$$

$$pd_3:=x_q\frac{V_\infty}{Z_e^2}(X_2\cos(\delta)+R_1\sin(\delta)) \tag{73}$$

$$pd_4:=x_d'\frac{V_\infty}{Z_e^2}(R_2\cos(\delta)-X_1\sin(\delta)) \tag{74}$$

$$pd_5:=Y_d \tag{75}$$

$$pd_6 := Y_q \tag{76}$$

$$pd_7 := x_q Y_q \tag{77}$$

$$pd_8 := 1 - x_d' Y_d \tag{78}$$

$$pd_9 := (v_d - x_d' i_q) pd_1 + (v_q + x_q' i_d) pd_2 \tag{79}$$

$$pd_{10} := Y_d (v_d - x_d' i_q) + Y_q (v_q x_q i_d) + i_q \tag{80}$$

$$pd_{11} := -\frac{1}{T_{do}'}(x_d - x_d') pd_1 \tag{81}$$

$$pd_{12} := -\frac{1}{T_{do}'} \tag{82}$$

$$pd_{13} := \frac{1}{T_{do}'} \tag{83}$$

$$pd_{14} := -\frac{1}{M} \omega_o pd_9 \tag{84}$$

$$pd_{15} := -\frac{1}{M}(pd_{15} f_1 + pd_{17} f_3 + pd_{10} pd_{19}) \tag{85}$$

$$pd_{16} := -\frac{1}{M}(pd_{16} f_1 + pd_{18} f_3 + pd_{10} pd_{20}) \tag{86}$$

$$pd_{17} := -\frac{1}{M}(pd_{10} pd_{21}) \tag{87}$$

$$pd_{18} := \frac{V_\infty}{Z_e^2}(R_2 \cos(\delta) + X_1 \sin(\delta)) \tag{88}$$

$$pd_{19} := \frac{V_\infty}{Z_e^2}(-X_2 \sin(\delta) + R_1 \cos(\delta)) \tag{89}$$

$$pd_{20} := (v_d - x_d' i_q) pd_{22} + 2(x_q - x_d')\, pd_1 pd_2 + (v_q + x_q i_d) pd_{23} \tag{90}$$

$$pd_{21} := Y_q (x_q - x_d') pd_1 + Y_d (x_q - x_d') pd_2 + pd_2 \tag{91}$$

$$pd_{22} := Y_q (x_q - x_d') pd_1 + pd_2 + Y_d (x_q - x_d') pd_2 \tag{92}$$

$$pd_{23} := 2 Y_q (1 + (x_q - x_d') Y_d) \tag{93}$$

The pd in above equations represents partial derivatives. The control input based on NFL is

$$u(t) = g(x(t), v(t)) := -\frac{L_f^3 h}{L_g L_f^2 h} + \frac{1}{L_g L_f^2 h} v(t) \tag{94}$$

$$= \frac{-[[\ pd_{11}\ \ pd_{12}\ \ pd_{13}\ \ pd_{14}] f - v(t)]}{[\ pd_{11}\ \ pd_{12}\ \ pd_{13}\ \ pd_{14}]\, g} \tag{95}$$

VI. Data Analysis

In this section, the data analysis is presented. The nominal data of the system, the operating conditions and the conventional PSS are listed in Appendix A1-A3. The values of A, B_1, B_2, C_1, C_2, D_{12}, D_{21} and D_{22} are

$$A = \begin{bmatrix} 0 & -0.0622 & -0.1052 & 0 \\ 376.991 & 0 & 0 & 0 \\ 0 & -0.0679 & -0.1957 & 0.1289 \\ 0 & 49.4 & -845.0 & -20 \end{bmatrix}$$

$B_1 = [0.108\ 0\ 0\ 0]^T$ $\quad B_2 = [0\ 0\ 0\ 1000]^T$
$C_1 = diag(1001, 0, 0, 0)$ $\quad C_2 = [1\ 0\ 0\ 0]$,
$D_{11} = diag(0, 0, 0, 0)$ $\quad D_{12} = [1\ 0\ 0\ 0]^T$
$D_{21} = [1\ 0\ 0\ 0]$ $\quad D_{22} = [0]$

A positive scalar γ is set to 1.2.
The H_∞ controller gain is $K_c = [1088.3,\ 1.30,\ -0.40,\ 0.0]$.
The H_∞ estimator gain is $K_e = [0.108,\ 0.0,\ 0.0,\ 0.0]$.
The sliding surface gain is
$G = 1.0e{+}5[-2.1529,\ -0.0201,\ 0.0383,\ 0.0000]^T$

VII. Nonlinear Time-domain Simulation Test

In this section, the nonlinear time-domain simulation studies are done to evaluate the performance of the proposed NFL- H_∞/SMC-PSS.

A fault is applied to verify the performance of the proposed controller under transient condition. The fault at about 2.0 sec is assumed to occur at the midpoint of the simple system in Fig. 2 and then is cleared after 0.05 sec and the line is reclosed. In Fig. 2, Z2 is the total impedance of faulted line and $d = 0.5$ puts the fault in the middle of the line.

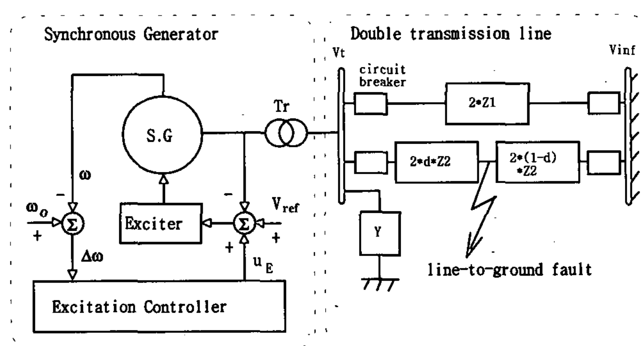
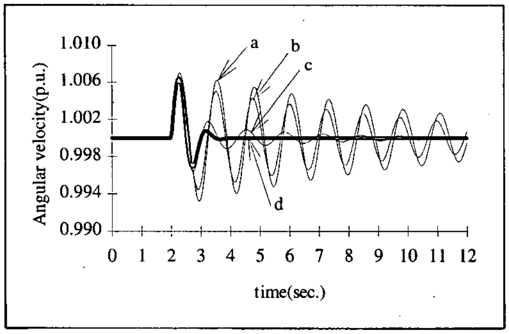
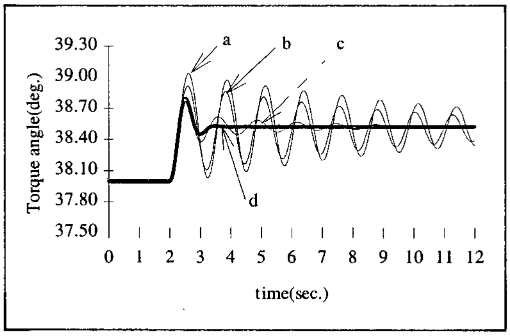


Fig. 2. Block diagram of an overall power system.

1. A 3-cycle line-to-ground fault simulation test



(1) Angual velocity



(2) Torque angle

Fig. 3. Normal load operation. (*a: no control b: conventional PSS c: NFL- H_∞C-PSS d: proposed NFL- H_∞/ SMC-PSS)*

Fig. 3 shows the angular velocity waveform in (1) and the torque angle waveform in (2) without any control, with the conventional Lead-Lag PSS, with the NFL- H_∞ C-PSS, and with the proposed NFL- H_∞/SMC-PSS with a 3-cycle line-to-ground fault under normal load operation.

Although the NFL- H_∞C-PSS can stabilize the system, it is shown that the proposed NFL- H_∞/SMC-PSS exhibits better damping properties.

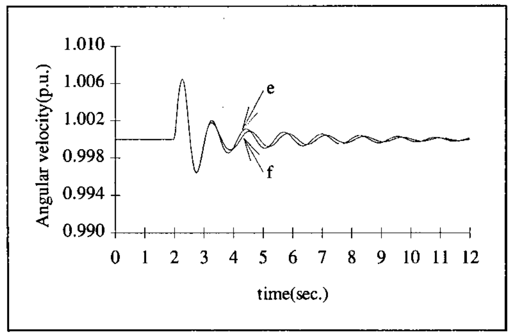
Because the conventional PSS in Fig. 3 (1)-(2) gives poorly damped response, the conventional PSS will not consider again in the following discussions.

2. Parameter variation test

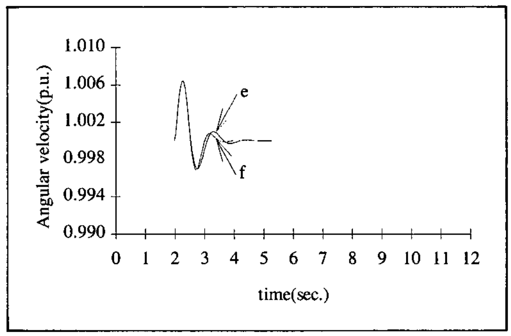
Case I : A parameter variations (20% over-estimation) of the AVR gain K_A

In Fig. 4, it is shown that these results demonstrate the insensitivity of the NFL- H_∞C-PSS in (1) and the proposed NFL- H_∞/SMC-PSS in (2) to parameter variation of the AVR gain K_A.

It is shown that the proposed NFL- H_∞/SMC-PSS exhibits better robust properties than the NFL- H_∞C-PSS.



(1) NFL- H_∞C-PSS



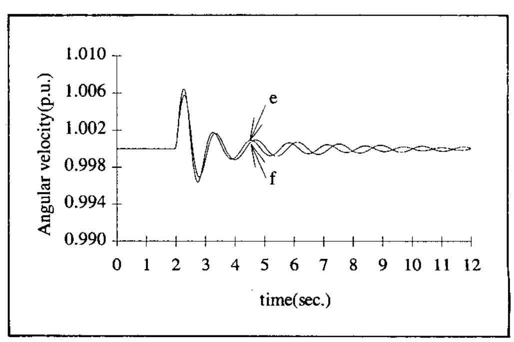
(2) proposed NFL- H_∞/SMC-PSS

Fig. 4. Angular velocity waveforms for parameter variation of the AVR gain KA.*(e : normal f : parameter variation)*

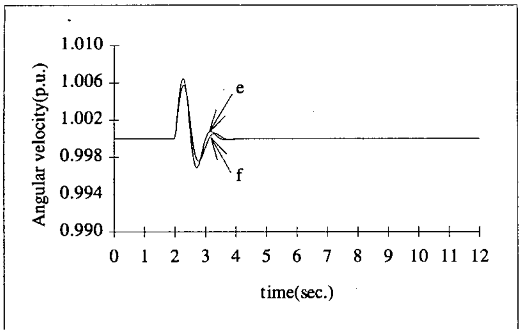
Case II : A parameter variations (20% over-estimation) of the inertia moment M.

Fig. 5 shows the angular velocity waveform in case of the parameter variations (20% over-estimation) of the inertia moment M of the generating unit in the system.

Also, it is shown that the proposed NFL- H_∞/SMC-PSS in Fig. 5 (2) exhibits better damping properties and is less sensitive to variations of the inertia moment M as compared to the NFL- H_∞C-PSS in Fig. 5 (1).



(1) NFL- H_∞C-PSS



(2) proposed NFL- H_∞/SMC-PSS

Fig. 5. Angular velocity waveforms for parameter variation of the inertia moment M. *(e : normal f : parameter variation)*

VIII. Conclusions

The standard DGKF's H_∞ controller (H_∞C) has been extended to the nonlinear feedback linearization- H_∞/sliding mode controller (NFL- H_∞/SMC) and has been applied as the nonlinear power system stabilizer (PSS) for the improvement of a transient stability in a nonlinear power system.

The main results are as follows :

1. Combining the H_∞ estimator with the nonlinear feedback linearization-sliding mode controller (NFL-SMC).
2. Obtaining a NFL- H_∞/SMC, to obtain smooth control as the linearized controller in a linear system, to tackle the problem of the unmeasurable state variables as in the conventional SMC, and to improve the time-domain performance under a worst case.
3. Improving in the sense of time-domain dynamic performance and robustness in case of a 3-cycle line-to-ground fault, and in case of the parameter variations for the AVR gain K_A and for the inertia moment M.

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Appendix

The nominal data of the system, the operating conditions and the conventional PSS are listed in Table A.1-A.3.

Table A.1 Generator data and initial condition data

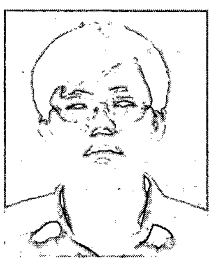
M	T'_{do}	D	x_d	x'_d	x_q	ω_o	P	Q	V_t
9.26	7.76	3.0	0.973	0.19	0.55	377	0.75	0.025	1.05

Table A.2 Excitation system data and line data

K_A	T_A	R	X	G	B
50.0	0.05	0.034	0.997	0.249	0.262

Table A.3 Conventional Lead-Lag PSS data

K	T_1	T_2	T_3	T_4
0.009	0.6851	0.1	0.06851	0.01



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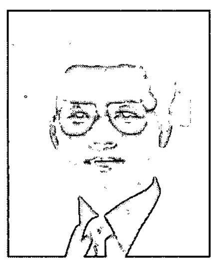

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