# Maximum Torque Operation of Interior Permanent Magnet Synchronous Motor in Flux-Weakening Control

Jang-Mok Kim, Hong-Woo Rhew, Jung-Lock Kwon and Seung-Ki Sul

#### Abstract

A new flux-weakening scheme for an Interior Permanent Magnet Synchronous Motor (IPMSM) is proposed. This control scheme enables the maximum torque operation for the fast acceleration in the constant power region according to the current and voltage limit condition. Especially the dynamic performance of the braking in the flux-weakening region is improved with the compensation of the stator resistance. Also since the onset of the flux weakening operation is adjusted according to the load conditions, the machine parameters, and whether motoring or braking region, the stable and precise transition operation into or out of the flux weakening region can be achieved. The effectiveness of the proposed scheme is verified through experiments with an IPMSM drive system.

#### I. Introduction

The interior permanent magnet synchronous motor (IPMSM) has gained an increasing popularity in recent years for a wide variety of industrial drive applications. The magnets are buried inside the rotor core with a steel pole piece in the IPMSM. Because of this geometry, IPMSM has a mechanically robust rotor construction, a rotor saliency, and the low effective airgap. These features permit this machine to be operated not only in the constant torque region but also in the constant power region up to a high speed by flux weakening[1,2,3].

In the high speed range the maximum torque capability of Interior Permanent Magnet Synchronous Motor (IPMSM) is limited by the voltage and current ratings of the machine as well as those of the inverter[1,2,3,4]. Therefore, under these two limit conditions, it is desirable to use a control scheme which can yield the maximum torque per ampere over the entire speed range including the flux-weakening region.

In the previous studies for flux-weakening the voltage drop of the stator resistance has been ignored or only its amplitude was compensated in high speed operation[1,2]. If the voltage drop of the stator resistance is ignored or only its amplitude is considered, the voltage equation of the stator is much simplified and the voltage limit ellipse of IPMSM is symmetrical to d-axis. But if the effect of the stator resistance is exactly considered and calculated in the voltage equation of the stator, the voltage limit ellipse is not symmetrical and slanted to d-axis. So the characteristics of the motoring and the braking region are different. In order to improve the dynamic performance of IPMSM the slanted voltage limit ellipse owing to the existence of the stator resistance must be exactly compensated. That is, the onset of flux-weakening operation and the trajectory producing the maximum torque in the constant power region are different in the motoring and the braking region.

In this paper, a new flux weakening scheme for IPMSM which ensures producing maximum torque per ampere over the entire flux weakening region is presented. In order to improve the dynamic performance of the braking region, the effect of the stator resistance is exactly compensated and considered in the flux weakening controller. Thus the stable and precise transition into the flux weakening operation is achieved by varying the onset point of the flux weakening according to the load conditions, the machine parameters and whether motoring or braking region.

Experimental results for an IPMSM drive system under various operating conditions are presented to verify the effectiveness of the proposed flux-weakening scheme.

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#### II. Modeling of IPMSM

The steady state voltage equation of IPMSM on the synchronous rotating d-q reference frame can be represented in the matrix form as followings.

$$\begin{bmatrix} V \stackrel{e}{c} \\ V \stackrel{e}{c} \\ V \stackrel{e}{c} \end{bmatrix} = \begin{bmatrix} R_s & -\omega_e L_{as} & R_s \end{bmatrix} \begin{bmatrix} i \stackrel{e}{c} \\ i \stackrel{e}{c} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_e \psi \end{bmatrix}$$
 (1)

(superscript 'e' denotes the variables on the synchronously rotating d-q reference frame)

where,  $V_{ds}^{e}$ ,  $V_{ss}^{e}$ : d, q axis motor terminal voltage,

 $i_{ds}^{e}$ ,  $i_{qs}^{e}$ : d, q axis stator current,

 $L_{ds}$ ,  $L_{qs}$ : d, q axis stator self-inductance,

 $R_s$ : stator resistance,

 $\phi_i$ : permanent magnet flux linkage,

 $\omega_{\epsilon}$ : electrical angular velocity.

And the developed torque equation of IPMSM is expressed as follows.

$$T_{e} = \frac{3}{2} p \left[ \psi_{f} i_{gs}^{e} + (L_{ds} - L_{gs}) i_{ds}^{e} i_{gs}^{e} \right]$$
 (2)

where, p: the number of pole pairs.

As shown in (2), the electrical torque consists of the magnetic torque produced by the flux linkage and the reluctance torque produced by the electric rotor saliency ( $L_{as}$ )  $L_{ds}$ ). So, it is desirable that the reluctance torque should be properly utilized in order to increase the whole efficiency of the IPMSM drive in the constant torque region.

In Fig.1, the maximum torque-per-current trajectory on the synchronously rotating d-q reference current plane is illustrated ( $O\rightarrow A$ ). When the machine is operated from the start up to an angular speed  $\omega_1$  in the constant torque region in Fig.1, the voltage limit ellipse reaches the cross point A of the maximum torque-per-ampere trajectory and the maximum current boundary represented by  $I_{smax}$  which is determined by the current rating of the machine or the maximum current capability of the inverter. Hence, the voltage limitation is not needed to consider in this constant torque region. The stator d- and q-axis current is to be controlled to use the reluctance torque fully and to maximize the machine efficiency in the constant torque region.

The magnitude of the output of the speed controller in the constant torque region  $|i|^{e}$  can be expressed as follows,

$$|i_{s}^{e}| = \sqrt{i_{ds}^{e^{2}} + i_{qs}^{e^{2}}}$$
 (3)

From (2) and (3), the d-q axis current components of the maximum torque-per-ampere are derived as follows[3]:

$$i \stackrel{e}{_{qs}} = sign(i \stackrel{e}{_s}) \sqrt{i \stackrel{e}{_s}^2 - i \stackrel{e}{_{ds}}^2}$$

$$\text{where,} \quad \begin{cases} \text{if } i \stackrel{e}{_s} \ge 0, \ sign(i \stackrel{e}{_s}) = 1 \\ \text{if } i \stackrel{e}{_s} < 0, \ sign(i \stackrel{e}{_s}) = -1 \end{cases}$$

$$i_{ds}^{e} = \frac{\psi_{f} - \sqrt{\psi_{f}^{2} + 8(L_{qs} - L_{ds})^{2} i_{s}^{e^{2}}}}{4(L_{qs} - L_{ds})}$$
 (5)

( $i \stackrel{e}{s}$  is the output of the speed controller)

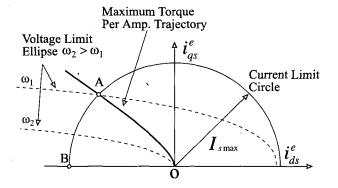


Fig. 1. Maximum torque-per-ampere trajectory and the voltage ellipse only considering the amplitude of the voltage drop of the stator resistance in the  $i \frac{e}{ds} - i \frac{e}{cs}$  plane.

## III. Maximum Torque Operation in Flux-Weakening Region

The maximum voltage  $V_{smax}$  is limited by DC link voltage and PWM strategy. The maximum current  $I_{smax}$  is determined by the inverter current rating and the machine thermal rating. Therefore the voltage and the current of the motor have the following limits [4]:

$$V_{ds}^{e^2} + V_{qs}^{e^2} \le V_{smax}^2$$
 (6)

$$i_{ds}^{e^2} + i_{ds}^{e^2} \le I_{smax}^2$$
 (7)

#### 1. Simple compensation method of the stator resistance

The maximum torque capability in flux-weakening region is determined by both of the voltage and the current limit conditions. In this region, the currents  $i \frac{e}{ds}$  and  $i \frac{e}{ds}$  producing the maximum output torque is the crossing point of the current limit circle and the voltage limit ellipse in the voltage equation (1). When (1), (3) and (4) are considered and the voltage drop of the stator resistance is simply compensated ( $V_c = V_{smax} - R_s I_{smax}$ ), the  $i \frac{e}{ds}$  and  $i \frac{e}{ds}$  for maximum torque operation in flux-weakening region can be derived to followings.

$$i_{qs}^{e} = sign(i_{sf}^{e})\sqrt{i_{sf}^{e} - i_{ds}^{e}^{2}}$$
(8)

where,  $\left\{ \begin{array}{ll} \text{if } i\stackrel{e}{s_f}\geq 0, \; \textit{sign}(i\stackrel{e}{s_f})=1\\ \\ \text{if } i\stackrel{e}{s_f}< 0, \; \textit{sign}(i\stackrel{e}{s_f})=-1 \end{array} \right.$ 

 $(i_{sf}^{e})$  is the output of the speed controller)

$$i \stackrel{e}{=} \frac{L_{ds} \phi_{f} - \sqrt{(L_{ds} \phi_{f})^{2} + (L_{ss}^{2} - L_{ds}^{2})(\phi_{f}^{2} + (L_{ds} i \stackrel{e}{\omega})^{2} - (\frac{V_{c}}{\omega_{e}})^{2})}}{L_{ss}^{2} - L_{ds}^{2}}$$
(9)

where,  $V_c = V_{smax} - R_s I_{smax}$ .

Without a proper flux weakening operation, the current regulator would be saturated and lose its controllability at a high speed. Since the onset of the current regulator saturation varies according to load condition and the machine parameters, the beginning point of flux weakening and the flux level should be changed. A late start of the flux weakening may result in undesired output torque drop according to the saturation of the current regulator, but an early start deteriorates the acceleration performance in the constant torque region[3].

Therefore it is desirable to change the onset point of flux weakening according to load condition and the machine parameters. From (1) and (6), the optimal starting point of the flux weakening can be deduced with simply considering the voltage drop of the stator resistance as follows.

$$\omega_{base} = \frac{V_c}{\sqrt{\left(L_{qs} i \frac{e}{qs}\right)^2 + \left(L_{ds} i \frac{e}{ds} + \psi_f\right)^2}}$$
 (10)

### 2. The actual effect of the stator resistance and its compensation

If the stator resistance is fully considered, the voltage limit ellipse is not symmetrical to d-axis as shown in Fig.2. The solid line is the voltage limit ellipse of IPMSM in which the effect of the stator resistance is considered. Generally the voltage drop of the stator resistance has been ignored or simply compensated in high speed operation because the voltage equation of the stator is much simplified and the solution of the equation for the flux weakening operation is

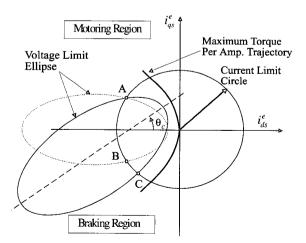


Fig. 2. Max. torque-per-Amp. trajectory and the slanted voltage ellipse considering the stator resistance in the  $i \frac{e}{ds} - i \frac{e}{ds}$  plane.

easily obtained. As shown in Fig.2 the trace of the flux weakening and the onset of that are not symmetrical to d-axis, that is, the characteristics of the motoring and the braking region are different.

In Fig.3 it can be seen that in IPMSM as the stator resistance increases, the slant angle  $\theta_c$  of the voltage limit ellipse increases near the starting point of the flux weakening operation. The nominal machine parameters used in Fig.3 are shown in Table 1.

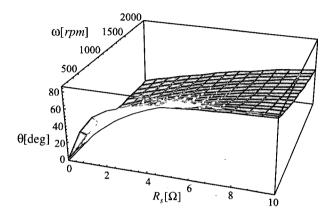


Fig. 3. Slant of the voltage limit ellipse according to the stator resistance and rotor speed.

This angle  $\theta_c$  is yielded by substituting (1), (11) into (6) and solving for  $\theta_c$  to satisfy  $i\frac{e}{ds}i\frac{e}{ds}$  term to zero. Arbitrary points,  $i\frac{e}{ds}$  and  $i\frac{e}{ds}$ , are on the synchronous d-q reference frame. As known from (12) the slanted voltage limit ellipse is related to stator resistance, stator self-inductance and rotor speed.

$$\begin{bmatrix} i \stackrel{e}{ds} \\ i \stackrel{e}{qs} \end{bmatrix} = \begin{bmatrix} \cos \theta_c & -\sin \theta_c \\ \sin \theta_c & \cos \theta_c \end{bmatrix} \begin{bmatrix} i \stackrel{e'}{ds} \\ i \stackrel{e'}{qs} \end{bmatrix}.$$
 (11)

$$\tan \theta_c = \frac{2 R_s}{\omega_c (L_{ds} + L_{as})}$$
 (12)

The onset of the flux-weakening in the motoring region is shown in Fig.4. From this figure as the stator resistance increases in the motoring region, that of the flux-weakening decreases as expected. But the starting point of the flux-weakening in the braking region is shown in Fig.5. From this figure as the stator resistance increases in the braking region, that of the flux-weakening increases to the negative direction. That is, the characteristics of the motoring and the braking region are different each other owing to the stator resistance. If the stator resistance is exactly considered in the flux weakening controller, the expansion of the constant torque region can be achieved in the braking region because there is much voltage margin in this region.

In order to improve the dynamic performance and expand the constant torque region, the current trajectory of the maximum torque-per-ampere and the optimal beginning point of the flux weakening are needed to meet the motoring operation and the braking operation in the flux weakening region.

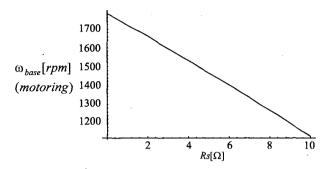


Fig. 4. Variation of the onset of flux weakening according to the stator resistance in the motoring region.

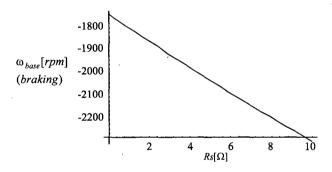


Fig. 5. Variation of the onset of flux weakening according to the stator resistance in the braking region.

Equation (1) can be written in the form:

$$V_{ds}^{e} - R_{s} i_{ds}^{e} = - \omega_{e} L_{qs} i_{qs}^{e} V_{ds}^{e} - R_{s} i_{ds}^{e} = \omega_{e} L_{ds} i_{ds}^{e} + \omega_{e} \psi_{I}$$
(13)

And the first terms of (13) are squared, added and rearranged as followings:

$$(V_{ds}^{e} - R_{s} i_{ds}^{e})^{2} + (V_{cs}^{e} - R_{s} i_{cs}^{e})^{2}$$

$$= V_{ds}^{e^{2}} + V_{cs}^{e^{2}} - 2 R_{s} (V_{ds}^{e} i_{ds}^{e} + V_{cs}^{e} i_{cs}^{e}) + R_{s}^{2} (i_{ds}^{e^{2}} + i_{cs}^{e^{2}}).$$
 (14)
$$= V_{smax}^{2} - 2 R_{s} (V_{cs}^{e} i_{ds}^{e} + V_{cs}^{e} i_{cs}^{e}) + R_{s}^{2} (i_{cs}^{e^{2}} + i_{cs}^{e^{2}}).$$

The underlined part of this equation is the input power of the motor and the inner product of the d-q voltage and d-q current. This power equation is rewritten as (15):

$$V_{ds}^{e} i_{ds}^{e} + V_{cs}^{e} i_{cs}^{e} = \sqrt{V_{ds}^{e^{2}} + V_{cs}^{e^{2}}} \sqrt{i_{ds}^{e^{2}} + i_{cs}^{e^{2}}} \cos \gamma.$$
 (15)

The power factor,  $\cos \gamma$ , is nearly unity in the motoring region, negative unity in the breaking region. So substituting (6) and (7) into (14), (16) is achieved as followings. But this is approximated but nearly exact equation.

$$V_{smax}^2 - 2 R_s V_{smax} I_{smax} \cos \gamma + R_s^2 I_{smax}^2$$
  
 $\approx V_{smax}^2 - 2 R_s V_{smax} I_{smax} + R_s^2 I_{smax}^2$  (16)

$$(-\omega_{e} L_{qs} i_{qs}^{e})^{2} + (\omega_{e} L_{ds} i_{ds}^{e} + \omega_{e} \psi_{f})^{2}$$

$$= (V_{ds}^{e} - R_{s} i_{ds}^{e})^{2} + (V_{qs}^{e} - R_{s} i_{qs}^{e})^{2}$$

$$\approx (V_{smax} - R_{s} I_{smax})^{2}$$
(17)

Using (16) and (17) and solving for  $i_{ds}^{e}$ , the current trajectory of the maximum torque-per-ampere is achieved not only in the motoring region but also in the braking region.

$$i_{d}^{s} = \frac{L_{d} \phi_{f} - \sqrt{(L_{d} \phi_{f})^{2} + (L_{d}^{2} - L_{d}^{2})(\phi_{f}^{2} + (L_{d} i_{s}^{2})^{2} - (\frac{V_{d}}{\omega_{s}})^{2})}}{L_{d}^{2} - L_{d}^{2}}.$$
 (18)

$$\text{where } \left\{ \begin{array}{ll} V_{cf} = V_{smax} - R_s \; I_{smax} & (\textit{If} \; T_e \; \omega_e \geq 0) \\ V_{cf} = V_{smax} + R_s \; I_{smax} & (\textit{If} \; T_e \; \omega_e \leq 0) \end{array} \right. .$$

$$i_{qs}^{e} = sign(i_{sf}^{e})\sqrt{i_{sf}^{e} - i_{ds}^{e}^{2}}.$$
 (19)

where, 
$$\begin{cases} \text{if } i \stackrel{e}{sf} \geq 0, \ sign(i \stackrel{e}{sf}) = 1 \\ \text{if } i \stackrel{e}{sf} < 0, \ sign(i \stackrel{e}{sf}) = -1 \end{cases}$$

From (1) and (6), the optimal starting point of the flux weakening can be derived as followings

$$\omega_{base} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (\omega_e \ge 0)$$
 (20)

where, 
$$\begin{cases} a = (L_{qs} i_{qs}^{e})^{2} + (L_{ds} i_{ds}^{e})^{2} + \psi_{f}^{2} + 2 L_{ds} \psi_{f} i_{ds}^{e} \\ b = 2(L_{ds} - L_{qs}) R_{s} i_{ds}^{e} i_{qs}^{e} + 2 R_{s} \psi_{f} i_{qs}^{e} \\ c = R_{s}^{2} (i_{ds}^{e} + i_{ds}^{e})^{2} - V_{smax}^{2} \end{cases}$$

In the motoring and braking region, the starting point of flux-weakening is given by (20). Although the beginning point of the flux weakening is different, the equation of that is the same.

To implement the maximum produced torque control and achieve the stable and precise transition into or out of the flux weakening operation in the motoring region as well as in the braking region, (18), (19) and (20) should be used in Fig.6. This is the block diagram of the motor controller including the proposed flux weakening controller.

#### IV. Experimental Results

A 5-Khz IGBT inverter-fed 900W IPMSM drive system was used in the test. The whole control algorithm was fully implemented in a software with a TMS320C31 DSP. The 900W IPMSM listed in Table I was used in the experimental test. The sampling time of current regulation loop is  $100~\mu sec$  and that of speed control loop and flux weakening control loop is 1msec.

Fig.7 shows the experimental results of the rotor speed, torque and d-q axis current waveforms when the voltage drop of the stator resistance is simply compensated in the flux weakening region.

In Fig. 7 the flat parts of the torque and d-q current wave forms are the constant torque operation, the both ends are the flux weakening operation not only in the motoring region but also in the braking region. And in this case the operation of the motoring region and braking region is symmetrical, that is, the motoring and braking operation are the same irrespective of the direction of the speed. Of course, the operation of the flux weakening is well behaved in the constant torque and constant power region as shown in Fig.7.

Fig.8 is the same condition of Fig.7 except the exact compensation of the voltage limit ellipse slanted by the stator resistance. The experimental results are the same in the

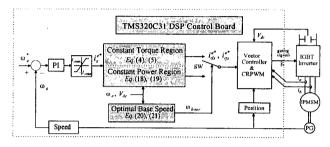


Fig. 6. Block diagram of the proposed flux weakening scheme.

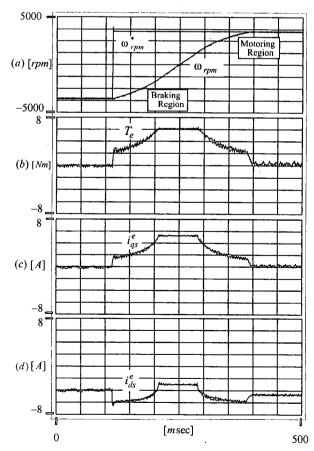


Fig. 7. Waveform of the simple compensating method of the stator resistance.

motoring region as shown in Fig.7, but different in the braking region. As known from Fig.9 in the initial part of the braking region the produced torque is increased by about 83% and the constant torque region and the base speed are enhanced by 71% over the conventional flux weakening method.

Therefore in this braking region the torque-speed characteristics is much improved by compensating the voltage limit ellipse slanted by the stator resistance.

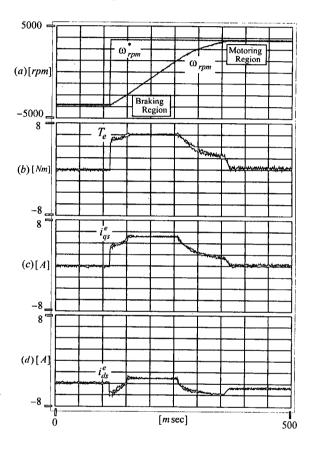


Fig. 8. Waveform of compensating the voltage limit ellipse slanted by the stator resistance.

#### V. Conclusions

A new flux weakening control scheme to utilize the maximum torque capability of IPMSM according to the current and voltage limit conditions over the entire flux-weakening region has been proposed. Especially the dynamic performance of the braking in the flux-weakening region is much improved by compensating the effect of the stator resistance. In the braking region the produced torque is increased by about 83% and the constant torque region is enhanced by 71% over the conventional flux weakening method without exactly compensating the effect of the stator

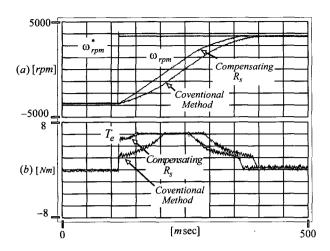


Fig. 9. Comparison of the conventional flux weakening method and the proposed method considering the stator resistance.

Table 1. Nominal parameters of IPMSM.

900[W], 220[V], 4[pole], 1700[rpm] 
$$R_s:4.3[], \quad \phi_s:0.272[\text{Wb}], \quad L_{ab}:27[\text{mH}], \quad L_{as}:67[\text{mH}],$$
 
$$V_{DC}=300[\text{V}], \quad I_{rate}=3[\text{A}], \quad I_{smax}=2 \ I_{rate},$$
 
$$I=0.000179[\text{Kg} \ m^2] \ (\text{motor and load inertia})$$

resistance. Also by varying the starting point of the flux weakening according to the load conditions, the machine

parameters and the direction of the speed, the stable and precise transition into or out of the flux weakening region was achieved. Experimental results for a laboratory IPMSM drive system has been presented to verify the effectiveness of the proposed flux weakening control scheme.

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