Time-Varying Signal Parameter Estimation by Variable Fading Memory Kalman Filtering

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Abstract

This paper proposes a VFM (Variable Fading Memory) Kalman filtering and applies it to the parameter estimation for time-varying signals. By adaptively calculating the fading memory, the proposed algorithm does not require any pre-determined fading memory when estimating the time-varying signal parameters. Moreover, the proposed algorithm has faster convergence speed than fixed fading memory one in case the signal contains an impulsive outlier. The performance of parameter estimation for time-varying signal is evaluated by computer simulations for two cases, one of which is the chirp signal whose frequency varies linearly with time and the other is the chirp signal with an impulsive outlier. The experimental results show that the VFM Kalman filtering estimates the parameter of the chirp signal more rapidly than the fixed fading memory one in the region of an outlier.

I. Introduction

Kalman filtering is an effective means of estimating the time-varying coefficients of an AR process[1]. However, the performance of Kalman filtering in estimating the time-varying signal parameters is degraded since it refers to the entire history of past observations. For this reason, the time-weighted-error Kalman filtering has been proposed which reduces the observation window size by introducing a fading memory [2]. However, the time-weighted-error Kalman filtering has the drawback that it is necessary to pre-determine the optimal fading memory according to the signal conditions. Recently, speech analysis via VFF-RLS algorithm has been reported, which generates the VFM according to the glottal pulse[3].

This paper presents a VFM Kalman filtering algorithm. This algorithm comes from the above VFF-RLS algorithm. In the comparison with the time-weighted-error Kalman filtering algorithm, the proposed algorithm doesn't require to pre-determine the fading memory since it adaptively calculates the fading memory at each update step.

In the next section, the derivation of the VFM Kalman parameter estimator will be presented. Then we will show some computer simulation results in case of a time varying chirp signal with and without an impulsive outlier and see how well the VFM Kalman filtering works in the estimation of AR parameters. This paper concludes in Section IV with a discussion of further applications of the algorithm.

II. VFM Kalman Filtering

The signal as an output of AR process can be represented as follows :

$$s(k) = \sum_{j=1}^{n} a_j(k)\, s(k-j) + \nu(k) \tag{1}$$

Expressing Eq.(1) to an equivalent state-space model, we get

$$\mathbf{a}(k) = \Phi(k, k-1)\, \mathbf{a}(k-1) \tag{2}$$

$$s(k) = \mathbf{S}^T(k-1)\ \mathbf{a}(k) + \nu(k) \tag{3}$$

where $\mathbf{a}(k)$ is an n-dimensional parameter vector, $\Phi(k, k-1)$ is the time-varying parameter transition matrix, and $\mathbf{S}(k-1)$ is an n-dimensional vector of past observation represented by $[(s(k-1), \cdots, s(k-n)]^T$. T denotes the vector transpose. $\nu(k)$ is a Gaussian, white noise and the driving signal of AR process and it follows the normal density $N(0, R(k))$, where $R(k)$ is assumed to be known [2]. The cost function of the VFM Kalman filter is given by

$$J_k = \sum_{j=1}^{k} w(j, k) R^{-1}(j)\, [s(j) - \mathbf{S}^T(j-1)\ \mathbf{a}(j)]^2 \tag{4}$$

Now define, $\overline{R}(j) = R(j)/w(j, k)$ then

$$J_k = \sum_{j=1}^{k} \overline{R}^{-1}(j)\, [s(j) - \mathbf{S}^T(j-1)\ \mathbf{a}(j)]^2 \, . \tag{5}$$

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The weighting coefficient is called VFM and is given by[3]

$$w(j,k)=\begin{cases}\prod_{i=j+1}^{k}\lambda(i) & 1\le j\le k-1\\ 1 & j\ge k\end{cases}\tag{6}$$

State space model given by Eqs. (2) and (3) defines a standard linear state estimation problem to which a Kalman filter can be applied to obtain the estimate of $\mathbf{a}(k)$. In the above model, Kalman filter can be thought of as a solution to the least-squares estimation problem. Considering a set of observations up to time k, Eqs. (2) and (3) can be rewritten as[5]

$$\mathbf{s}(k)=H(k-1)\,\mathbf{a}(k)+\mathbf{v}(k)\tag{7}$$

where

$\mathbf{s}(k)=[s(k),\cdots,s(1)]^T$, $\mathbf{v}(k)=[\nu(k),\cdots,\nu(1)]^T$, $H(k-1)=[\mathrm{S}^T(k-1)\ \mathrm{S}^T(k-2)\,\Phi(k-1,k)\cdots\mathrm{S}^T(0)\,\Phi(0,k)]^T$. The VFM has properties that $w(k,k)=1$, $w(k-1,k)=\lambda(k)$.

The coefficient $\lambda(k)$ decreases the weight of past estimation errors provided $0\le\lambda(k)\le 1$. Note that if $\lambda(k)=\lambda$ for all k, then the fading memory is fixed to $w(j,k)=\lambda^{k-j}$ and the cost function coincides with that of fixed fading memory one. Decreasing λ increases the reliability of recent measurements. Therefore, when the signal to be estimated is time-varying, we have to set λ appropriately small so that the dependence of past observation is lowered.

Since the cost function given by Eq. (5) is quadratic in $\mathbf{a}(k)$, the optimum least-squares estimate $\mathbf{a}(k|k)$ becomes

$$\mathbf{a}(k|k)=[H^T(k-1)\,V^{-1}(k)\,H(k-1)]^{-1}\cdot H^T(k-1)\,V^{-1}(k)\,\mathbf{s}(k)\tag{8}$$

where

$$V(k)=E[\mathbf{v}(k)\,\mathbf{v}^T(k)]=\begin{bmatrix}\bar{R}(k) & & 0\\ & \ddots & \\ 0 & & \bar{R}(1)\end{bmatrix}=\begin{bmatrix}\frac{R(k)}{w(k,k)} & & 0\\ & \ddots & \\ 0 & & \frac{R(1)}{w(1,k)}\end{bmatrix}$$

The corresponding error is given by

$$\bar{\mathbf{a}}(k|k)=[H^T(k-1)\,V^{-1}(k)\,H(k-1)]^{-1}\cdot H^T(k-1)\,V^{-1}(k)\,\mathbf{v}(k)\tag{9}$$

The error covariance is derived by defining

$$\mathbf{P}(k|k)=E\{\bar{a}(k|k)\,\bar{a}^T(k|k)\,|\,\mathbf{s}(j),\ j=1,2,\cdots,k\}.\tag{10}$$

After substituting Eq.(9) into Eq.(10) and simplifying, we can obtain

$$\mathbf{P}(k|k)=[H^T(k-1)\,V^{-1}(k)\,H(k-1)]^{-1}.\tag{11}$$

If we take the next observation, we set

$$\begin{aligned}&\mathbf{P}(k+1|k+1)\\&=[H^T(k)\,V^{-1}(k+1)\,H(k)]^{-1}\\&\cdot\left\{\begin{bmatrix}\mathrm{S}(k)\\ \Phi^T(k,k+1)H^T(k-1)\end{bmatrix}\begin{bmatrix}R^{-1}(k+1) & 0\\ 0 & \lambda(k)\,V^{-1}(k)\end{bmatrix}\begin{bmatrix}\mathrm{S}^T(k)\\ H(k-1)\,\Phi(k,k+1)\end{bmatrix}\right\}^{-1}\\&=[\mathrm{S}(k)\,R^{-1}(k+1)\,\mathrm{S}^T(k)+\lambda(k)\,\Phi^T(k,k+1)\\&\quad\cdot H^T(k-1)\,V^{-1}(k)\,H(k-1)\,\Phi(k,k+1)]^{-1}\end{aligned}\tag{12}$$

or, using a well-known matrix inversion lemma,

$$\begin{aligned}\mathbf{P}(k+1|k+1)&=\mathbf{P}(k+1|k)-\mathbf{P}(k+1|k)\,\mathrm{S}(k)\\&\cdot[\mathrm{S}^T(k)\,\mathbf{P}(k+1|k)\,\mathrm{S}(k)+R(k+1)]^{-1}\,\mathrm{S}^T(k)\,\mathbf{P}(k+1|k)\end{aligned}\tag{13}$$

where

$$\begin{aligned}\mathbf{P}(k+1|k)&=[\lambda(k)\,\Phi^T(k,k+1)\,H^T(k-1)\,V^{-1}(k)\,H(k-1)\,\Phi(k,k+1)]^{-1}\\&=\lambda(k)^{-1}\,\Phi^T(k,k+1)\,\mathbf{P}(k|k)\,\Phi(k,k+1).\end{aligned}$$

For a fixed fading memory $\lambda(k)=\lambda$, the Kalman filter comes to the same result in [2] as Table 1. If we set the noise variance $R(k)=1$ and time-varying parameter transition matrix $\Phi(k,k-1)=\mathbf{I}$, the fixed fading memory Kalman filtering algorithm is equal to the RLS (Recursive Least Square) algorithm with fading memory. It is reported that VFF-RLS algorithm is used in the estimation of speech parameter[3].

Table 1. Fixed fading memory Kalman filtering algorithm

$\mathbf{a}(k|k-1)=\Phi(k,k-1)\,\mathbf{a}(k-1|k-1)$,

$\mathbf{a}(k|k)=\mathbf{a}(k|k-1)+K(k)\,[s(k)-S^T(k-1)\,\mathbf{a}(k|k-1)]$,

$K(k)=\mathbf{P}(k|k-1)\,\mathrm{S}(k-1)\,[\mathrm{S}^T(k-1)\,\mathbf{P}(k|k-1)\,\mathrm{S}(k-1)+R(k)]^{-1}$,

$\mathbf{P}(k|k-1)=\lambda^{-1}\,\Phi(k,k-1)\,\mathbf{P}(k-1|k-1)\,\Phi(k,k-1)$,

$\mathbf{P}(k|k)=(\mathbf{I}-K(k)\,\mathrm{S}^T(k-1))\,\mathbf{P}(k|k-1)$.

VFM for Kalman filter can to be derived by following procedure[3]. First, J_{k+1} is given by

$$J_{k-1} = \sum_{j=1}^{k-1} w(j, k-1) R^{-1}(j) [s(j) - \mathbf{S}^T(j-1)\, \mathbf{a}(j)]^2. \quad (15)$$

To obtain the recursive equation for J_k, multiplying $\lambda(k)$ to Eq. (15) leads to

$$\begin{aligned}\lambda(k) J_{k-1} &= \lambda(k) \sum_{j=1}^{k-1} w(j, k-1) R^{-1}(j) [s(j) - \mathbf{S}^T(j-1)\, \mathbf{a}(j)]^2 \\ &= \sum_{j=1}^{k-1} w(j, k) R^{-1}(j) [s(j) - \mathbf{S}^T(j-1)\, \mathbf{a}(j)]^2. \end{aligned} \quad (16)$$

Therefore J_k is related to J_{k-1} by following recursive equation

$$\begin{aligned} J_k &= \sum_{j=1}^{k-1} w(j, k) R^{-1}(j) [s(j) - \mathbf{S}^T(j-1)\, \mathbf{a}(j)]^2 \\ &\quad + w(k, k) R^{-1}(k) [s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k)]^2 \quad (17) \\ &= \lambda(k) J_{k-1} + w(k, k) R^{-1}(k) [s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k)]^2 \end{aligned}$$

On the other hand, estimation error is given by

$$e(k) = s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k|k-1). \quad (18)$$

However, since it is impossible to have the parameter vector $\mathbf{a}(k|k-1)$ beforehand, the prediction errors will be used instead of the estimation errors[5], which is given by

$$e(k) = s(k) - \mathbf{S}^T(k-1)\, (k-1|k-1). \quad (19)$$

Combining from Eq. (17) to Eq. (19), VFM $\lambda(k)$ for Kalman filter can be derived as follows.

$$\begin{aligned} \lambda(k) &= \frac{J_k}{J_{k-1}} - \frac{R^{-1}(k)}{J_{k-1}} [s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k)]^2 \\ &= \frac{J_k}{J_{k-1}} - \frac{R^{-1}(k)}{J_{k-1}} \times [s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k|k-1) \\ &\quad - \mathbf{S}^T(k-1) K(k) e(k)]^2 \quad (20) \\ &\approx \frac{J_k}{J_{k-1}} - \frac{R^{-1}(k)}{J_{k-1}} [e(k) - \mathbf{S}^T(k-1) K(k) e(k)]^2 \\ &= \frac{J_k}{J_{k-1}} - \frac{R^{-1}(k)}{J_{k-1}} e^2(k) [1 - \mathbf{S}^T(k-1) K(k)]^2 \end{aligned}$$

A simplifying strategy to compute $\lambda(k)$ can be defined if we require that $J_k = J_{k-1} = \cdots = J_1$[4]. The meaning of this is that the fading memory compensates at each step k for the new error information in the latest measurement. Therefore the final equation of VFM for Kalman filter can be rewritten as

$$\lambda(k) = 1 - \frac{R^{-1}(k)}{J_1} e^2(k) [1 - \mathbf{S}^T(k-1) K(k)]^2 \quad (21)$$

The minimun value of VFM is empirically given by

$$\lambda_{\min} = \frac{1}{5 N_{AR}}, \quad \text{if } \lambda(k) < \lambda_{\min} \text{ then } \lambda(k) = \lambda_{\min} \quad (22)$$

where N_{AR} is the model order of AR process.

The VFM Kalman filtering algorithm can be summarized as Table 2. The initial value of the VFM is set to be 1, which means that the VFM Kalman filter refers to the entire history of past data at first. The algorithm described in Table 2., which we call the VFM Kalman filter, looks similar to the time-weighted-error Kalman filter version. The main difference between Table 1. and Table 2. is that there is an additional stage to calculate the VFM $\lambda(k)$ before updating error covariance $\mathbf{P}(k|k)$.

Table 2. VFM Kalman filtering algorithm

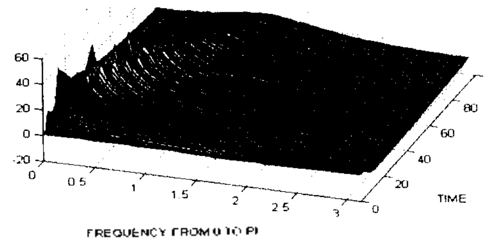
$$\begin{aligned} &\mathbf{a}(k|k) = \mathbf{a}(k|k-1) + K(k)[s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k|k-1)], \\ &\mathbf{a}(k|k-1) = \Phi(k, k-1)\, \mathbf{a}(k-1|k-1), \\ &K(k) = \mathbf{P}(k|k-1) \mathbf{S}(k-1) [\mathbf{S}^T(k-1) \mathbf{P}(k|k-1) \mathbf{S}(k-1) + R(k)]^{-1}, \\ &\lambda(k) = 1 - \frac{R^{-1}(k)}{J_1} e^2(k) [1 - \mathbf{S}^T(k-1) K(k)]^2, \\ &e(k) = s(k) - \mathbf{S}^T(k-1)\, \mathbf{a}(k-1|k-1), \\ &\mathbf{P}(k|k-1) = \lambda(k)^{-1} \Phi(k, k-1) \mathbf{P}(k-1|k-1) \Phi(k, k-1), \\ &\mathbf{P}(k|k) = [\mathbf{I} - K(k) \mathbf{S}^T(k-1)]\, \mathbf{P}(k|k-1). \end{aligned}$$

III. Results

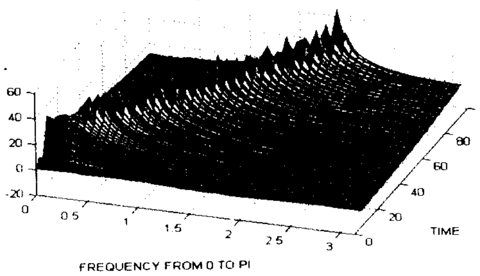
The proposed Kalman filter has been tested on a chirp signal and a chirp signal with an impulsive outlier. The filter equations used are given by Table 2. In each test the following values are used:

$$\Phi(k, k-1) = \mathbf{I}, \quad \mathbf{a}(0|0) = \mathbf{0},$$
$$\mathbf{P}(0|0) = diag[100 \ldots 100], \quad R(k) = 1.0.$$

A chirp defined by $s(k) = \sin(a\pi k^2/N)$, N = number of samples is considered as test signal. The chirp signal is a familiar signal whose frequency varies linearly with time. The simulated chirp signal uses $a = 0.318$ and $N = 100$. Fig. 1(a)-(c) shows three-dimensional plots of the estimated log-magnitude spectra. These spectra are estimated using a second-order AR process that is updated at each sample and plotted on every other sample. Fig. 1(a) is the signal estimate via fixed fading memory Kalman filter with $\lambda = 1$. This estimator quickly resolves the signal initially but cannot track the signal as it changes with time. Fig. 1(b) is for $\lambda = 2/3$ and we can see that the ability to tack the signal is much more improved. On the other hand, Fig. 1(c) is the signal estimate via VFM Kalman



(a)



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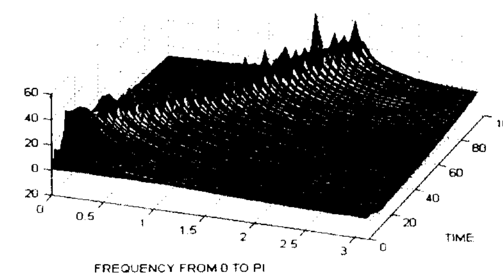
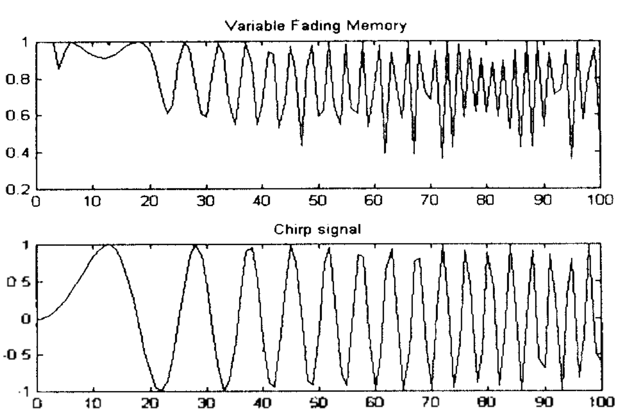


(c)

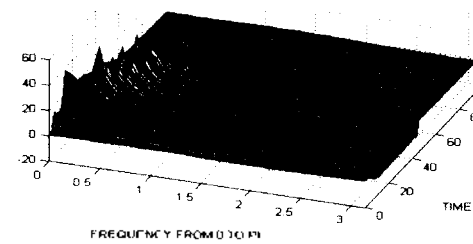


(d)

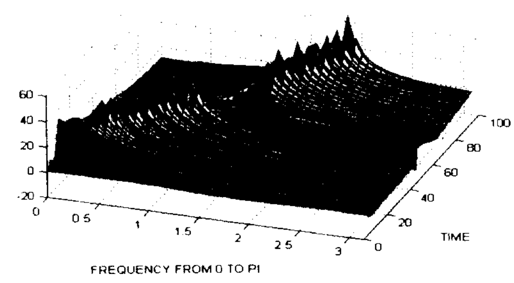
Figure 1. Spectra of estimated AR parameters ; (a) fixed fading memory Kalman filter($\lambda = 1$) (b) fixed fading memory Kalman filter ($\lambda = 2/3$) (c) VFM Kalman filter (d) VFM and chirp signal.

filtering. Fig. 1(d) is the corresponding VFM where the minimum value of VFM is set to be 0.1. We can see that the VFM Kalman filter has the comparable performance with that of the fixed fading memory one.

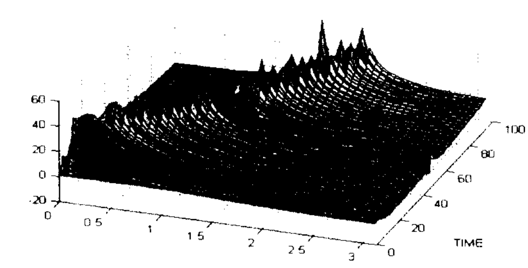
Fig. 2 shows the results for the parameter estimation when the chirp signal contains an impulsive outlier with amplitude 10 at time 50. The parameter estimation is not performed at all after an impulsive outlier when $\lambda = 1$ as in Fig. 2(a) and by optimizing fading memory with $\lambda = 2/3$ the parameter estimation is much more improved as in Fig. 2(b). On the other hand, the VFM Kalman filter estimates the AR parameter more rapidly even after the impulsive outlier than fixed fading memory one as in Fig. 2(c). Fig. 2(d) shows the corresponding fading memory and we can see that in the region of outlier fading me-



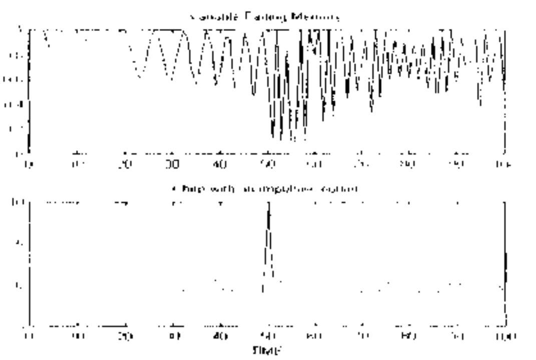
(a)



(b)



(c)



(d)

Figure 2. Spectra of estimated AR parameters with an outlier (a) fixed fading memory Kalman filter($\lambda = 1$) (b) fixed fading memory Kalman filter ($\lambda = 2/3$) (c) VFM Kalman filter (d) VFM and chirp with an impulsive outlier.

mory is set to its minimum value of 0.1 so that the effect of outlier is considerably reduced.

IV. Conclusion

This paper has presented a VFM Kalman filter to effectively estimate the parameters of time-varying signal. The proposed algorithm calculates the fading memory at each parameter update step and reflects this to covariance update so that it can efficiently estimate the time varying signal parameter. The VFM Kalman filter has been tested against the chirp signal with or without an impulsive noise by computer simulations. From this, faster tracking performance with VFM Kalman filter was observed than with fixed fading memory one. This is because the VFM Kalman filter estimates the AR parameter more effectively by varying the fading memory adaptively according to the signal condition. However, it would be better to switch the algorithm in case the signal contains a large impulsive noise since the large error variance affects the estimation performance. To effectively cope with the aforementioned problem, a modified approach to reduce the effect of the large error variance is under work.

This algorithm can also be applicable to the adaptive antenna processing and the adaptive equalizer in fast fading environments.

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