Effects of Error Path Delay on Stability of the Filtered-x/Constrained Filtered-x LMS Algorithm

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Abstract

Many of the active noise control system utilize a form of the least mean square(LMS) algorithm. This paper discusses the dependence of the convergence rate on the acoustic error path in the popular algorithm which is conventional "filtered-x LMS" and introduces new algorithm "constrained filtered-x LMS". The proposed method increase the convergence region regardless of the time-delay in the acoustic error path. In the algorithms, coefficients of the controller are adapted using the residuals of constrained structure which are defined in such a way that the control process become stationary. Advantages of constrained filtered-x LMS algorithm is illustrated by convergence analysis in the mean sense.

I. Introduction

In the active control of noise in ducts, it is common practice to locate the error microphone at a reasonable distance from the control source to avoid the near-field effects by evanescent waves. Such a distance between the control source and the error microphone makes a certain level of time delay inevitable and, hence, yields undesirable effects on the convergence of the filtered-x LMS algorithm. This paper discusses the effects of the time-delay on the convergence analysis of the filtered-x and constrained filtered-x LMS algorithm which is proposed recently to overcome the aforementioned deficiency of filtered-x LMS algorithm. Advantages of constraint filtered-x LMS algorithm will be further illustrated by convertgence analysis in the mean sense. Robustness of the constrained filtered-x LMS algorithm to the error path delay is demonstrated through a numerical analysis.

II. The Filtered-x and Constrained Filtered-x LMS algorithm

Figure 1 shows a block diagram of an adaptive control system based on the filtered-x LMS algorithm for the cancellation of noise. The filtered-x LMS algorithm is described by the following equations [1] :

$$w_i(k+1) = w_i(k) + 2\eta e(k) \sum_{j=0}^{p} h_j x(k-j-i), \qquad (1)$$

$$e(k) = d(k) - \sum_{j=0}^{p} \sum_{i=0}^{q} h_j w_i(k-j)\, x(k-j-i), \qquad (2)$$

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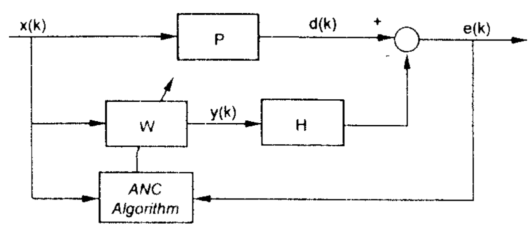


Figure 1. Block diagram of noise cancellation system.

where $d(k)$ is the desired signal and $x(k)$ is the primary noise source signal. η is the convergence rate which determines the speed of the adaptation. The controller W is represented by a FIR filter of order q. Assuming slow time varying weights, i.e. $w_i(k-j) = w_i(k)$, the filtered-x LMS algorithm is derived based on the steepest descent method. The H, FIR filter of order p, represents the error path consisting of acoustic path between the control source and the error microphone. Such an error path makes a certain level of time delay inevitable and, hence, yield undesirable effects on the convergence speed of the filtered-x LMS algorithms which is popular because of its simplicity.

Let us consider another set of error $\epsilon(k)$ defined by

$$\epsilon(k) = d(k) - \sum_{j=0}^{p} \sum_{i=0}^{q} h_j w_i(k)\, x(k-j-i), \qquad (3)$$

which is obtained by imposing the *constraint* $w_i(k-j) = w_i(k)$ on the original error $e(k)$ of the filtered-x LMS algorithm. It is derived to modify the adaptation rule given in eqs. (1-2) as follows[2,3,4] :

$$w_i(k+1) = w_i(k) + 2\eta\epsilon(k) \sum_{j=0}^{p} h_j x(k-j-i), \qquad (4)$$

This is so called the constrained filtered-x LMS algorithm. Modified Error e(k) can be represented in terms of the original error from Eqs. (2) and (3) as follows :

$$\varepsilon(k) = e(k) - \sum_{j=1}^{p} \sum_{i=0}^{q} h_j w_i(k) x(k-j-i) + \sum_{j=1}^{p} \sum_{i=0}^{q} h_j w_i(k-j) x(k-j-i). \tag{5}$$

III. Convergence Analysis of the Constrained Filtered-x LMS Algorithm in the Mean Sense

In this section, we discuss the convergence properties of the constrained filtered-x LMS algorithm in the sense of mean value of weight. It is very difficult to examine the stability of the constrained filtered-x LMS algorithm in general. We derive the region of h which makes the controller stable in a specific case where the error path is a very simple n-step time-delay system. Let's assume that error path H and its model $\hat{H}$ are represent as follows :

$$H = [0, \ldots, 0, h_n, 0, \ldots, 0]^T,$$
$$\hat{H} = [0, \ldots, 0, h_{n+m}, 0, \ldots, 0]^T. \tag{6}$$

$\hat{H}$, the estimated model of H, is introduced to accommodate the real-world situation where true error path model can not be identified exactly. Therefore actual constrained filtered-x LMS algorithm can be written in a vector form as follows :

$$W(k+1) = W(k) + 2\eta\varepsilon(k)\, h_{n+m} X(k-m), \tag{7}$$

$$e(k) = d(k) - h_n X(k)^T W(k-n), \tag{8}$$

$$\begin{aligned} \varepsilon(k) &= d(k) - h_{n+m} X(k-m)^T W(k) \\ &= e(k) + h_{n+m} X(k-m)^T W(k-n-m) \\ &\quad - h_{n+m} X(k-m)^T W(k) \\ &= d(k) - h_n X(k)^T W(k-n) \\ &\quad + h_{n+m} X(k-m)^T \{W(k-n-m) - W(k)\}, \end{aligned} \tag{9}$$

where

$$W(k) = [w_0(k), w_1(k), \ldots, w_q(k)]^T,$$
$$X(k) = [x(k-n), x(k-n-1), \ldots, x(k-n-q)]^T.$$

Let's take expectation on the eq.(7) then

$$E\{W(k+1)\} = E\{W(k)\} + 2\eta E\{h_{n+m} X(k-m)\, \varepsilon(k)\}. \tag{10}$$

$2\eta E\{h_{n+m} X(k-m)\varepsilon(k)\}$ can be rewritten as

$$\begin{aligned} &2\eta E\{h_{n+m} X(k-m)\varepsilon(k)\} \\ &= 2\eta[\hat{h} R_{dx} - \hat{h} h R_{xx} E\{W(k-n)\} \\ &\quad + \hat{h}^2 R_{xx} E\{W(k-n-m) - W(k)\}], \end{aligned} \tag{11}$$

where $X(k) \equiv X$, $X(k-m) \equiv \hat{X}$, $h_{n+m} \equiv \hat{h}$, $h_n \equiv h$

$$R_{dx} = E\{d(k) X(k-m)\},\ R_{\hat{x}x} = E\{X(k-m) X(k)^T\},$$
$$R_{x\hat{x}} = E\{X(k) X(k-m)^T\}.$$

The R_{xx} and R_{xy} are the auto-correlation matrix and the cross-correlation vector, respectively. Note that elements of matrix R_{xx} are complex numbers in general. The shur decomposition of R_{xx} gives matrix T whose diagonal terms are eigenvalues of R_{xx} as follows :

$$\begin{aligned} &R_{xx} = UTU^{-1} = UTU^T, \\ &UU^{-1} = I, \\ &T_{ii} = \lambda_i = \sigma_i + j\omega_i. \end{aligned} \tag{12}$$

The optimum weight vector, W*, that results in a gradient of zero, is

$$W^* = \frac{R_{xx}^{-1} R_{dx}}{h}. \tag{13}$$

Substituting eqs.(11-13) into eq(10) gives

$$\begin{aligned} U^{-1}E(W(k+1) - W^*) = U^{-1}E(W(k) - W^*) + 2\eta U^{-1}[\hat{h} R_{dx} \\ - \hat{h} h R_{xx} E\{W(k-n)\} + \hat{h}^2 R_{xx} E\{W(k-n-m) - W(k)\}]. \end{aligned} \tag{14}$$

By introducing a vector V(k), which represents the difference between W(k) and its steady state value W*, we can rewrite as

$$W(k) = W^* + V(k). \tag{15}$$

By substituting eq(15), the update form of constrained filtered-x LMS algorithm can be written as follows :

$$\begin{aligned} V(k+1) = (I - 2\eta\hat{h}^2 T) V(k) + 2\eta\hat{h}^2 T\, V(k-n-m) \\ - 2\eta\hat{h}\, h\, T\, V(k-n). \end{aligned} \tag{16}$$

To find the analytical stability condition, let's apply Z-transform to eq(16).

$$V(z) = \frac{z^{n+m+1} V(0)}{I z^{n+m+1} - (I - 2\eta\hat{h}^2 T) z^{n+m} + 2\eta\hat{h} h T z^m - 2\eta\hat{h}^2 T}. \tag{17}$$

All poles of V(z), the roots of the following characteristic equation, must be located inside the unit circle on z-plane to stabilize the update process.

$$\det\left(I-(I-2\eta\hat{h}^2 T)z^{-1}+2\eta\hat{h}hTz^{-n-1}-2\eta\hat{h}^2 Tz^{-n-m-1}\right)=0. \quad (18)$$

Then using $\det[U]\det[U^{-1}]=1$, eq(18) is rewritten as

$$\prod_{i=0}^{N}\left(I-(I-2\eta\hat{h}^2\lambda_i)e^{-j\theta}+2\eta\hat{h}h\lambda_i e^{-j\theta(n+1)}-2\eta\hat{h}^2\lambda_i e^{-j\theta(n+m+1)}\right)=0. \quad (19)$$

We investigate the convergence ranges in detail for the following special cases.

1) $m=0,\ h=\hat{h}$

This is the case when H and $\hat{H}$ are n step delays with same gains. From eq(19), the stable range of η can be obtained as follows:

$$0<\eta<\frac{1}{h^2\lambda_{max}}, \quad (20)$$

where λ_{max} represents the maximum eigenvalue of the auto correlation matrix of the input signal. It should be noted that the convergence region of η becomes independent of the delay. In case of the conventional filtered-x LMS algorithm, reference [5,6] showed that the stable range of η as follows:

$$0<\eta<\frac{\sin\frac{\pi}{4n+2}}{\hat{h}h\lambda_{max}}. \quad (21)$$

Hence, the convergence speed of the filtered-x LMS algorithm is expected to decrease with increasing delay n. So we should consider the error path delay when we choose the convergence rate η, otherwise the update process becomes unstable.

2) $m=0,\ h\neq\hat{h}$

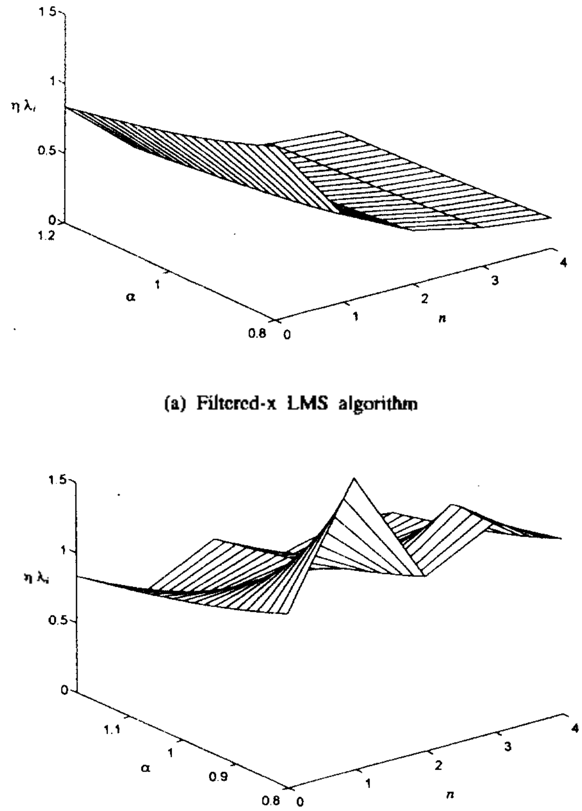
This is the case when H and $\hat{H}$ are n step delays with different gains. From eq(19), the stable range of η can be obtained as follows:

$$0<\eta<\frac{1}{\lambda_{max}}\ \frac{\cos(n+1)\theta-\cos n\theta}{2\hat{h}^2-2h\hat{h}-2\hat{h}^2\cos n\theta} \quad (22)$$

We can find stable bound from eq.(22) in a numerical way. In case of conventional filtered-x LMS algorithm, reference [4,5] showed that the stable range of η as follows:

$$0<\eta<\frac{\sin\frac{\pi}{4n+2}}{\hat{h}h\lambda_{max}}. \quad (23)$$

By the numerical simulation, eqs.(22-23) is illustrated in the Figure 2 ($\alpha=\hat{h}/h$). It is noted that convergence bound of constrained filtered-x LMS algorithm is less sensitive to the system delay than filtered-x LMS algorithm.



(a) Filtered-x LMS algorithm

(b) Constrained Filtered-x LMS algorithm

Figure 2. Convergence bounds of the Filtered-x LMS algorithm and Constrained Filtered-x LMS algorithm.

IV. Conclusion

We introduce the convergence analysis of the constrained filtered-x LMS algorithm in the mean sense. We derive the region of the convergence rate, η which makes the controller stable for a special case where the error path is a simple n-step time-delay and true error path model can not be identified exactly. Also it is demonstrated by the numerical simulation that convergence bound of the constrained filtered-x LMS algorithm is less sensitive to the time delay than filtered-x LMS algorithm.

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