Elastic Wave Propagation in Monoclinic System Due to Harmonic Line Load

*Yong-Yun Kim

Abstract

An analysis of dynamic responses is carried out on monoclinic anisotropic system due to a buried harmonic line source. The load is in the form of a normal stress acting along an arbitrary axis on the plane of symmetry within the orthotropic materials: In case that the line load is acting along the symmetry axis normal to the plane of symmetry, plane wave equation is coupled with vertical shear wave and longitudinal wave. However, if the line load is acting along an arbitrary axis normal to the plane of symmetry, plane wave equation is coupled with vertical shear wave, longitudinal wave and horizontal shear wave. We first considered the equation of motion in a reference coordinate system, where the line load is coincident with a symmetry axis of the orthotropic material. Then the equation of motion is transformed into the one with respect to general coordinate system with azimuthal angle by using transformation tensor. Plane wave solutions of monoclinic systems are derived for infinite media. Finally complete solutions for the plane harmonic wave are obtained by calculating the inverse of the integral transforms, in which bulk wave poles are avoided by deforming the contour of the integration to the complex plane. Numerical results for examples of orthotropic material belonging to monoclinic symmetry are demonstrated.

I. Introduction

Acoustic vibrations in solid structures essentially involve the propagation of wave motion throughout the supporting media. In dealing with acoustic vibrations of systems involving the coupling of compressible fluids with plate and shell structures, it is important to appreciate the "wave view" of vibration. Understanding the response of elastic solids to internal mechanical sources, then, has long been of interest to researchers in classical fields such as acoustics, vibration, seismology, as well as modern fields of application like ultrasonics and acoustic emission.

Plane harmonic wave interaction with homogeneous elastic anisotropic media, in general, and with layered anisotropic media, in particular, have been extensively investigated in the past decade or so. This advancement has been prompted at least from a mechanics point of view, by the increased use of advanced composite materials in many structural applications. A quick review of available literature on this subject reveals that most of the work done so far is carried out on isotropic media. The effect of imposed line load in homogeneous isotropic media has been discussed by several investigators since Lord Rayleigh discovered the existence of surface waves on the surfaces of solids [1]. An account of the literature dealing with this problem through 1957 can be found in Ewing, Jardetzky and Press [2]. Most of the earlier work [3-4] followed Lamb [5], who apparently was the first to consider the motion of half space caused by a vertically applied line load on the free surface or within the medium. He was able to show that displacements at large distance consists of a series of events which corresponds to the arrival of longitudinal, shear, and Rayleigh surface wave.

In this paper, the formal developments in previous works are rigorously followed [6-9] and the response of several anisotropic systems to buried harmonic line loads are studied. The problem is mathematically formulated based on the equations of motion in the constitutive relations. The load is in the form of a normal stress acting along an arbitrary axis on the plane of symmetry within the orthotropic material. In case that the line load is acting along the symmetry axis (azimuthal angle $\phi = 0$) normal to the plane of symmetry, plane wave equation is coupled with vertical shear wave and longitudinal wave. However, if the line load is acting with arbitrary axis (azimuthal angle $\phi \neq 0$) normal to the plane of symmetry, plane wave equation is coupled with vertical shear wave, longitudinal wave and horizontal shear wave. We first considered the equation of motion in a reference coordinate system, where the line load is coincident with a symmetry axis of the orthotropic material. Then the equation of motion is transformed into the one with respect to general coordinate system with azimuthal angle by using transformation tensor. Plane wave solutions of

* Samsung Aerospace Industries Ltd., Precision Instruments R & D Center

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monoclinic systems are derived for infinite media. Finally complete solutions for the plane harmonic wave are obtained by calculating the inverse of the integral transforms, in which bulk wave poles are avoided by deforming the contour of the integration to the complex plane. Numerical results for examples of orthotropic material belonging to monoclinic symmetry are demonstrated.

II. Problem Formulation

Consider an infinite anisotropic elastic medium possessing orthotropic symmetry. The medium is oriented with respect to the reference cartesian coordinate system $x_i' = (x_1', x_2', x_3')$ such that the x_3' is assumed normal to its plane of symmetry as shown in Fig. 1. The plane of symmetry defining the orthotropic symmetry is thus coincident with the $x_1' - x_2'$ plane. With respect to this coordinate system, the equations of motion in the medium are given by

$$\frac{\partial \sigma_{ij}'}{\partial x_i'} + f_i' = \rho' \frac{\partial^2 u_i'}{\partial t^2} \tag{2.1}$$

and, from the general constitutive relations for anisotropic media,

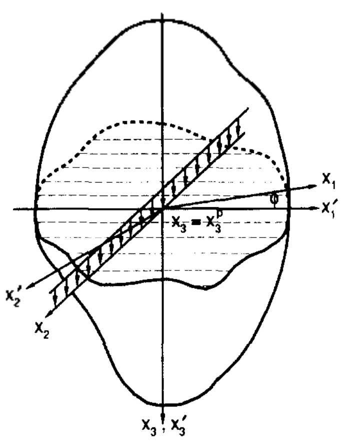

Figure 1. Line load in orthotropic infinite media.

$$\sigma_{ij}' = c_{ijkl}' \, e_{kl}', \qquad i, j, k, l = 1, 2, 3 \tag{2.2}$$

or in the matrix form to specialized orthotropic media

$$\begin{Bmatrix} \sigma_{11}' \\ \sigma_{22}' \\ \sigma_{33}' \\ \sigma_{23}' \\ \sigma_{13}' \\ \sigma_{12}' \end{Bmatrix} = \begin{bmatrix} c_{11}' & c_{12}' & c_{13}' & 0 & 0 & 0 \\ c_{12}' & c_{22}' & c_{23}' & 0 & 0 & 0 \\ c_{13}' & c_{23}' & c_{33}' & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}' \end{bmatrix} \begin{Bmatrix} e_{11}' \\ e_{22}' \\ e_{33}' \\ \gamma_{23}' \\ \gamma_{13}' \\ \gamma_{12}' \end{Bmatrix} \tag{2.3}$$

Where we used the standard contracted subscript notations $1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 13$ and $6 \to 12$, to replace fourth order tenser $c_{ijkl}\,(i, j, k, l = 1, 2, 3)$ by c_{pq} $(p, q = 1, 2, \ldots, 6)$. Thus, c_{45} stands for c_{2313}, for example. Here σ_{ij}, e_{ij} and u_i are the components of stress, strain and displacement, respectively, and ρ is the material density. In equation (2.3), $\gamma_{ij} = 2\,e_{ij}\,(i \neq j)$ defines the engineering shear strain components.

In what follows, we study the response of the infinite medium to a uniform harmonic line load that is applied along a direction that makes an arbitrary azimuthal angle ϕ with the x_2' axis. The analysis is then conducted in a transformed coordinate system x_i formed by a rotation of the plane $x_1' - x_2'$ through the angle ϕ about the x_3' axis. Thus, the direction x_2 will coincide with the line load direction.

Since c_{ijkl} is a fourth order tensor, then for any orthogonal transformation of the primed to the unprimed coordinates, i.e., it transforms according to

$$c_{ijkl} = \beta_{im} \beta_{jn} \beta_{ko} \beta_{lp} c_{mnop}' \tag{2.4}$$

where β_{ij} is the cosine of the angle between x_i and x_j', respectively. For a rotation of angle ϕ in the $x_1' - x_2'$ plane, the transformation tensor β_{ij} reduces to

$$\beta_{ij} = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{2.5}$$

If the transformation is applied to Eq.(2.3), one gets

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} \tag{2.6}$$

The orthotropic system is transformed to monoclinic with nonzero constants of c_{16}, c_{26}, c_{36}, and c_{45}. And the secular equations (2.1) is written in the expanded form in terms of displacement components by applying the tensor transformation equations, Eq.(2.4)-Eq.(2.6)

$$[c_{11}\frac{\partial^2}{\partial x_1^2} + c_{55}\frac{\partial^2}{\partial x_3^2}]u_1 + [c_{16}\frac{\partial^2}{\partial x_1^2} + c_{45}\frac{\partial^2}{\partial x_3^2}]u_2 + \frac{\partial}{\partial x_3}(c_{13} + c_{55})\frac{\partial u_3}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2} - f_1 \tag{2.7a}$$

$$[c_{16}\frac{\partial^2}{\partial x_1^2}+c_{45}\frac{\partial^2}{\partial x_3^2}]u_1+[c_{66}\frac{\partial^2}{\partial x_1^2}+c_{44}\frac{\partial^2}{\partial x_3^2}]u_2 + \frac{\partial}{\partial x_3}(c_{36}+c_{45})\frac{\partial u_3}{\partial x_1}=\rho\frac{\partial^2 u_2}{\partial t^2}-f_2 \quad (2.7b)$$

$$\frac{\partial}{\partial x_3}(c_{13}+c_{55})\frac{\partial u_1}{\partial x_1}+\frac{\partial}{\partial x_3}(c_{36}+c_{45})\frac{\partial u_2}{\partial x_1} + [c_{55}\frac{\partial^2}{\partial x_1^2}+c_{33}\frac{\partial^2}{\partial x_3^2}]u_3=\rho\frac{\partial^2 u_3}{\partial t^2}-f_3 \quad (2.7c)$$

where f_i is defined as $f_1=f_2=0,\ f_3=Q\delta(x_1)\delta(x_3-x_3^p)e^{j\omega t}$ for the harmonic line load.

III. Solutions in the infinite media

The steps leading to formal solutions of Eqs 2.7a-c for each of the two semi-infinite spaces (See Fig. 1) will be outlined. Since the body force has been replaced by an "artificial interface condition", f_i is deleted from Eqs. 2.7a-c[10]. Next, assume harmonic solutions and apply the Fourier transformation to these equations in accordance with

$$u_q=\overline{u_q}\,e^{j\omega t},\qquad \widehat{u_q}=\int_{-\infty}^{\infty}\overline{u_q}\,e^{-j\xi x_1}dx_1 \quad (3.1)$$

The general solution of the resulting differential equations is then sought in the form

$$\widehat{u_i}=U_i e^{-\alpha x_3},\qquad i=1,2,3 \quad (3.2)$$

leading to the characteristic equation

$$\begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{12} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{13} & \Lambda_{23} & \Lambda_{33}\end{pmatrix}\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix}=0 \quad (3.3)$$

where the various entries Λ_{ij} are given in Appendix. Note from equation (3.3) that the Λ_{ij} matrix is symmetric. For the existence of nontrivial solutions in U_i the determinant of the Λ_{ij} matrix in Eq.(3.3) must vanish, thereby leading to an algebraic equation which relates α to ω. This is obviously an alternative presentation of Christoffel's equation. A significant difference is that α is being found in terms of ω as compared with solving for the phase velocity for a given propagation direction. Upon setting the determinant equal to zero, one obtains a fourth order equation which is written symbolically as

$$A_1\alpha^6+A_2\alpha^4+A_3\alpha^2+A_4=0 \quad (3.4)$$

with its coefficients given in Appendix. Eq.(3.4) admits six solutions for α. These α's have the following properties

$$\alpha_2=-\alpha_1,\quad \alpha_4=-\alpha_3,\quad \alpha_6=-\alpha_5 \quad (3.5)$$

Furthermore for each α, equation (3.3) yields the displacement amplitude ratio,

$$\begin{aligned} V_q= V_{q+1}=U_{2q}/U_{1q}&=-[\Lambda_{11}\Lambda_{23}-\Lambda_{12}\Lambda_{13}]/[\Lambda_{12}\Lambda_{23}-\Lambda_{22}\Lambda_{13}]\\ W_q=-W_{q+1}=U_{3q}/U_{1q}&=-[\Lambda_{12}\Lambda_{23}-\Lambda_{13}\Lambda_{22}]/[\Lambda_{23}\Lambda_{23}-\Lambda_{22}\Lambda_{33}]\end{aligned} \quad (3.6)$$

Finally, using superposition, the formal solutions can be written for the displacements of Eq. (2.7a,b) and their associated stress components using Eq.(3.2) as

$$\begin{aligned}(\widehat{u_1},\widehat{u_2},\widehat{u_3})&=\sum_{q=1}^{6}(1,V_q,W_q)U_{1q}e^{-p\alpha_q(x_3-x_3^p)}\\ (\widehat{\sigma_{33}},\widehat{\sigma_{13}},\widehat{\sigma_{23}})&=\sum_{q=1}^{6}p(D_{1q},D_{2q},D_{3q})U_{1q}e^{-p\alpha_q(x_3-x_3^p)}\end{aligned} \quad (3.7)$$

where D_{iq} is given in Appendix.

The above solutions with their various properties can now be specialized to both artificial semi-infinite spaces by the following steps. Inspection of the above solutions indicates that each consists of three pairs of wave components, each pair propagating in the mirror image fashion with respect to the interface, namely along positive and negative x_3 directions. Since propagation is expected to extend from the interface into both media, one arbitrarily reserves α_1, α_3 and α_5 for the lower half-space; the others, namely α_2, α_4 and α_6 for the upper one. The formal solutions are listed in lower and upper half-spaces according to

$$\begin{aligned}(\widehat{u_1},\widehat{u_2},\widehat{u_3})&=\sum_{q=1,3,5}(1,V_q,W_q)U_{1q}e^{\alpha_q(x_3-x_3^p)},\\ (\widehat{\sigma_{33}},\widehat{\sigma_{13}},\widehat{\sigma_{23}})&=\sum_{q=1,3,5}(D_{1q},D_{2q},D_{3q})U_{1q}e^{-\alpha_q(x_3-x_3^p)}\\ &x_3\ge x_3^p\end{aligned} \quad (3.8a)$$

$$\begin{aligned}(\widehat{u_1},\widehat{u_2},\widehat{u_3})&=\sum_{q=2,4,6}(1,V_q,W_q)U_{1q}e^{\alpha_q(x_3-x_3^p)},\\ (\widehat{\sigma_{33}},\widehat{\sigma_{13}},\widehat{\sigma_{23}})&=\sum_{q=2,4,6}(D_{1q},D_{2q},D_{3q})U_{1q}e^{-\alpha_q(x_3-x_3^p)}\\ &x_3\le x_3^p\end{aligned} \quad (3.8b)$$

At this point, a formal solution of the field equation in orthotropic media has been presented. The amplitudes U_{1q} are the unknowns. The amplitudes U_{1q} will be determined by imposing the artificial interface conditions

$$u_1 = u_2 = 0,\ c_{33}\frac{\partial u_3}{\partial x_3} = -\frac{1}{2}Q\delta(x_1)e^{i\omega t}\quad \text{for } x_3 \geq x_3^p,\ \text{at } x_3 = x_3^p$$
$$u_1 = u_2 = 0,\ c_{33}\frac{\partial u_3}{\partial x_3} = \frac{1}{2}Q\delta(x_1)e^{i\omega t}\quad \text{for } x_3 \geq x_3^p,\ \text{at } x_3 = x_3^p \qquad (3.9)$$

To this end, if Eq.(3.8a,b) is subjected to the conditions Eq.(3.9), one finally solves the displacement amplitudes as

$$U_{11} = -U_{12} = (V_5 - V_3)Q/(2c_{33}D_{um})$$
$$U_{13} = -U_{14} = (V_1 - V_5)Q/(2c_{33}D_{um}) \qquad (3.10)$$
$$U_{15} = -U_{16} = (V_3 - V_1)Q/(2c_{33}D_{um})$$

where

$$D_{um} = V_1(\alpha_3 W_3 - \alpha_5 W_5) + V_3(\alpha_5 W_5 - \alpha_1 W_1) + V_5(\alpha_1 W_1 - \alpha_3 W_3) \quad (3.11)$$

It is interesting to note that $D_{um} = 0$ defines an equivalent Christoffel characteristic equation for the propagation of bulk waves in the medium. With these solutions for the wave amplitudes, solutions for infinite space can be written in terms of $q = 1, 3, 5$

$$(\widehat{u_1}, \widehat{u_2}, \widehat{u_3}) = \sum_{q=1,3,5}(1, V_q, W_q)U_{1q}e^{\alpha_q|x_3 - x_3^p|}$$
$$(\widehat{\sigma_{33}}, \widehat{\sigma_{13}}, \widehat{\sigma_{23}}) = \sum_{q=1,3,5}(D_{1q}, D_{2q}, D_{3q})U_{1q}e^{\alpha_q|x_3 - x_3^p|}$$
$$\text{for} \quad x_3 \geq x_3^p \qquad (3.12a)$$

$$(\widehat{u_1}, \widehat{u_2}, \widehat{u_3}) = \sum_{q=1,3,5}(-1, -V_q, W_q)U_{1q}e^{-\alpha_q|x_3 - x_3^p|}$$
$$(\widehat{\sigma_{33}}, \widehat{\sigma_{13}}, \widehat{\sigma_{23}}) = \sum_{q=1,3,5}(-D_{1q}, D_{2q}, D_{3q})U_{1q}e^{\alpha_q|x_3 - x_3^p|}$$
$$\text{for} \quad x_3 \leq x_3^p \qquad (3.12b)$$

In summary, solutions (3.12) define the propagation fields in the infinite space.

IV. Numerical Results and Discussion

Numerical illustrations are presented which verify the previous analysis. For illustrative purposes, a graphite-epoxy composite material is selected. Based upon a 60% graphite fiber volume fraction, these effective properties are given with respect to the reference coordinate system x_i' as

$c_{11} = 155.6\times10^{10}\ \mathrm{N/m^2}$, $c_{22} = c_{33} = 15.95\times10^{10}\ \mathrm{N/m^2}$,
$c_{12} = c_{13} = c_{23} = 3.7\times10^{10}\ \mathrm{N/m^2}$, $c_{44} = 3.7\times10^{10}\ \mathrm{N/m^2}$,
$c_{55} = c_{66} = 7.46\times10^{10}\ \mathrm{N/m^2}$ and $\rho = 1.6\ \mathrm{g/cm^3}$.

For a rotation of $\phi = 45°$ for example, these properties transform to

$c_{11} = c_{22} = 51.20\times10^{10}\ \mathrm{N/m^2}$, $c_{33} = 16.0\times10^{10}\ \mathrm{N/m^2}$,
$c_{12} = 36.28\times10^{10}\ \mathrm{N/m^2}$, $c_{13} = c_{23} = 4.0\times10^{10}\ \mathrm{N/m^2}$,
$c_{44} = c_{55} = 6.6\times10^{10}\ \mathrm{N/m^2}$, $c_{66} = 40.0\times10^{10}\ \mathrm{N/m^2}$,
$c_{16} = c_{26} = 36.1\times10^{10}\ \mathrm{N/m^2}$, $c_{36} = 0.3\times10^{10}\ \mathrm{N/m^2}$

which confirm the earlier conclusion that the transformed matrix takes the form of monoclinic symmetry. Having chosen the material, the procedure for our subsequent calculations can now be summarized. After specifying the azimuthal angle ϕ (namely, the line load direction), the first step is to evaluate the set of α's for given values of the wavenumber, ξ. The various α's are then sorted in accordance with the required format of Eqs.(3.5). In the second step, the displacement ratios are calculated. Now, the inverse transforms can be determined in accordance with

$$u_i = \frac{1}{2\pi}\int_{-\infty}^{\infty}\widehat{u_i}\,e^{i\xi x_1}\,d\xi, \quad i = 1, 2, 3 \qquad (4.1)$$

Depending upon whether $\widehat{u}_i$ is symmetric or antisymmetric, computational effort could be greatly reduced. It follows that $\widehat{u}_1$ and $\widehat{u}_2$ are antisymmetric with respect to ξ while $\widehat{u}_3$ is symmetric. Accordingly, Eq.(4.1) simplifies to

$$\overline{u_i} = \frac{1}{\pi}\int_0^{\infty} j\,\widehat{u}_i\,\sin(\xi x_1)\,d\xi, \quad i = 1, 2 \qquad (4.2a)$$

$$\overline{u_3} = \frac{1}{\pi}\int_0^{\infty}\widehat{u}_3\,\cos(\xi x_1)\,d\xi \qquad (4.2b)$$

Calculation of displacement and stress components can then be done in a straightforward manner, except at singularities. These singularities are poles corresponding to the zeroes of D_{um}, namely, the characteristic equation for bulk wave in the infinite space. The poles do not exist in the second and the fourth quadrants of Fig. 2. Two different procedures have been used in dealing with these poles in the integration process. The first requires the removal of these singularities from the integrals. The second merely deforms the contour of integration below the real ξ-axis, as shown in Fig. 2, so that no poles occur on the path of integration. In order to optimize accuracy, judicious choices of the parameter, ξ_i, are required. The value of ξ_i, imaginary part of ξ, needs to be large enough to offset the contributions of singular points, but not too large since it reduces the accuracy. The value $\xi_i = 0.1$ was selected for the calculation. The azimuthal angle was arbitrarily chosen $\varphi = 45°$ and the angular frequency, $\omega = 3.0\times10^6$ rad/sec.

Fig. 3 depicts the displacements and a normal stress component, $\overline{u_1}$, $\overline{u_2}$, $\overline{u_3}$, $\overline{\sigma_{33}}$ in the media at the location $x_3 = -3$ mm. The location of the line load is chosen at the origin of the coordinate system for infinite geometry : thus $\overline{u_3}$ and $\overline{\sigma_{33}}$ are symmetric with respect to x_3 axis while $\overline{u_1}$ and $\overline{u_2}$ are antisymmetric.

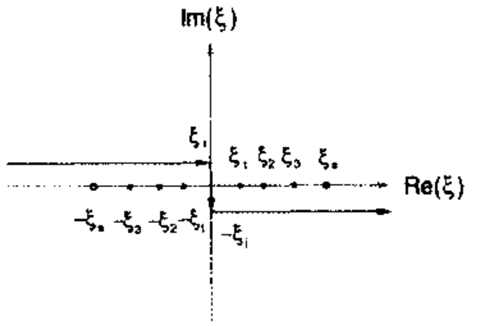

Figure 2. complex integral contour.

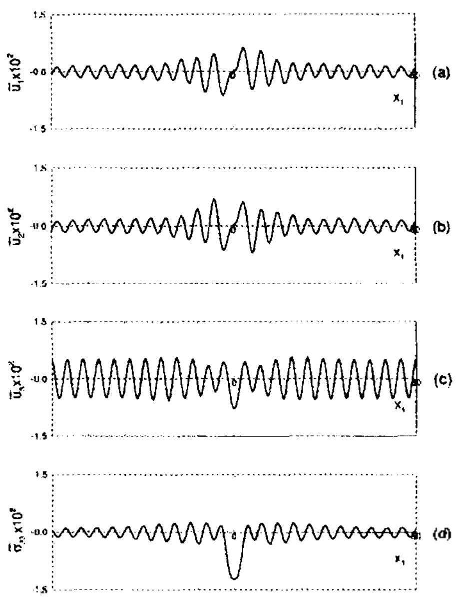

Figure 3. Displacements and stress in graphite-epoxy at $x_3 = -3$.

Note from the figures that the amplitudes of the displacement and stress component slowly decrease with oscillations for long distances from the location of the line load in the infinite space. Fig. 4 presents the displacement modes ($\overline{u_1}$, $\overline{u_3}$) at various locations. The deformation fields are calculated at the locations along the three broken line circles having radii of 2.4, 5.55, and 7.2 mm away from the source location. For these investigations, the line load is located at the origin of the x_i coordinate system in the infinite media. Solid curves represent deformations defined by $x_1 + \overline{u_1} \times 10^2$, $x_3 + \overline{u_3} \times 10^2$. The line load is located at (0,0,1) Note that in the infinite media, $\overline{u_1}$ is zero on the (x_1, 0, 1) and that $\overline{u_3}$ has maximum amplitude on the x_3 axis.

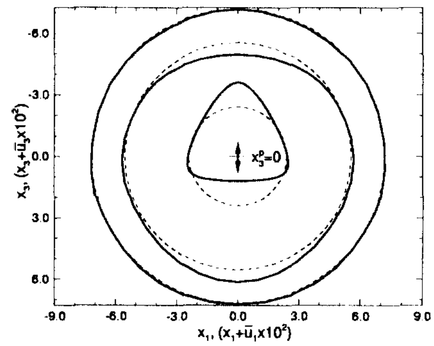

Figure 4. Displacement modes of graphite-epoxy on $x_1 - x_3$ plane.

IV. Conclusion

An analysis of dynamic response is carried out on monoclinic anisotropic system due to a buried harmonic line source. Plane wave equation is coupled with vertical shear wave, longitudinal wave and horizontal shear wave. As the azimuthal angle approaches to zero, horizontal shear wave components disappear and other wave components have numerical results close to those in orthotropic system. But all the wave components can't be numerically calculated by using final solutions in monoclinic system since the displacement ratio V_q have always zero value in case of the orthotropic system which banish all the displacement amplitudes. So analytical solutions for orthotropic system have to be driven by decoupling horizontal shear wave, which will be adquate for the material system possessing orthotropic or higher symmetry, transversely isotropic, cubic, and isotropic symmetry. The solutions of the system with orthotropic symmetry will be simplified to those of isotropic systems by exploiting elastic properties of λ and μ, also.

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Appendix

$$A_{11} = c_{55}\alpha^2 - c_{11}\xi^2 + \rho\omega^2$$
$$A_{12} = c_{45}\alpha^2 - c_{16}\xi^2$$
$$A_{13} = -j\xi\alpha(c_{13} + c_{55})$$
$$A_{22} = c_{44}\alpha^2 - c_{66}\xi^2 + \rho\omega^2$$
$$A_{23} = -j\xi\alpha(c_{36} + c_{45})$$
$$A_{33} = c_{33}\alpha^2 - c_{55}\xi^2 + \rho\omega^2$$

$$A_1 = c_{33}c_{45}^2 - c_{33}c_{44}c_{55}$$
$$A_2 = [c_{11}c_{33}c_{44} + c_{55}F_1 - c_{16}c_{33}c_{45} - c_{45}F_2 + (c_{13} + c_{55})F_3]\xi^2 + [c_{55}(c_{33} + c_{44}) + c_{33}c_{44} - c_{45}^2]\rho\omega$$
$$A_3 = \{(c_{13} + c_{55})F_4 + c_{45}c_{16}c_{55} + c_{16}F_2 - c_{55}c_{66} - c_{11}F_1\}\xi^4 + [(c_{13} + c_{55})^2 + 2c_{45}c_{16} - F_1 - c_{55}(c_{55} + c_{66}) - c_{11}(c_{33} + c_{44})]\rho\omega\xi^2 - (c_{33} + c_{44} + c_{55})\rho^2\omega^2$$
$$A_4 = (c_{11}c_{55}c_{66} - c_{16}^2c_{55})\xi^6 + [c_{11}(c_{55} + c_{66}) - c_{16}^2]\rho\omega\xi^4 + (c_{11} + c_{55} + c_{66})\rho^2\omega^2\xi^2 + \rho^3\omega^3$$

with

$$F_1 = c_{33}c_{66} + c_{44}c_{55} - (c_{36} + c_{45})^2$$
$$F_2 = c_{33}c_{16} + c_{45}c_{55} - (c_{36} + c_{45})(c_{13} + c_{55})$$
$$F_3 = c_{45}(c_{36} + c_{45}) - c_{44}(c_{13} + c_{55})$$
$$F_4 = c_{66}(c_{13} + c_{55}) - c_{16}(c_{36} + c_{45})$$

$$D_{1q} = j\xi(c_{13} + c_{36}V_q) - c_{33}\alpha_q W_q$$
$$D_{2q} = c_{55}(j\xi W_q - \alpha_q) - c_{45}\alpha_q V_q$$
$$D_{3q} = c_{45}(j\xi W_q - \alpha_q) - c_{44}\alpha_q V_q, \quad q = 1, 2, 3, 4, 5, 6$$

$$\alpha_2 = -\alpha_1, \quad \alpha_4 = -\alpha_3, \quad \alpha_6 = -\alpha_5$$
$$V_2 = V_1, \quad V_4 = V_3, \quad V_6 = V_5$$
$$W_2 = -W_1, \quad W_4 = -W_3, \quad W_6 = -W_5$$
$$D_{12} = D_{11}, \quad D_{14} = D_{13}, \quad D_{16} = D_{15}$$
$$D_{22} = -D_{21}, \quad D_{24} = -D_{23}, \quad D_{24} = -D_{25}$$
$$D_{32} = -D_{31}, \quad D_{34} = -D_{33}, \quad D_{36} = -D_{35}$$

▲Yong Yun Kim

Yong Yun Kim received the B.S. degree in mechanical engineering from Yonsei University, Seoul, Korea in 1982 and the M.S. degree in mechanical engineering from University of Toledo, Ohio, U.S.A. in 1989. He received the Ph.D. degree in 1993 and worked as a post doctoral researcher for about one year in aerospace engineering and engineering mechanics from University of Cincinnati, Ohio, U.S.A. Since 1994, he has been with Samsung Aerospace Industries, Kyhung, Korea, where he has been developing several types of interconnecters for semiconductor package. His researchinterests are mathematical and experimental modeling of field problems with regard to plastic deformation of thin plate, elastic wave propagation, and vibration and noise.