Dynamic Responses in Orthotropic Media Due to Pulsating Line Source

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Abstract

The analysis of dynamic responses are carried out on several anisotropic systems due to buried pulsating line sources. These include infinite, semi-infinite spaces. The media possess orthotropic or higher symmetry. The load is in the form of a normal stress acting with parallel to symmetry axis on the plane of symmetry within the materials. The results are first derived for infinite media. Subsequently the results for semi-infinite are derived by using superposition of the solution in the infinite medium together with a scattered solution from the boundaries. The sum of both solutions has to satisfy stress free boundary conditions, thereby leading to the complete solutions. The solutions are simplified to the systems possessing of higher symmetry, such as orthotropic, transversely isotropic, cubic, and isotropic symmetry.

I. Introduction

Acoustic vibrations in solid structures essentially involve the propagation of wave motion throughout the supporting media. In dealing with acoustic vibrations of systems involving coupling of compressible fluids with plate and shell structures, it is important to possess an appreciation of the "wave view" of vibration. Understanding the response of elastic solids to internal mechanical sources, then, has long been of interest to researchers in classical fields such as acoustics, vibration, seismology, as well as modern fields of application like ultrasonics and acoustic emission.

Plane harmonic wave interaction with homogeneous elastic anisotropic media, in general, and with layered anisotropic media, in particular, have been extensively investigated in the past decade or so. This advancement has been prompted at least from a mechanics point of view, by the increased use of advanced composite materials in many structural applications. A quick review of available literature on this subject reveals that most of the work done so far is carried out on isotropic media. The effect of imposed line load in homogeneous isotropic media has been discussed by several investigators ever since Lord Rayleigh discovered the existence of surface waves on the surfaces of solids [11]. An account of the literature dealing with this problem through 1957 can be found in Ewing, Jardetzky and Press [7]. Most of the earlier work [7-9] followed Lamb [10], who apparently was the first to consider the motion of half space caused by a vertically applied line load on the free surface or within the medium. He was able to show that displacements at large distance consists of a series of events which corresponds to the arrival of longitudinal, shear, and Rayleigh surface wave.

In this paper, the formal developments in previous works are rigorously followed [1-4] and study the response of several anisotropic systems to buried pulsating line loads. These include infinite and semi-infinite, systems. The problem is mathematically formulated based on the equations of motion in the constitutive relations. The internal line load will be in the form of a normal stress load, acting at a symmetry direction within the materials in the plane of symmetry. The load is first described as a body force in the equations of the motion for the infinite media and then it is mathematically characterized as "artificial interface conditions" for each semi-infinite spaces. A building block approach is utilized in which the analysis is begun by deriving the results for an infinite media. Subsequently the results for semi-infinite spaces are drived by using superposition of the infinite medium solution together with a scattered solution from the free surface. The sum of both solutions has to satisfy the stress free boundary conditions, thereby leading to a complete solution. The plane harmonic wave is studied in anisotropic media possessing so low as orthotropic symmetry. The solutions then reduce to the case with isotropic symmetry, which agrees with the classical solutions of the potential equations by the separation of variables technique [5].

II. Problem Formulation

Consider an infinite anisotropic elastic medium possessing orthotropic symmetry. The medium is oriented with respect to the reference cartesian coordinate system $x_i = (x_1, x_2, x_3)$ such that the x is assumed normal to

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its plane of symmetry as shown in Figure 1. The plane of symmetry defining the orthotropic symmetry is thus coincident with the $x_1 - x_2$ plane. With respect to this coordinate system, the equations of motion in the medium are given by [1]

$$\frac{\partial \sigma_{ij}}{\partial x_i} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{2.1}$$

and, from the general constitutive relations for anisotropic media,

$$\sigma_{ij} = c_{ijkl}\, e_{kl}, \qquad i, j, k, l = 1, 2, 3 \tag{2.2}$$

by the specialized expanded matrix form to orthotropic media

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{pmatrix} \tag{2.3}$$

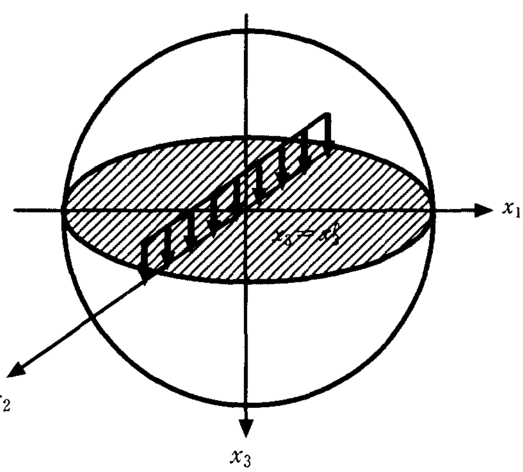

Figure 1. An applied line load in Orthotropic Infinite Media.

Where we used the standard contracted subscript notations $1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 13$ and $6 \to 12$, to replace fourth order tenser c_{ijkl} $(i, j, k, l = 1, 2, 3)$ with c_{pq} $(p, q = 1, 2, \ldots, 6)$. Thus, c_{45} stands for c_{2313}, for example. Here σ_{ij}, e_{ij} and u_i are the components of stress, strain and displacement, respectively, and ρ is the material density. In equation (2.3), $\gamma_{ij} = 2 e_{ij} (i \neq j)$ defines the engineering shear strain components.

In what follows, one studies of the infinite medium to a uniform harmonic line load that is applied along a direction on the symmetry axis of the material system. One would like to solve the case of orthotropic symmetry by applying constitutive equation (2.3) to the secular equations (2.1) that is written in the expanded form in terms of displacement components

$$\left[c_{11} \frac{\partial^2}{\partial x_1^2} + c_{55} \frac{\partial^2}{\partial x_3^2}\right] u_1 + (c_{13} + c_{55}) \frac{\partial^2 u_3}{\partial x_3^2} = \rho \frac{\partial^2 u_1}{\partial t^2} - f_1 \tag{2.3a}$$

$$\frac{\partial}{\partial x_3} \left[(c_{13} + c_{55}) \frac{\partial}{\partial x_1}\right] u_1 + \left[c_{55} \frac{\partial^2}{\partial x_1^2} + c_{33} \frac{\partial^2}{\partial x_3^2}\right] u_3 = \rho \frac{\partial^2 u_3}{\partial t^2} - f_3 \tag{2.3b}$$

$$\left[c_{66} \frac{\partial^2}{\partial x_1^2} + c_{44} \frac{\partial^2}{\partial x_3^2}\right] u_2 = \rho \frac{\partial^2 u_2}{\partial t^2} - f_2 \tag{2.3c}$$

where f_i is defined as $f_1 = f_2 = 0$, $f_3 = Q\,\delta(x_1)\,\delta(x_3 - x_3^0)\, e^{i\omega t}$. Equation(2.3c) represents a horizontal shear wave equation that is independent of a vertical shear wave and a longitudinal wave. Since the line load has only vertical component, equations(2.3a,b) need to be considered.

III. Solutions in the Infinite Media

The steps leading to formal solutions of equations (2.3a,b) for each of the two semi-infinite spaces (See Figure 1) will be outlined. Since the body force has been replaced by an "artificial interface condition", f_i deleted from Equations (2.3a,b). Next, assume harmonic solutions followed by applying the Fourier transform to these equations in accordance with

$$u_q = \overline{u_q}\, e^{i\omega t}, \quad \widehat{u_q} = \int_{-\infty}^{\infty} \overline{u_q}\, e^{-i\xi x_1}\, dx_1 \tag{3.1}$$

The general solution of the resulting differential equations is then sought in the form

$$\hat{u}_i = U_i e^{-\alpha x_3}, \quad i = 1, 2, 3 \tag{3.2}$$

leading to the characteristic equation

$$\begin{pmatrix} \Lambda_{11} & \Lambda_{13} \\ \Lambda_{13} & \Lambda_{33} \end{pmatrix} \begin{pmatrix} U_1 \\ U_3 \end{pmatrix} = 0 \tag{3.3}$$

where the various entries Λ_{ij}, are given by

$$\begin{aligned} \Lambda_{11} &= C_{55}\alpha^2 - c_{11} + \rho\omega^2 \\ \Lambda_{13} &= -j\xi\alpha(c_{13} + c_{55}) \\ \Lambda_{33} &= c_{33}\alpha^2 - c_{55}\xi^2 + \rho\omega^2 \end{aligned} \tag{3.4}$$

Note from equation (3.3) that the Λ_{ij} matrix is symmetric. For the existence of nontrivial solutions in U, the determinant in equation (3.3), must vanish, there by leading to an algebraic equation which relates α to ω. This is obviously an alternative presentation of Christoffel's equation [1]. The difference is that α is being found in terms of ω as compared with solving for the phase velocity for a given propagation direction. Upon setting the determinant equal to zero, one obtains a fourth order equation in α which is written symbolically as

$$A_1\alpha^4 + A_2\alpha^2 + A_3 = 0 \tag{3.5}$$

with its coefficients given by

$$\begin{aligned} A_1 &= c_{33}c_{55} \\ A_2 &= [(c_{13}+c_{55})^2 - c_{11}c_{33} - c_{55}^2]\xi^2 - (c_{33}+c_{55})\rho\omega^2 \\ A_3 &= c_{11}c_{55}\xi^4 + (c_{11}+c_{55})\rho\xi^2 + \rho^2\omega^4 \end{aligned} \tag{3.6}$$

Equation (3.5) admits four solutions for α. These α's have the further properties that

$$\alpha_2 = -\alpha_1, \quad \alpha_4 = -\alpha_3 \tag{3.7}$$

Furthermore for each α, equation (3.3) yields the displacement amplitude ratio,

$$W_q = U_{3q}/U_{1q} = \frac{j\xi\alpha_q(c_{13}+c_{55})}{c_{33}\alpha_q^2 - c_{55}\xi^2 + \rho\omega^2} \tag{3.8}$$

Finally, using superposition, the formal solutions can be written for the displacements of equations (2.3a,b) and their associated stress components using equation (3.2) as

$$(\widehat{u_1}, \widehat{u_3}) = \sum_{q=1}^{4} (1, W_q)\, U_{1q}\, e^{-\alpha_q(x_3 - x_3^s)} \tag{3.9a}$$

$$(\widehat{\sigma_{33}}, \widehat{\sigma_{13}}) = \sum_{q=1}^{4} (D_{1q}, D_{2q})\, U_{1q}\, e^{-\alpha_q(x_3 - x_3^s)} \tag{3.9b}$$

where

$$D_{1q} = j\xi c_{13} - c_{33}\alpha_q W_q \tag{3.10a}$$

$$D_{2q} = c_{55}(j\xi W_q - \alpha_q), \quad q = 1, 2, 3, 4 \tag{3.10b}$$

The above solutions with their various properties can now be specialized to both artificial semi-infinite spaces by the following steps. Inspection of the above solutions indicate that each consists of two pairs of wave components, each pair propagating in mirror image fashion with respect to the interface, namely along positive and negative x_3 directions. Since propagation is expected to emanate from the interface into both media, one arbitrary reserves α_1 and α_3 for the lower half-space; the remaining one's, namely described with α_2 and α_4 for the upper one. The formal solutions are listed in lower and upper half-spaces according to

$$\left.\begin{aligned} (\widehat{u_1}, \widehat{u_3}) &= \sum_{q=1,3} (1, W_q) U_{1q} e^{-\alpha_q(x_3 - x^{p})} \\ (\widehat{\sigma_{33}}, \widehat{\sigma_{13}}) &= \sum_{q=1,3} (D_{1q}, D_{2q}) U_{1q} e^{-\alpha_q(x_3 - x^{p})} \end{aligned}\right\}, \; x_3 \geq x_3^p \tag{3.11a}$$

$$\left.\begin{aligned} (\widehat{u_1}, \widehat{u_3}) &= \sum_{q=2,4} (1, W_q) U_{1q} e^{-\alpha_q(x_3 - x^{p})} \\ (\widehat{\sigma_{33}}, \widehat{\sigma_{13}}) &= \sum_{q=2,4} (D_{1q}, D_{2q}) U_{1q} e^{-\alpha_q(x_3 - x^{p})} \end{aligned}\right\}, \; x_3 \leq x_3^p \tag{3.11b}$$

At this point, a formal solution of the field equation in orthotropic media has been presented. The amplitudes U_{1q} are the unknowns. The amplitudes U_{1q} will be determined by implementing the artificial interface conditions

$$\begin{aligned} c_{33}\frac{\partial u_3}{\partial x_3} &= -\frac{1}{2}Q\delta(x_1)e^{j\omega t}, \quad \text{for } x_3 = x_3^p \text{ at } x_3 \geq x_3^p \\ c_{33}\frac{\partial u_3}{\partial x_3} &= \frac{1}{2}Q\delta(x_1)e^{j\omega t}, \quad \text{for } x_3 = x_3^p \text{ at } x_3 \geq x_3^p \end{aligned} \tag{3.12}$$

To this end, if (3.11a,b) is subjected to the conditions (3.12), one finally solves the displacement amplitudes as

$$U_{11} = -U_{12} = \frac{Q}{(2c_{33}D_{uo})}, \quad U_{11} = -U_{12} = \frac{-Q}{(2c_{33}D_{uo})} \tag{3.13}$$

where

$$D_{uo} = \alpha_1 W_1 - \alpha_3 W_3 \tag{3.14}$$

It is interesting to note that $D_{uo} = 0$ defines an equivalent Christoffel characteristic equation for the propagation of bulk waves in the medium. With these solutions for the wave amplitudes, solutions for infinite space can be written in terms of $q = 1, 3$

$$\begin{aligned} 2c_{33}D_{uo}\widehat{u_1} &= Q[e^{-\alpha_1|x_3 - x_3^s|} - e^{-\alpha_3|x_3 - x_3^s|}] \\ 2c_{33}D_{uo}\widehat{u_3} &= Q[W_1 e^{-\alpha_1|x_3 - x_3^s|} - W_3 e^{-\alpha_3|x_3 - x_3^s|}] \end{aligned} \tag{3.15}$$

In summary, solutions (3.15) define the propagation fields in the infinite spaces.

IV. Solution in Semi-infinite Spaces

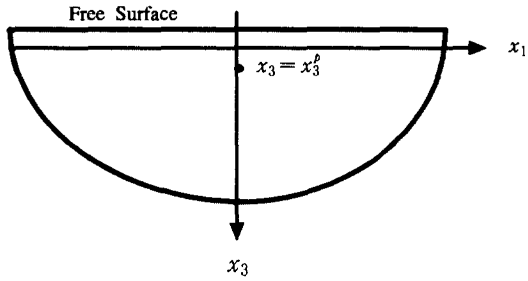

Figure 2. Semi-infinite Media.

The solutions will now be specialized to the infinite media (3.11a,b) to solve for the case where the free boundary intercepts the propagating pulse at some arbitrary location parallel to the plane $x_3 = 0$. It is assumed that the free boundary is located at $x_3 = -a$ as depicted in Figure 2. This implies that the free boundary is located in the upper region and thus can only interfere with the propagation fields in the negative x_3 direction. For this case, the solution (3.15) will constitute an incident wave on a free surface. As a result, waves will reflect from the free boundary and propagate in the positive x_3 direction. Thus, appropriate formal solutions for the reflected waves can be adapted from the solution (3.11a,b) in accordance with (Note now that x_3^β does not appear because solutions are referred to the origin $x_3 = 0$)

$$(\widehat{u_1^r}, \widehat{u_3^r}) = \sum_{q=1,3} (1, W_q) U_{1q}^r e^{-a_q x_3} \tag{4.1a}$$

$$(\widehat{\sigma_{33}^r}, \widehat{\sigma_{13}^r}) = \sum_{q=1,3} (D_{1q}, D_{2q}) U_{1q}^r e^{a_q x_3} \tag{4.1b}$$

With this, the total solution for the semi-infinite space (designated with superscript 's') which is required to satisfy the stress free boundary condition is obtained by superposing the incident waves and reflected waves in accordance with

$$(\widehat{u_1^s}, \widehat{u_3^s}) = \sum_{q=1,3} (1, W_q) U_{1q}^r e^{-a_q x_3} + \sum_{q=1,3} (-1, W_q) U_{1q}^{(i)} e^{a_q (x_3^\beta - x_3)} \tag{4.2a}$$

$$(\widehat{\sigma_{33}^s}, \widehat{\sigma_{13}^s}) = \sum_{q=1,3} (D_{1q}, D_{2q}) U_{1q}^r e^{-a_q x_3} + \sum_{q=1,3} (-D_{1q}, D_{2q}) U_{1q}^{(i)} e^{-a_q (x_3^\beta - x_3)} \tag{4.2b}$$

The boundary condition is given by

$$\widehat{\sigma_{13}^s} = \widehat{\sigma_{33}^s} = 0 \quad \text{at} \quad x_3 = -d \tag{4.3}$$

By imposing the boundary conditions (4.5) on equation (4.4), a linear system is obtained, which is expressible as

$$\begin{pmatrix} D_{11} & D_{13} \\ D_{21} & D_{23} \end{pmatrix} \begin{pmatrix} U_{11}^r E_1 \\ U_{13}^r E_3 \end{pmatrix} = \begin{pmatrix} R_1 \\ R_3 \end{pmatrix} \tag{4.4}$$

where

$$\begin{aligned} R_1 &= \quad (U_{11}^{(i)} D_{11} E_1^r + U_{13}^{(i)} D_{13} E_3^r) \\ R_3 &= -(U_{11}^{(i)} D_{21} E_1^r + U_{13}^{(i)} D_{23} E_3^r) \end{aligned} \tag{4.5}$$

Using the standard Cramer's rule, the solutions for the reflected amplitudes are

$$\begin{aligned} U_{11}^r &= (R_1 D_{23} - R_3 D_{13})/(D_{so} E_1) \\ U_{13}^r &= (R_3 D_{11} - R_1 D_{21})/(D_{so} E_3) \end{aligned} \tag{4.6}$$

with

$$E_q = e^{a_q d}, \quad E_q^r = e^{a_q (x_3^\beta + d)}, \quad q = 1, 3 \tag{4.7}$$

and

$$D_{so} = D_{11} D_{23} - D_{21} D_{13} \tag{4.8}$$

The simplified solutions are obtained by combining (4.7) with (4.8) as

$$\begin{aligned} \widehat{u_1} = \frac{Q}{2c_{33} D_{\mu o} D_{so}} [& \text{sign}\, D_{so} (e^{-a_1 |x_3 - x_3^\beta|} - e^{-a_3 |x_3 - x_3^\beta|}) \\ & + ((D_{11} D_{23} + D_{21} D_{13}) E_1^r - 2 D_{13} D_{23} E_3^r) e^{-a_1 (x_3 + d)} \\ & + ((D_{11} D_{23} + D_{21} D_{13}) E_3^r - 2 D_{11} D_{21} E_1^r) e^{-a_3 (x_3 + d)}] \end{aligned} \tag{4.9a}$$

$$\begin{aligned} \widehat{u_3} = \frac{Q}{2c_{33} D_{\mu o} D_{so}} [& D_{so} (W_1 e^{-a_1 |x_3 - x_3^\beta|} - W_3 e^{-a_3 |x_3 - x_3^\beta|}) \\ & + ((D_{11} D_{23} + D_{21} D_{13}) E_1^r - 2 D_{13} D_{23} E_3^r) W_1 e^{-a_1 (x_3 + d)} \\ & + ((D_{11} D_{23} + D_{21} D_{13}) E_3^r - 2 D_{11} D_{21} E_1^r) W_3 e^{-a_3 (x_3 + d)}] \end{aligned} \tag{4.9b}$$

with $\text{sign} = (x_3 - x_3^\beta)/|x_3 - x_3^\beta|$

V. Reduction of Results to Isotropic Media

Solutions for the isotropic symmetry can be obtained from the orthotropic symmetry by exploiting the degeneracy's of the elastic properties c_{mn}. Replacing c_{11} by $\lambda + 2\mu$, c_{55} by μ, setting $c_{33} = c_{11}$ and $c_{13} = c_{11} - 2c_{55} = \lambda$ results in an isotropic medium. Although these simplifications are adequate to reduce all of the previous results to ones pertaining to isotropic media, nevertheless they lead to much simpler expressions for the various propagation parameters that are given by equations (3.7, 3.8, 3.10a,b) [6]. The characteristic equation (3.5) reduces to

$$[\alpha^2-(\xi^2-\xi_L^2)][\alpha^2-(\xi^2-\xi_T^2)]=0 \qquad (5.1)$$

Specially, implementing the isotropic restrictions reduces these parameters to the following.

$$\begin{array}{ll} \xi_L=\sqrt{\rho\omega^2/(\lambda+2\mu)}, & \xi_L=\sqrt{\rho\omega^2/(\lambda+2\mu)} \\ \alpha_1=\sqrt{\xi^2-\xi_L^2}, & \alpha_3=\sqrt{\xi^2-\xi_T^2} \\ \omega_1=j\alpha_1/\xi, & \omega_3=j\xi/\alpha_3 \\ D_{11}=j\mu(2\xi^2-\xi_T^2)/\xi, & D_{13}=2j\xi\mu \\ D_{21}=2\alpha_1, & D_{23}=(2\xi^2-\xi_T^2)/\alpha_3 \end{array} \qquad (5.2)$$

By substituting (5.2) into (3.15), the solutions for orthotropic infinite media are reduced to the solutions for those of isotropic media [5]

$$\begin{aligned} \widehat{u_1} &= \frac{\text{sign } jQ\xi}{2\mu\xi_T^2}\left[e^{-\alpha_1|x_3-x_3'|}-e^{-\alpha_3|x_3-x_3'|}\right] \\ \widehat{u_3} &= \frac{Q}{2\mu\xi_T^2\alpha_3}\left[-\alpha_1\alpha_3 e^{-\alpha_1|x_3-x_3'|}+\xi^2 e^{-\alpha_3|x_3-x_3'|}\right] \end{aligned} \qquad (5.3)$$

with the aid of

$$D_u=\frac{j\xi_L^2}{\xi} \qquad (5.4)$$

The response to the internal line load in the semi-infinite space with orthotropic symmetry reduces to the solution for a media with isotropic symmetry. Equations (4.11, 4.12) then become

$$\begin{aligned} \widehat{u_{1s}} &= \frac{Q}{2\mu\xi_T^2 D_s}[\text{ sign } j\xi D_s(e^{-\alpha_1|x_3-x_3'|}-e^{-\alpha_3|x_3-x_3'|}) \\ &\quad + S_1 e^{-\alpha_1(x_3+d)}+S_3 e^{-\alpha_3(x_3+d)}] \\ \widehat{u_{3s}} &= \frac{Q}{2\mu\xi_T^2\alpha_3 D_s}[D_s(-\alpha_1\alpha_3 e^{-\alpha_1|x_3-x_3'|}+\xi^2 e^{-\alpha_3|x_3-x_3'|}) \\ &\quad - S_1\alpha_1\alpha_3 e^{-\alpha_1(x_3+d)}-S_3\xi^2 e^{-\alpha_3(x_3+d)}] \end{aligned} \qquad (5.5)$$

where

$$\begin{aligned} S_1 &= \overline{D_s}E_1'-4\xi^2(2\xi^2-\xi_T^2)E_3' \\ S_3 &= \overline{D_s}E_3'-4\alpha_1\alpha_3(2\xi^2-\xi_T^2)E_1' \end{aligned} \qquad (5.6)$$

with

$$\begin{aligned} D_s &= (2\xi^2-\xi_T^2)^2-4\xi^2\alpha_1\alpha_3 \\ \overline{D_s} &= (2\xi^2-\xi_T^2)^2+4\xi^2\alpha_1\alpha_3 \end{aligned} \qquad (5.7)$$

VI. Conclusion

The analysis of dynamic responses due to pulsating line load are carried out on orthotropic system which include infinite, semi-infinite spaces. These analytical solutions are adquate for the material system possessing orthotropic or higher symmetry, transversely isotropic, cubic, and isotropic contained implicitly in the analysis. The solutions of the system with orthotropic symmetry have simplified to those of isotropic systems by exploiting elastic properties of λ and μ.

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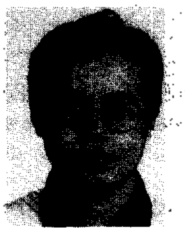

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