Complex Fuzzy Logic Filter and Learning Algorithm for Signal Processing Application

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Abstract

A fuzzy logic filter is constructed from a set of fuzzy IF-THEN rules which change adaptively to minimize some criterion function as new information becomes available. This paper generalizes the fuzzy logic filter and it's adaptive filtering algorithm to include complex parameters and complex signals. Using the complex Stone-Weierstrass theorem, we prove that linear combinations of the fuzzy basis functions are capable of uniformly approximating any complex continuous function on a compact set to arbitrary accuracy. Based on the fuzzy basis function representations, a complex orthogonal least-squares (COLS) learning algorithm is developed for designing fuzzy systems based on given input-output pairs. Also, we propose an adaptive algorithm based on LMS which adjust simultaneously filter parameters and the parameter of the membership function which characterize the fuzzy concepts in the IF-THEN rules. The modeling of a nonlinear communications channel based on a complex fuzzy filter is used to demonstrate the effectiveness of these algorithm.

Ⅰ. Introduction

In recent, the fuzzy logic filter have ever-increasingly been applied to many diverse fields in signal and control processing[1][2][3][4]. The fuzzy adaptive filter, which is constructed from a set of changeable fuzzy IF-THEN rules has drawn a great deal of attention because of its universal approximation ability. These fuzzy rules come either from human experts or by matching input-output pairs through an adaptation procedure[5]. We therefore would like to apply this non-linear adaptive algorithm to the signal processing problems. Some examples of application of fuzzy filter to signal processing include classification and signal prediction, communications channel equalisation, and nonlinear systems modeling and identification. Most available fuzzy filters are real-valued and are suitable for signal processing in real multidimensional space.

In some applications, however, signals are complex valued and processing is done in complex multi-dimensional space[6][7][8]. An example is the equalisation of digital communications channels with complex signaling schemes such as quadrature amplitude modulation (QAM). For complex signal processing problems, many existing fuzzy filters cannot directly be applied.

The present study proposes a complex fuzzy adaptive filter with changeable fuzzy IF-THEN rules, which is an extension of the real fuzzy filter. Specifically, membership function of this is real. The inputs and outputs as well as parameters of the filter are all complex-valued. The filter can be viewed as a mapping from the complex multi-input onto the complex one-output. When both the filter inputs and desired outputs are reduced to real-valued, this complex fuzzy filter degenerates naturally into the real fuzzy filter.

An advantage of this complex fuzzy filter is that linear learning laws can be derived as in the real case. Two learning algorithms, the complex orthogonal least squares algorithm and complex LMS algorithm are presented. The COLS algorithm is a batch learning algorithm and it constructs fuzzy rule in a rational way until an adequate performance is achieved. The complex LMS algorithm can conveniently be implemented as a recursive learning algorithm. These two algorithm are derived as natural extensions of their real counterparts. The adaptive algorithm based on least mean squares (LMS) adjust filter coefficient and the parameters of the membership functions which characterize the fuzzy concepts in the IF-THEN rules. The modeling of a nonlinear communications channel based on a complex fuzzy filter is used to demonstrate

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the effectiveness of this algorithm. In this practical application, the complex fuzzy filter realizes the Bayesian equalizer.

Ⅱ. Complex Fuzzy Filter

We consider n-input and single-output fuzzy systems which deal with complex values: $U \subset R^n \rightarrow R$, where U and R are respective complex space of the input and output of filter. As usual a complex quantity is defined as

$$z = z_R + jz_I = (z_R,\ z_I) \tag{1}$$

where z_R and z_I are the real and imaginary parts of z, respectively, and $j = \sqrt{-1}$.

Define M fuzzy sets in the product space of real intervals $[C_i^-,\ C_i^+]$ and imaginary intervals $[B_i^-,\ B_i^+]$ of the complex input space U, which are labeled as F_i^l ($i=1, ..., n$, $l=1, ..., M$), in the following way: the M membership functions $\mu_{F_i^l}$ cover the space $[C_i^-,\ C_i^+]\times[B_i^-,\ B_i^+]$ in the sense that for each $z_{Ri}\in[C_i^-,\ C_i^+]$ and $z_{Ii}\in[B_i^-,\ B_i^+]$, there exists at least one $\mu_{F_i^l}[z_{Ri},\ z_{Ii}]\neq 0$. We can construct a set of changeable complex fuzzy IF-THEN rules in the following form:

$$\begin{aligned} R^l: &\text{IF } z_1=(z_{R1},\ z_{I1}) \text{ is } F_1^l \text{ and } ... \\ &\text{and } z_n=(z_{Rn},\ z_{In}) \text{ is } F_n^l,\ \text{THEN } d \text{ is } G^l \end{aligned} \tag{2}$$

where $z=(z_1 ... z_n)^T\in U$, d is the complex output variable of the fuzzy system, and G^l are linguistic terms characterized by fuzzy membership functions μ_{G^l} with center $\theta^l=\theta_R^l+j\theta_I^l$.

If we choose the Gaussian fuzzy membership function, $\mu_{F_i^l}(z_{Ri},\ z_{Ii})$ is defined in terms of the mean and variance pairs of z_R and z_I, $(m_R^l,\ \sigma_R^{l^2})$ and $(m_I^l,\ \sigma_I^{l^2})$ by eq.(1):

$$\begin{aligned} \mu_{F_i^l}(z_{Ri},\ z_{Ii}) &= \mu_{F_i^l}(z_{Ri})\,\mu_{F_i^l}(z_{Ii}) \\ &= exp\left[-\frac{1}{2}\left(\frac{z_{Ri}-m_{Ri}^l}{\sigma_{Ri}^l}\right)^2\right] exp\left[-\frac{1}{2}\left(\frac{z_{Ii}-m_{Ii}^l}{\sigma_{Ii}^l}\right)^2\right] \end{aligned} \tag{3}$$

Then, we construct the complex fuzzy adaptive filter which is equivalent to a fuzzy basis function expansion in a fuzzy adaptive filter [1]:

$$f_k(z) = \frac{\sum_{l=1}^{M} \theta^l \prod_{i=1}^{n} \mu_{F_i^l}(z_i(k))}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{F_i^l}(z_i(k))} \tag{4}$$

where $\mu_{F_i^l}$ are the Gaussian membership functions defined as

$$\mu_{F_i^l}(z_i) = exp\left[-\frac{1}{2}\left(\frac{z_{Ri}-m_{Ri}^l}{\sigma_{Ri}^l}\right)^2 - \frac{1}{2}\left(\frac{z_{Ii}-m_{Ii}^l}{\sigma_{Ii}^l}\right)^2\right] \tag{5}$$

and $\theta^l\in R$ is any point at which μ_{G^l} achieves its maximum value.

Defining fuzzy basis functions as

$$p_j(z) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(z_i(k))}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{F_i^l}(z_i(k))}$$

the fuzzy system (4) is equivalent to an complex FBF expansion:

$$f(z) = \sum_{j=1}^{M} p_j(z)\,\theta^j. \tag{6}$$

The following theorem shows the complex fuzzy system based on the FBF expansion are universal approximators.

Theorem: Let Y be the set of all the FBF expansions. Then, Y is dense in the set of all complex continuous functions on a compact set. Therefore, for any given complex continuous function g on the compact set $U\subset k^n$, k is complex domain and arbitrary $\varepsilon>0$, there exists $f\in Y$ such that

$$sup_{x\in U}\,|g(x)-f(x)|<\varepsilon. \tag{7}$$

A proof of this theorem is given in the Appendix. This theorem shows that the FBF expansions (6) are "universal approximators."

Ⅲ. Complex Orthogonal Least Squares Algorithm

In order to describe how the complex orthogonal least-squares (COLS) learning algorithm works, it is essential to view the complex fuzzy basis function expansion (6) as a special case of the linear regression model

$$d(t) = \sum_{j=1}^{M} p_j(t)\,\theta_j + e(t) \tag{8}$$

where $d(t)$ is system output, θ_j are complex parameters, $p_j(t)$ are known as regressors which are fixed functions of system inputs $z(t)$, i.e.,

$$p_j(t) = p_j(z(t)) \tag{9}$$

and $e(t)$ is an error signal which is assumed to be uncorrelated with regressors. Suppose that we have N input-output pairs: $(z^0(t), d^0(t))$, $t=1, 2, \ldots, N$. Our task is to design an FBF expansions $f(z)$ such that some error function between $f(z^0(t))$ and $d^0(t)$ is minimized.

In order to present the Complex OLS algorithm, we arrange (8) from $t=1$ to N in the following matrix form:

$$D = P\Theta + e \tag{10}$$

where $D=[d(1), \ldots, d(N)]^T$, $P=[p_1, \ldots, p_M]$ with $p_i=[p_i(1), \ldots, p_i(N)]^T$, and $\Theta=[\theta_1, \ldots, \theta_M]^T$, $e=[e(1), \ldots, e(N)]^T$.

The complex OLS algorithm transforms the set of p_i into a set of orthogonal basis vectors and uses only the significant basis vectors to form the final FBF expansion. In order to perform the COLS procedure, we first need to fix the parameters (m^l_{Ri}, m^l_{Ii}) and $(\sigma^l_{Ri}, \sigma^l_{Ii})$ in the FBF $p_i(z)$ based on the input-output pairs. We propose the following scheme:

Initial FBF Determination,

Choose N initial $p_j(z)$'s in the form of (6), with the parameters determined as follows: $(m^j_{Ri}, m^j_{Ii})=(z^0_{Ri}(j), z^0_{Ii}(j))$,

$\sigma^j_{Ri}=\left[\max_j(z^0_{Ri}(j))-\min_j(z^0_{Ri}(j))\right]/M_s$, and $\sigma^j_{Ii}=\left[\max_j(z^0_{Ii}(j))-\min_j(z^0_{Ii}(j))\right]/M_s$

where $i=1, 2, \ldots, n$, $j=1, 2, \ldots, N$, and M_s is the number of FBF's in the final FBF expansion. We assume that M_s is given based on practical constraints; in general, $M_s \ll N$.

The fuzzy membership functions can be assumed to achieve unity membership value at some center. We choose the centers to be the input points in the given input-output pairs. Finally, the above choice of should make the final FBF's "uniformly" cover the input region spanned by the input points in the given input-output pairs.

Next, we use the COLS algorithm, similar to that in [7] and [8]. To select the significant FBF's from the N FBF's determined by the initial FBF determination method:

At the first step, for $1 \le i \le N$, compute

$$\Gamma^i_1 = p_i, \tag{11}$$

$$g^i_{1q} = (\Gamma^i_1)^T d_q/((\Gamma^i_1)^T(\Gamma^i_1)), \quad 1 \le q \le n_0 \tag{12}$$

$$[err]^i_1 = \left(\sum_{q=1}^{n_0} |g^i_{1q}|^2\right) \cdot ((\Gamma^i_1)^T(\Gamma^i_1))/\mathrm{trace}(D^T D) \tag{13}$$

Find

$$[err]_1 = [err]^{i_1}_1 = \max\{[err]^i_1, \ 1 \le i \le N\} \tag{14}$$

and select $\Gamma_1 = \Gamma^{i_1}_1 = p_{i_1}$.

At the k-th step where $k \ge 2$, for $1 \le i \le N$, $i \ne i_1, \ldots, i \ne i_{k-1}$, compute

$$w^i_{lk} = \Gamma^T_l p_i/((\Gamma_l)^T(\Gamma_l)), \quad 1 \le l \le k, \tag{15}$$

$$\Gamma^i_k = p_i - \sum_{l=1}^{k-1} w^i_{lk}\Gamma_l, \tag{16}$$

$$g^i_{kq} = (\Gamma^i_k)^T d_q/((\Gamma^i_k)^T(\Gamma^i_k)), \quad 1 \le q \le n_0 \tag{17}$$

$$[err]^i_k = \left(\sum_{q=1}^{n_0} |g^i_{kq}|^2\right) \cdot ((\Gamma^i_k)^T(\Gamma^i_k))/\mathrm{trace}(D^T D) \tag{18}$$

Find

$$[err]_k = [err]^{i_k}_k = \max\{[err]^i_k, \ 1 \le i \le N, \ i \ne i_1, \ldots, i \ne i_{k-1}\} \tag{19}$$

and select

$$\Gamma_k = \Gamma^{i_k}_k = p_{i_k} - \sum_{l=1}^{k-1} w^i_{lk}\Gamma_l \ \text{ where } \ w_{lk} = w^{i_k}_{lk}, \quad 1 \le l \le k. \tag{20}$$

The procedure is terminated at the n_h-th step when

$$1 - \sum_{l=1}^{n_h} [err]_l < \zeta \tag{21}$$

where $0 < \zeta < 1$ is a chosen tolerance. This gives rise to a subset logic filter containing n_h significant rule.

Solve the triangular system

$$A^{(M_s)}\Theta^{(M_s)} = g^{(M_s)} \tag{22}$$

where $A^{(M_s)} = \begin{bmatrix} 1 & w^{i_2}_{12} & w^{i_3}_{13} & \cdots & w^{iM_s}_{1M_s} \\ 0 & 1 & w^{i_3}_{23} & \cdots & w^{iM_s}_{2M_s} \\ \cdots & \cdots & \cdots & 1 & w^{iM_s}_{M_s-1M_s} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$

$$g^{(M_s)} = [g_1, \ldots, g_{M_s}]^T \Theta^{(M_s)} = [\theta^{M_s}_1, \ldots, \theta^{M_s}_{M_s}]^T.$$

The final FBF expansion is

$$f(x) = \sum_{j=1}^{M_s} p_{i_j}(x)\theta^{M_s}_j \tag{23}$$

where make up the subset of the FBF's determined by the initial FBF determination method with determined by the above steps.

The $[err]^i_k = \left(\sum_{q=1}^{n_0} |g^i_{kq}|^2\right) \cdot ((\Gamma^i_k)^T(\Gamma^i_k))/\mathrm{trace}(D^T D)$ represents the error reduction ratio caused by Γ^i_k. Hence our COLS algorithm selects significant FBF's based on their error reduction ratio; i.e., the FBF's with largest error reduction ratios are retained in the final FBF expansion.

Ⅳ. Adaptive Algorithm based on LMS

Because the θ as well as (m^l_{Ri}, m^l_{Ii}) and $(\sigma^l_{Ri}, \sigma^l_{Ii})$ are free parameter, the filter of (4) is nonlinear in the parameter. Given the desired complex signal $d(k)$, the adaptive algorithm adjusts the filter coefficients $\{\theta^l, m^l_{Ri}, m^l_{Ii}, \sigma^l_{Ri}, \sigma^l_{Ii}\}$ to minimize the cost function L given by

$$L(k) = E\{e(k)e^*(k)\} \tag{24}$$

where E is statistical expectation, the superscript * denotes complex conjugate, and $e(k) = d(k) - f_k(z(k))$.

i] Estimation of $\theta^l(k)$

Because the filter coefficients $\theta^l(k)$ are complex, it is necessary to simultaneously adapt both the real and imaginary parts. Let the coefficient be expressed in terms of its real and imaginary part as

$$\theta^l(k) = \theta^l_R(k) + j\theta^l_I(k) \tag{25}$$

According to complex LMS algorithm [5], the adaptation rule for each part of the p arameter θ^l is

$$\theta^l_R(k+1) = \theta^l_R(k) - \frac{1}{2}\alpha\frac{\partial L}{\partial\theta^l_R(k)} \tag{26}$$

$$\theta^l_I(k+1) = \theta^l_I(k) - \frac{1}{2}\alpha\frac{\partial L}{\partial\theta^l_I(k)} \tag{27}$$

where α is a scalar step size that controls the stability and convergence rate of the algorithm.

Combining (26) and (27) according to (25), we obtain the update expression for $\theta^l(k+1)$ in terms of $\theta^l(k)$ as

$$\theta^l(k+1) = \theta^l(k) - \frac{1}{2}\alpha\left(\frac{\partial L}{\partial\theta^l_R(k)} + j\frac{\partial L}{\partial\theta^l_I(k)}\right) \tag{28}$$

Thus, the next step is to find some expression for the partial derivative,

$$\frac{\partial L}{\partial\theta^l_R(k)} = \frac{\partial L}{\partial f_k}\cdot\frac{\partial f_k}{\partial\theta^l_R(k)} + \frac{\partial L}{\partial f^*_k}\cdot\frac{\partial f^*_k}{\partial\theta^l_R(k)} \tag{29}$$

Evaluating all the partial derivatives in (29), we get

$$\frac{\partial L}{\partial\theta^l_R(k)} = -e^*(k)b^l(k) - e(k)b^l(k) \tag{30}$$

Similarly,

$$\frac{\partial L}{\partial\theta^l_I(k)} = je^*(k)b^l(k) - je(k)b^l(k) \tag{31}$$

where $b^l(k)$ have a real value defined as

$$b^l(k) = \frac{\prod_{i=1}^{n} exp\left[-\frac{1}{2}\left(\frac{z_{Ri}(k)-m^l_{Ri}(k)}{\sigma^l_{Ri}(k)}\right)^2 - \frac{1}{2}\left(\frac{z_{Ii}(k)-m^l_{Ii}(k)}{\sigma^l_{Ii}(k)}\right)^2\right]}{\sum_{l=1}^{M}\prod_{i=1}^{n} exp\left[-\frac{1}{2}\left(\frac{z_{Ri}(k)-m^l_{Ri}(k)}{\sigma^l_{Ri}(k)}\right)^2 - \frac{1}{2}\left(\frac{z_{Ii}(k)-m^l_{Ii}(k)}{\sigma^l_{Ii}(k)}\right)^2\right]}.$$

Combining (30) and (31), we have

$$\frac{\partial L}{\partial\theta^l_R(k)} + j\frac{\partial L}{\partial\theta^l_I(k)} = -2e^*(k)b^l(k) \tag{32}$$

Substituting (32) into (28), we obtain the following compact expression for adaptation rule of the θ^l:

$$\theta^l(k+1) = \theta^l(k) + \alpha\cdot e^*(k)b^l(k). \tag{33}$$

ii] Estimation of and m^l_i and σ^l_i

We now will estimate the parameter (m^l_{Ri}, m^l_{Ii}) and $(\sigma^l_{Ri}, \sigma^l_{Ii})$ in membership function. We have define as

$$m^l_i(k) = [m^l_{Ri}(k), m^l_{Ii}(k)]^T, \tag{34}$$

$$\sigma^l_i(k) = [\sigma^l_{Ri}(k), \sigma^l_{Ii}(k)]^T. \tag{35}$$

We consider the adaptation rule of parameter m^l_i and σ^l_i,

$$m^l_i(k+1) = m^l_i(k) - \frac{1}{2}\alpha\frac{\partial L}{\partial m^l_i(k)} \tag{36}$$

$$\sigma^l_i(k+1) = \sigma^l_i(k) - \frac{1}{2}\alpha\frac{\partial L}{\partial\sigma^l_i(k)} \tag{37}$$

where $\frac{\partial L}{\partial m^l_i(k)} = \left[\frac{\partial L}{\partial m^l_{Ri}(k)}, \frac{\partial L}{\partial m^l_{Ii}(k)}\right]^T$

$$\frac{\partial L}{\partial\sigma^l_i(k)} = \left[\frac{\partial L}{\partial\sigma^l_{Ri}(k)}, \frac{\partial L}{\partial\sigma^l_{Ii}(k)}\right]^T$$

Thus, the next step is to find some expression for the partial derivative,

$$\frac{\partial L}{\partial m^l_{Ri}(k)} = \frac{\partial L}{\partial f}\cdot\frac{\partial f}{\partial m^l_{Ri}(k)} + \frac{\partial L}{\partial f^*}\cdot\frac{\partial f^*}{\partial m^l_{Ri}(k)}. \tag{38}$$

Evaluating all the partial derivatives in (38), we get

$$\begin{aligned}\frac{\partial L}{\partial m^l_{Ri}(k)} &= -e^*(k)(\theta^l(k)-f(k))b^l(k)\cdot\frac{z_{Ri}(k)-m^l_{Ri}(k)}{\sigma^{l^2}_{Ri}(k)} \\ &\quad -e(k)(\theta^l(k)-f(k))^*b^l(k)\cdot\frac{z_{Ri}(k)-m^l_{Ri}(k)}{\sigma^{l^2}_i(k)},\end{aligned} \tag{39}$$

where

$$f(k) = \frac{\sum_{l=1}^{M}\theta^l(k)\prod_{i=1}^{n} exp\left[-\frac{1}{2}\frac{(z_i-m^l_i(k))^T(z_i-m^l_i(k))}{\sigma^2_i(k)}\right]}{\sum_{l=1}^{M}\prod_{i=1}^{n} exp\left[-\frac{1}{2}\frac{(z_i-m^l_i(k))^T(z_i-m^l_i(k))}{\sigma^2_i(k)}\right]}.$$

Since $e^*(k)(\theta^l(k)-f(k))$ and $e(k)(\theta^l(k)-f(k))^*$ are conjugate pairs, we define

$$e^*(k)(\theta^l(k)-f(k))-e(k)(\theta^l(k)-f(k))^* = 2\,Re[e^*(k)(\theta^l(k)-f(k))]$$
$$= 2\,Re[e(k)(\theta^l(k)-f(k))^*]$$
$$= 2h^l(k) \quad (40)$$

Therefore, (39) can be written as

$$\frac{\partial L}{\partial m^l_{Ri}(k)} = -h^l(k)b^l(k)\cdot\frac{z_{Ri}(k)-m^l_{Ri}(k)}{\sigma_i^{l^2}(k)}. \quad (41)$$

Similarly,

$$\frac{\partial L}{\partial m^l_{Ii}(k)} = -h^l(k)b^l(k)\cdot\frac{z_{Ii}(k)-m^l_{Ii}(k)}{\sigma_{Ii}^{l^2}(k)}, \quad (42)$$

$$\frac{\partial L}{\partial \sigma^l_{Ri}(k)} = -h^l(k)b^l(k)\cdot\frac{(z_{Ri}(k)-m^l_{Ri}(k))^2}{\sigma_{Ri}^{l^3}(k)}, \quad (43)$$

$$\frac{\partial L}{\partial \sigma^l_{Ii}(k)} = -h^l(k)b^l(k)\cdot\frac{(z_{Ii}(k)-m^l_{Ii}(k))^2}{\sigma_{Ii}^{l^3}(k)}. \quad (44)$$

Substituting (41)-(44) into (36)-(37), the final adaptation rule of parameters m^l_i and σ^l_i is obtained as

$$m^l_i(k+1) = m^l_i(k) - \frac{1}{2}\,\alpha\cdot h^l(k)b^l(k)p^l(k), \quad (45)$$

$$\sigma^l_i(k+1) = \sigma^l_i(k) - \frac{1}{2}\,\alpha\cdot h^l(k)b^l(k)q^l(k) \quad (46)$$

where we have defined

$$p^l(k) = \left[\frac{z_{Ri}(k)-m^l_{Ri}(k)}{\sigma_{Ri}^{l^2}(k)}\;\;\frac{z_{Ii}(k)-m^l_{Ii}(k)}{\sigma_{Ii}^{l^2}(k)}\right]^T \text{ and}$$

$$q^l(k) = \left[\frac{(z_{Ri}(k)-m^l_{Ri}(k))^2}{\sigma_{Ri}^{l^3}(k)}\;\;\frac{(z_{Ii}(k)-m^l_{Ii}(k))^2}{\sigma_{Ii}^{l^3}(k)}\right]^T.$$

Equations (33), (45) and (46) are the update equation for the filter parameters. We see that the initial complex LMS fuzzy adaptive filter is constructed based on linguistic rules from human experts and some arbitrary. The membership functions in these rules will change during the LMS adaptation procedure. Because minimizing (24) can be viewed as matching the input and outputs, our complex LMS fuzzy adaptive filter combines both linguistic and numerical information in its design. Also, since the initial complex LMS fuzzy adaptive filter is constructed based on linguistic rules from human experts, the adaptation procedure should converge quickly.

V. Experimental Results

In this study, the transmitted symbols s(k) are assumed to be the 4-QAM scheme, i.e., the constellation of $s(k)$ is given by $s(k)=s_R(k)+js_I(k)$; $1+j$, $-1+j$, $1-j$, $-1-j$. The task of the equalizer is to reconstruct the transmitted symbols $s(k)$ based on noisy channel observations $z(k)=\bar{z}(k)+n(k)$ where $\bar{z}(k)$ is a channel output sequence under noise free condition and $n(k)$ is the noise. Then, the equalizer is defined by

$$s(k-m) = sgn(f_k(z)) \quad (47)$$

where m is lag and $sgn(\cdot)$ is the complex signum function defined as

$$sgn(f) = \begin{cases} 1+j, & Re[f]\ge 0 \text{ and } Im[f]\ge 0 \\ -1+j, & Re[f]<0 \text{ and } Im[f]\ge 0 \\ 1-j, & Re[f]\ge 0 \text{ and } Im[f]<0 \\ -1-j, & Re[f]<0 \text{ and } Im[f]<0 \end{cases}$$

Now consider the nonlinear channel

$$H(z) = (1.0119 - j0.7589) + (-0.3796 + j0.5059)\,z^{-1}$$

and the additive noise $n(k)=n_R(k)+jn_I(k)$, where both and are white Gaussian noise component with zero mean and identical variance $\sigma^2=\sigma_R^2=\sigma_I^2$. To solve this specific equalization problem, we chose M = 64, α = 0.05. We now use the complex LMS fuzzy adaptive filters without any linguistic information with the initialand $\theta^l_R(0)$ and $\theta^l_I(0)$ randomly in [-0.5, 0.5], $m^l_{Ri}(0)$ and $m^l_{Ii}(0)$'s randomly in [-2, 2], and $\sigma^2(0)$'s randomly in [0.1, 0.3]. Next, we used the complex LMS fuzzy filter and incorporated the fuzzy rules shown in Table 1 and the fuzzy membership function shown in Fig. 1, as initial parameters.

We have 64 rules in table 1: for example, (P8, P1; N8, N1) corresponds to the rule;

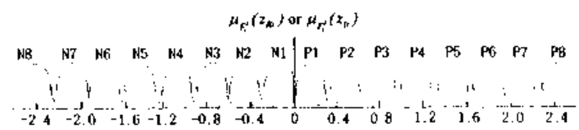

Figure 1. The membership functions.

IF (z_{R1} is P8 and z_{I1} is P1) and (z_{R2} is N8 and z_{I2} is N1), THEN f_k is G

where the filter output is f_k and the center of μ_G is 0.4 + j0.4. Because the filter output f_k is a weighted average of these centers, the number 0.4 + j0.4, 0.4 − j0.4, −0.4 + j0.4, −0.4 − j0.4 in table 1 reflect our belief that input points should corresponds to one of four catalog $\pm 1 \pm j$. Fig. 2

Table 1. Fuzzy rules about the decision

Input	Center
(P8, P1;N8, N1), (P8, P1;N7, N5), (P8, P1;N6, P3), (P8, P1;N5, P6) (P7, P4;P1, N8), (P7, P4;P2, N4), (P7, P4;P3, P5), (P7, P4;P5, N7) (P6, N3;N3, N6), (P6, N3;N2, P4), (P6, N3;N1, P8), (P6, N3;P3, P5) (P5, N7;P5, N7), (P5, N7;P6, N3), (P5, N7;P7, P4), (P5, N7;P8, P1)	0.4 + j0.4
(P4, P2;N8, N1), (P8, P2;N7, N5), (P8, P2;N6, P3), (P8, P2;N5, B6) (P3, P5;P5, N7), (P3, P5;P6, N3), (P3, P5;P7, P4), (P3, P5;P8, P1) (P2, N4;P1, N8), (P2, N4;P2, N4), (P2, N4;P3, P5), (P2, N4;P5, N7) (P1, N8;N3, N6), (P1, N8;N2, P4), (P1, N8;N1, P8), (P1, N8;P3, B5)	0.4 − j0.4
(N2, P4;N3, N6), (N2, P4;N2, P4), (N2, P4;N1, P8), (N2, P4;P3, P5) (N3, N6;P5, N7), (N3, N6;P6, N3), (N3, N6;P7, P4), (N3, N6;P8, P1) (N4, N2;N8, N1), (N4, N2;N7, N5), (N4, N2;N6, P3), (N4, N2;N5, P6) (N1, P8;P1, N8), (N1, P8;P3, P5), (N1, P8;P3, P5), (N1, P8;P5, N7)	−0.4 + j0.4
(N5, P6;N8, N1), (N5, P6;N7, N5), (N5, P6;N6, P3), (N5, P6;N5, P6) (N6, P3;P1, N8), (N6, P3;P2, N4), (N6, P3;P3, P5), (N6, P3;P5, N7) (N7, N5;N3, N6), (N7, N5;N2, P4), (N7, N5;N1, P8), (N7, N5;P3, P5) (N8, N1;P5, N7), (N8, N1;P6, N3), (N8, N1;P7, P4), (N8, N1;P8, P1)	−0.4 − j0.4

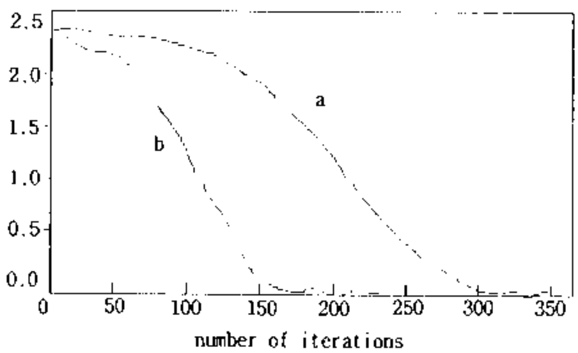

Figure 2. Learning characteristics of fuzzy adaptive filter; (a) without using any linguistic information, (b) after incorporating some linguistic information.

shows the learning characteristics of the fuzzy adaptive filters under SNR = 15dB. In Fig. 2, we see that the learning speed can be greatly improved by incorporating these fuzzy rules.

We compared the bit error rates achieved by the optimal equalizer, radial basis function (RBF) equalizer with 64 centers [4], and the complex LMS fuzzy adaptive equalizers, for different signal-to-noise ratios, for the given channel with equalizer order $n=2$ and $m=1$. These equalizer were first trained with 1000 symbols from the output of the channel and then we evaluated the bit error rates (BER) based on 10^6 more received symbols, for each realization. We see from Fig. 3 that the BER of the fuzzy equalizers are very close to the optimal values.

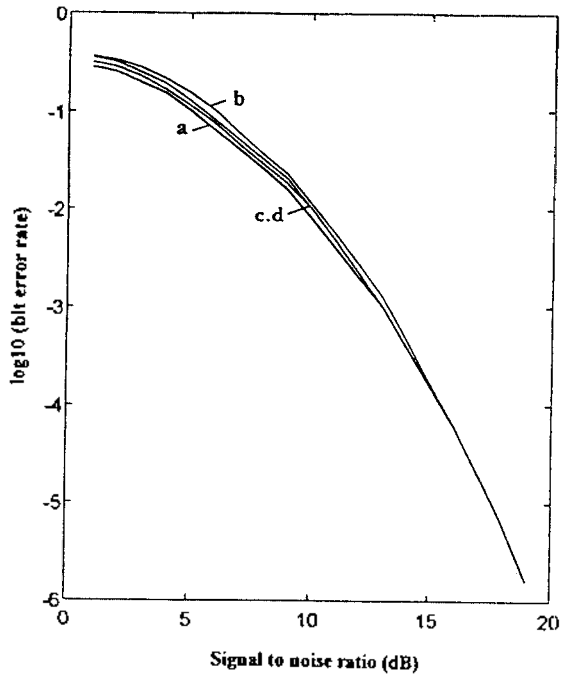

Figure 3. Comparison of bit error rates achieved by; (1) optimal equalizer, (b) complex RBF equalizer with 64 states, (c) complex LMS fuzzy equalizer.

Ⅵ. Conclusions

In this paper, we developed a complex fuzzy filter and proved that a complex fuzzy basis functions are capable of uniformly approximating any complex continuous function on a compact set to arbitrary accuracy, i.e., they are universal approximators. Also, We developed an complex orthogonal least squares algorithm to select the significant fuzzy basis functions. Applying the fuzzy filter to nonlinear channel equalization problems with complex components, we showed the usefulness of the complex fuzzy filter. From simulations, we show that the bit error rates of the fuzzy equalizers were close to that of the optimal equalizer.

APPENDIX

To prove the theorem, we use the complex version of Stone-Weierstraus theorem [9].

Complex Stone-Weierstraus Theorem: Let Z be a set of complex continuous functions on a compact set U. If 1) Z is a complex algebra, i.e., the set Z is closed under addition, multiplication, and complex constant multiplication, 2) Z is self-adjoint, i.e., there exists $\bar{f} \in Z$ whenever $f \in Z$, where $\bar{f}$ is the complex conjugate of f. 3) Z separates points on U, i.e., for any every x, $y \in U$, $x \neq y$ there existtts $f \in Z$ such that, $f(x) \neq f(y)$ and 4) Z vanishes at no point of U, i.e., for each $x \in U$ there exists such that $f(x) \neq 0$, then, the uniform closure of Z consists of all complex continuous functions on U; i.e., Z is dense in set of all complex continuous functions on a compact set.

Proof) 1)Therefore, first we prove that Y is a complex algebra. Let $f_1(z(k))$, $f_2(z(k)) \in Y$, so that we can write them as

$$f_1(z(k)) = \frac{\sum_{l=1}^{M_1} (\theta_{R,1}{}^l + j\theta_{I,1}{}^l)\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\right)}{\sum_{l=1}^{M_1}\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\right)}, \quad (a.1)$$

$$f_2(z(k)) = \frac{\sum_{p=1}^{M_2} (\theta_{2,R}{}^p + j\theta_{2,I}{}^p) \prod_{i=1}^{n} \mu_{2,F_i^p}(z_i(k))}{\sum_{p=1}^{M_2} \prod_{i=1}^{n} \mu_{2,F_i^p}(z_i(k))}. \quad (a.2)$$

Then,

$$f_1(z(k)) + f_2(z(k)) =$$

$$\frac{\sum_{l=1}^{M_1}\sum_{p=1}^{M_1} ((\theta_{R,1}^l + \theta_{2,R}^p) + j(\theta_{1,I}^l + \theta_{2,I}^p))\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\cdot \mu_{2,F_i^p}(z_i(k))\right)}{\sum_{l=1}^{M_1}\sum_{p=1}^{M_2}\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\cdot \mu_{2,F_i^p}(z_i(k))\right)} \quad (a.3)$$

since $\mu_{1,F_i^l}(\cdot)$ and $\mu_{2,F_i^p}(\cdot)$ are Gaussian in form, their product $\mu_{1,F_i^l}(\cdot)\mu_{2,F_i^p}(\cdot)$ is also Gaussian in form; hence (a.3) is in the same form as (6), so that $f_1(z(k)) + f_2(z(k)) \in Y$.

Similiary, we have (a.4)

$$f_1(z(k))\cdot f_2(z(k)) =$$

$$\frac{\sum_{l=1}^{M_1}\sum_{p=1}^{M_1} ((\theta_{1,R}^l\theta_{2,R}^p - \theta_{1,I}^l\theta_{2,I}^p) + j(\theta_{1,I}^l\theta_{2,R}^p + \theta_{1,R}^l\theta_{2,I}^p))\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\cdot \mu_{2,F_i^p}(z_i(k))\right)}{\sum_{l=1}^{M_1}\sum_{p=1}^{M_1}\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\cdot \mu_{2,F_i^p}(z_i(k))\right)}, \quad (a.4)$$

which is also in the same form of (4); hence $f_1(z(k))\cdot f_2(z(k)) \in Y$

Finally, for arbitrary complex constant $c = a + jb \in k$,

$$c\cdot f_1(z(k)) =$$

$$\frac{\sum_{l=1}^{M} ((a\cdot\theta_{1,R}{}^l - b\cdot\theta_{1,I}{}^l) + j((a\cdot\theta_{1,I}{}^l + b\cdot\theta_{1,R}{}^l))\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\cdot \mu_{2,F_i^p}(z_i(k))\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))\cdot \mu_{2,F_i^p}(z_i(k))\right)} \quad (a.5)$$

which is again in the form of (4); hence $c\cdot f_1(z(k)) \in Y$

2) we have complex conjugate of (4) as

$$\bar{f}_1(z(k)) = \frac{\sum_{l=1}^{M} (\theta_{1,R}{}^l - j\theta_{1,I}{}^l) \prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))}{\sum_{l=1}^{M}\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))} = \frac{\sum_{l=1}^{M} (\theta_{1,R}{}^l + j(-\theta_{1,I}{}^l)) \prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))}{\sum_{l=1}^{M}\prod_{i=1}^{n} \mu_{1,F_i^l}(z_i(k))} \quad (a.6)$$

which is in the same form of (4); hence $\bar{f}_1(z(k)) \in Y$. There, Y is self-adjoint.

3) we prove this by constructing a required f; i.e., we specify $f(x) \in Y$ such that $f(x) \neq f(y)$ for arbitrarily given x, $y \in U$ with. Since Y is self-adjoint, we have real and imaginary term of f as

$$\mathrm{Re}f(x) = \frac{f(x) + \bar{f}(x)}{2} = \frac{\sum_{l=1}^{M} (\theta_R{}^l) \prod_{i=1}^{n} \mu_{1,F_i^l}(x_i(k))}{\sum_{l=1}^{M}\prod_{i=1}^{n} \mu_{1,F_i^l}(x_i(k))}, \quad (a.7)$$

$$\mathrm{Im}f(x) = \frac{j(f(x) + \bar{f}(x))}{2} = \frac{\sum_{l=1}^{M} (\theta_I{}^l) \prod_{i=1}^{n} \mu_{1,F_i^l}(x_i(k))}{\sum_{l=1}^{M}\prod_{i=1}^{n} \mu_{1,F_i^l}(x_i(k))}. \quad (a.8)$$

Then, $\mathrm{Re}f(x)$ and $\mathrm{Im}f(x)$ is a real-valued function. Using stone-Weistrass theorem [1], we can easily see that $\mathrm{Re}f(x)$ and $\mathrm{Im}f(x)$ is real-valued continuous function, i.e., $\mathrm{Re}f(x) \neq \mathrm{Re}f(y)$ or $\mathrm{Im}f(x) \neq \mathrm{Im}f(y)$, for $x \neq y$; hence $f(x) \neq f(y)$ for arbitrily given x, $y \in U$ with $x \neq y$. Therefore Y separates points on U.

4) we prove that Y vanishes at no point of U. We simply choose all $|\theta^l| = |\theta_R^l + j\theta_I^l| > 0$; i.e., any $f \in Y$ with $|\theta^l| > 0$ serves as the required f.

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