

A Design of the Robust Controller for an Active Noise Control

*Tae-Jin Chung, *Hak-Joon Oh, *Chi-Hyoung Yu, and *Chan-Soo Chung

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Abstract

In this paper, a robust active noise controller is designed to reduce noise in a small cavity. Noise characteristics in the small cavity are nonlinear and we could get its model with considerable modelling errors. The objective of this paper is to minimize the effects of these modelling errors and maximize the noise reduction performance. The solution could be obtained by the H_∞ robust control theory. The resulting feedback controller minimizes the H_∞ norm of the mixed sensitivity function, which means the effects of uncertainties of the model are suppressed in the sense of stability and the performance is enhanced as a given specification. The designed controller is realized with analog devices such as Op. Amps and experimental results show that the controller reduces noise signal sufficiently.

1. Introduction

The noise reduction problem in a small cavity was introduced by Olson(1956) and studied in detail by Wheeler(1986) and Carne(1987)[1]. Their approach was to design a feedback high gain controller. The difficulty in their approach was that when they increased a controller's gain to reduce noise, the stability tended to be worse. Carne introduced a simple R-C compensator to enhance the stability of his controller. But his approach was based on an ideal case-he assumed he knew the exact electroacoustic transfer function of the cavity system. Unfortunately, in practice, we can't be sure the exactness of the transfer function we have measured. There exist situations where nonlinear characteristics in the cavity system cannot be measured. So, we can't guarantee stability and performance of the control system, unless we consider these characteristics in the first step of designing our controller. The approach we tried in this paper is the H_∞ robust control method. The H_∞ robust control method had been proposed by G. Zames in 1980s and has been studied by many researchers like J. C Doyle, B. A Fransis, and K. Glover. In this method, a feedback controller minimizes the mixed sensitivity function of the closed system, following the designer's specification. So we can specify uncertainties of the electroacoustic model we measured and design a controller which would remain

stable within the pre-determined uncertainty level and maximize noise reduction performance. When considering the H_∞ robust regulating problem, this method could be easily applied to the noise control problem in a small cavity.

II. Scheme of noise control in a small cavity

The diagram of the small cavity is shown in Figure 2.1. In this system, the input is the noise signal induced by external noise sources and the output is the error signal detected by the error microphone. The control signal is fed back into the cavity through the control speaker. Then, the control scheme can be simplified as in Figure 2.2.

In Figure 2.2, G is the transfer function of the electroacoustic path from the control speaker to the error microphone. K is the controller which will be designed. d is noise signal and e is error signal. The closed loop transfer function from noise signal to error signal is given by

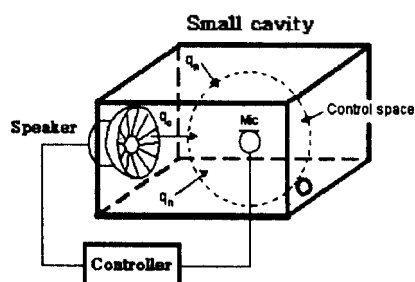


Figure 2.1 The Small Cavity System(q_n : noise signal, q_c : control signal).

*Department of Electrical Engineering, Soongsil University, Korea

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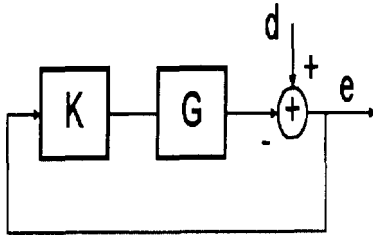


Figure 2.2 Block diagram of the ANC system for the small cavity.

$$\frac{E(s)}{D(s)} = \frac{1}{1 + K(s) \cdot G(s)} \quad (2-1)$$

In this paper, the objective is to minimize $\|E(s)/D(s)\|_{\infty}$. This seems to be easily obtained by Eq. (2-1). That is, when $|K(s)|$ is increased to infinity $\|E(s)/D(s)\|_{\infty}$ goes to zero. But, just the increase of the gain of the controller leads to the poor stability of the closed system. Thus, another approach is needed for more reliable controller. Let's re-interpret the design objective. When considering the sensitivity function of the system, Eq. (2-1) is equal to the sensitivity function of the closed system. This is expressed in Eq.(2-2).

$$\frac{E(s)}{D(s)} = \frac{1}{1 + K(s) \cdot G(s)} = S(s) \quad (2-1)$$

Then, the control objective is to minimize $\|S(s)\|_{\infty}$ and it means that our noise reduction problem is identical to the well-known regulating problem. Finally, we will discuss about uncertainties in the system. In practices, we cannot avoid modelling errors when modelling the electroacoustic path, which includes nonlinear characteristics of the speaker, time delays of the signal, and many eigenmodes of the cavity. One method to deal with this is that, after we get a model of the path as exact as possible, we try to suppress the effects of the unmodelled dynamics. The H_{∞} robust regulating method solves this problem.

III. The H_{∞} Robust Control Problem and its Solution: Theoretical Background[4, 5, 6]

Most systems have unmodeled characteristics. In this paper, the model is expressed as a multiplicative uncertainty model as shown in Eq.(3-1).

$$\tilde{G} = (1 + \Delta W_2) \cdot G \quad (3-1)$$

Theorem 1 A necessary and sufficient condition for robust performance is

$$\| |W_1 \cdot S| + |W_2 \cdot T| \|_{\infty} < 1 \quad (3-2)$$

where S and T are sensitivity and complementary sensitivity functions, W_1 and W_2 are weight functions.

Proof: See the references[5].

The above theorem is the combined version of two theorem. One is for the stability, $\|W_1 \cdot S\|_{\infty} < 1$, and the other is for the robustness, $\|W_2 \cdot T\|_{\infty} < 1$. That is, the desired controller would shape the sensitivity and the complementary sensitivity functions based on the associated weight functions. The left-hand side of the inequality is called the mixed sensitivity function and the problem to minimize this function is called the H_{∞} robust optimal control problem expressed as follows

$$\| |W_1 \cdot S| + |W_2 \cdot T| \|_{\infty} < \gamma \quad (3-3)$$

Let's represent the block diagram of the standard feedback system as shown in Figure 3.1.

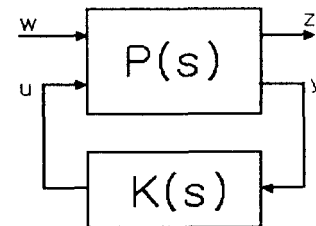


Figure 3.1 Block Diagram of Standard Feedback System.

The plant, P is represented in state space form as

$$\begin{aligned} \dot{x}(t) &= A \cdot x(t) + B_1 \cdot w(t) + B_2 \cdot u(t) \\ z(t) &= C_1 \cdot x(t) + D_{11} \cdot w(t) + D_{12} \cdot u(t) \\ y(t) &= C_2 \cdot x(t) + D_{21} \cdot w(t) + D_{22} \cdot u(t) \end{aligned} \quad (3-4)$$

and its transfer function will be denoted as

$$P(s) = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ \dots & \dots & \dots \\ C_1 & D_{11} & D_{12} \\ \dots & \dots & \dots \\ C_2 & D_{21} & D_{22} \end{bmatrix}^{-1}$$

If $y = -K(s)u$ is connected from y to u , the closed-loop transfer function from w to z will be denoted

1) $\begin{bmatrix} A & B \\ \dots & \dots \\ C & D \end{bmatrix}$ means $D + C(sI - A)^{-1}B$.

$$T(P, K) := P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}. \quad (3-5)$$

The H_∞ control problem is to find a controller that makes the closed-loop system internally stable and minimizes $\|T(P, K)\|_\infty$. We will find a controller K , such that

$$\|T(P, K)\|_\infty < \gamma \quad (3-6)$$

and reduce γ until K will not exist.

For the controller satisfying Eq.(3-6), the following conditions should be satisfied.

C1. (A, B_2, C_2) is stabilizable and detectable.

C2. D_{12} and D_{21} are full rank.

C3. A scaling and a unitary transformation enable us to assume

$$D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_{21} = [0 \ I]$$

$$D_{11} = \begin{bmatrix} D_{111} & D_{112} \\ D_{121} & D_{122} \end{bmatrix}$$

C4. $\text{rank} \begin{pmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{pmatrix} = \text{rank}(x) + \text{rank}(u)$, for all ω

C5. $\text{rank} \begin{pmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{pmatrix} = \text{rank}(x) + \text{rank}(y)$, for all ω

C1 is for stabilizability of P by output feedback. C2 is for nonsingular control problem. C3 is for computational simplicity, so it is not necessary. C4 and C5 are for stability solution of Riccati equations.

Now, define

$$R = D_1' D_1 - \begin{pmatrix} \gamma^2 I & 0 \\ 0 & 0 \end{pmatrix} \text{ where } D_1 = [D_{11} \ D_{12}]$$

$$\tilde{R} = D_{-1} D_{-1}' - \begin{pmatrix} \gamma^2 I & 0 \\ 0 & 0 \end{pmatrix} \text{ where } D_{-1} = [D_{11} \ D_{12}]^T$$

and suppose there exist solutions to the following Algebraic Riccati Equations

$$X_\infty = \text{Ric} \left\{ \begin{pmatrix} A & 0 \\ -C_1' C_1 & -A' \end{pmatrix} - \begin{pmatrix} B \\ -C_1' D_1 \end{pmatrix} R^{-1} [D_1' C_1, B'] \right\}$$

$$Y_\infty = \text{Ric} \left\{ \begin{pmatrix} A' & 0 \\ -B_1 B_1' & -A \end{pmatrix} - \begin{pmatrix} C \\ -B_1 D_1' \end{pmatrix} \tilde{R}^{-1} [D_{-1} B_1', C] \right\}$$

The state feedback matrix, F and output estimation matrix, H are defined as

$$F = -R^{-1} [D_1' C_1 + B' X_\infty] = [F_{11}, F_{12}, F_{21}]^T$$

$$H = -[B_1 D_1' + Y_\infty C'] \tilde{R}^{-1} = [H_{11}, H_{12}, H_2]$$

The main result is stated in terms of the above matrices.

Theorem 2. For the system described by $P(s)$ and satisfying the conditions C1-C5:

(a) There exists an internally stabilizing controller $K(s)$ such that $\|T(P, K)\|_\infty < \gamma$ if and only if

(i) $\gamma > \max(\bar{\sigma}[D_{111}, D_{112}], \bar{\sigma}[D_{111}', D_{112}'])$ and

(ii) $X_\infty \geq 0$ and $Y_\infty \geq 0$ such that $\rho(X_\infty Y_\infty) < \gamma^2$. ($\rho(\cdot)$ denotes the largest eigenvalue.)

(b) Given that the conditions of part (a) are satisfied, then all rational internally stabilizing controller $K(s)$ satisfying $\|T(P, K)\|_\infty < \gamma$ are given by

$K = T(K_a, \Phi)$ where Φ is any stable, proper, and rational function such that $\|\Phi\|_\infty < \gamma$, where

$$K_a = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hat{C}_1 & \hat{D}_{11} & \hat{D}_{12} \\ \hat{C}_1 & \hat{D}_{21} & 0 \end{bmatrix}$$

$$\hat{D}_{11} = -D_{121} D_{111}' (\gamma^2 I - D_{111} D_{111}')^{-1} D_{112} - D_{1122}$$

$$\hat{D}_{12} \hat{D}_{12}' = I - D_{112}' (\gamma^2 I - D_{111} D_{111}')^{-1} D_{112}$$

$$\hat{D}_{21}' \hat{D}_{21} = I - D_{112} (\gamma^2 I - D_{111} D_{111}')^{-1} D_{112}$$

and

$$\hat{B}_2 = (B_2 + B_{12}) \hat{D}_{12}$$

$$\hat{C}_2 = -\hat{D}_{21} (C_2 + F_{12}) Z$$

$$\hat{B} = -H_2 + \hat{B}_2 \hat{D}_{12}^{-1} \hat{D}_{11}$$

$$\hat{C} = F_2 Z + \hat{D}_{11} \hat{D}_{21}^{-1} \hat{C}_2$$

$$\hat{A} = A + HC + \hat{B}_2 \hat{D}_{12}^{-1} \hat{C}_1$$

and

$$Z = (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$$

Proof: See the reference[4, 6].

The above theorem solves the H_∞ robust control problem and we will use the result to design the desired controller to minimize noise signal. The left problem is to form the standard feedback system with the block diagram shown in Figure 2.2 and will be discussed in the next section.

IV. The design of the active noise controller

4.1 Modelling of the transfer function G

To get a model of the electroacoustic path of the cavity system, this paper used series of sinusoidal signals which

excite the system and saved the magnitude responses measured with the error microphone. We divided the frequency range logarithmically from 100Hz to 5kHz into 200 frequencies and used sinusoid signals of these frequencies. Figure 4.1 shows the measured data. Based on the measured data, the model of the system can be obtained by Yule-Walker algorithms[9], one of the least square methods.

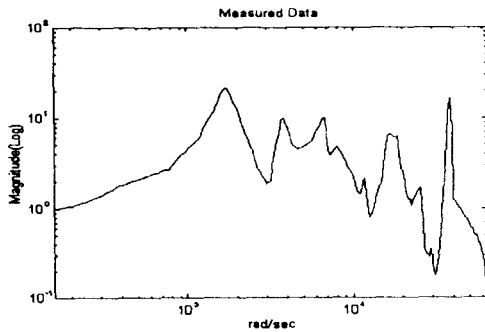


Figure 4.1 Frequency characteristic of G.

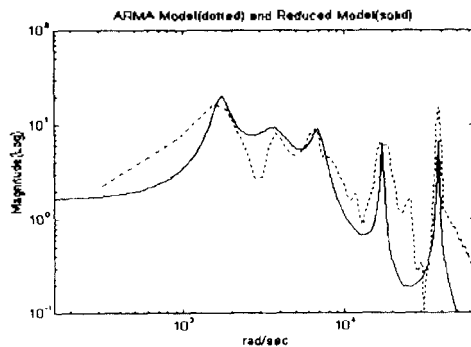


Figure 4.2 Frequency characteristics of the model and the reduced-order model.

As shown in Figure 4.1, the characteristic of a small cavity have the complex characteristic at the frequency over 1kHz. There are five dominant modes which play important role on the characteristic. The modified Yule-Walker method is used to model the transfer characteristic.

The obtained model is shown in Figure 4.2. But, its degree is too high to handle. So, we reduced the order of the model to 11th.

The reduced-order model is expressed in Eq.(4-1),

$$\hat{G}(s) = \frac{\text{NumG}(s)}{\text{DenG}(s)} \tag{4-1}$$

where

$$\begin{aligned} \text{NumG}(s) &= 94.5e6 \cdot s^8 + 5.48e12 \cdot s^7 + 1.65e17 \cdot s^6 \\ &+ 3.38e21 \cdot s^5 + 3.43e29 \cdot s^4 + 1.57e33 \cdot s^3 + 4.02e36 \cdot s^2 \\ &+ 5.2e39 \cdot s + 2.615e42 \\ \text{DenG}(s) &= s^{11} + 5.42e3 \cdot s^{10} + 1.81e9 \cdot s^9 + 9.38e12 \cdot s^8 \\ &+ 5.52e17 \cdot s^7 + 2.56e21 \cdot s^6 \\ &+ 3.27e25 \cdot s^5 + 1.09e29 \cdot s^4 + 4.68e32 \cdot s^3 + 9.39e35 \cdot s^2 \\ &+ 1.21e39 \cdot s + 1.61e42 \end{aligned}$$

4.2 The design of the controller for the small cavity

As explained in the section 2, the purpose of this paper is to minimize the mixed sensitivity function of the system keeping a robustness. For noise control in the small cavity, we defined specifications of the controller as follows

First: there exist modelling errors and nonlinear characteristics in the cavity model, especially in the high frequency ranges, so the magnitude of the complementary sensitivity function should be lower than 20 dB at those frequencies to reduce their effect.

Second: most of the noise signal induced in the small cavity is below 1kHz. So the magnitude of the sensitivity function is below as far as possible below 1kHz.

These specifications are expressed as weight functions as following.

$$W_1 = \frac{100 \left(\frac{1}{100000} s + 1 \right)^3}{\left(\frac{1}{10000} s + 1 \right)^4} \tag{4-2a}$$

$$W_2 = \frac{\left(\frac{1}{10000} s + 1 \right)^4}{1600 \left(\frac{1}{300000} s + 1 \right)^3} \tag{4-2b}$$

Now, we will form the standard feedback system to solve the problem. In the cavity system, the input and output signal is the noise signal and the measured error

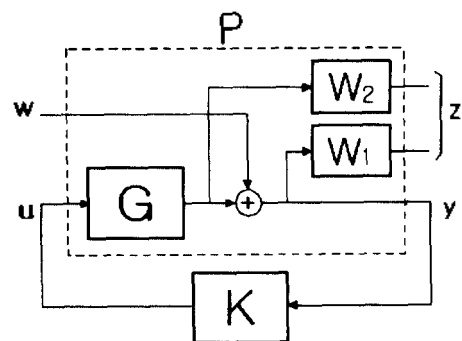


Figure 4.3 Standard feedback System for the small cavity.

signal, respectively, and they are related to w and y in the standard feedback system, respectively. u is the controller's output. The problem is z . This should include the information of our specifications, which is related to the sensitivity and the complementary sensitivity functions. Therefore, we formed the standard feedback system shown in Figure 4.3.

Based on the above system, we solved the H_∞ robust control problem using *Robust Control Toolbox of Matlab* [7]. The obtained controller is

$$K(s) = \frac{\text{NumK}(s)}{\text{DenK}(s)} \quad (4-3)$$

where

$$\begin{aligned} \text{NumK}(s) &= 3.51e5 \cdot s^{17} + 9.31e13 \cdot s^{13} + 9.02e19 \cdot s^{15} + 3.13e25 \cdot s^{14} \\ &+ 4.06e30 \cdot s^{13} + 2.79e35 \cdot s^{12} + 1.25e40 \cdot s^{11} + 4.25e44 \cdot s^{10} \\ &+ 1.05e49 \cdot s^9 + 1.51e53 \cdot s^8 + 2.55e57 \cdot s^7 + 1.58e61 \cdot s^6 \\ &+ 1.37e65 \cdot s^5 + 4.83e68 \cdot s^4 + 1.83e72 \cdot s^3 + 3.31e75 \cdot s^2 \\ &+ 3.14e78 \cdot s + 1.29e82 \end{aligned}$$

$$\begin{aligned} \text{DenK}(s) &= s^{17} + 6.42e6 \cdot s^{16} + 9.00e15 \cdot s^{15} + 4.25e21 \cdot s^{14} \\ &+ 8.41e26 \cdot s^{13} + 8.81e31 \cdot s^{12} + 4.91e36 \cdot s^{11} + 1.74e41 \cdot s^{10} \\ &+ 4.34e45 \cdot s^9 + 7.92e49 \cdot s^8 + 1.05e54 \cdot s^7 + 6.74e61 \cdot s^6 \\ &+ 3.12e65 \cdot s^5 + 9.53e68 \cdot s^4 + 1.81e72 \cdot s^3 + 1.90e75 \cdot s \end{aligned}$$

The frequency response of the designed controller is shown in Figure 4.4. In Figure 4.5, the sensitivity function below 1 kHz is less than 10^{-2} which means that the noise signal can be reduced to 1/100 times. In the region of high frequency, the magnitude of the complement sensitivity function is getting smaller. This means that the effect of uncertainties by the modeling error would be reduced.

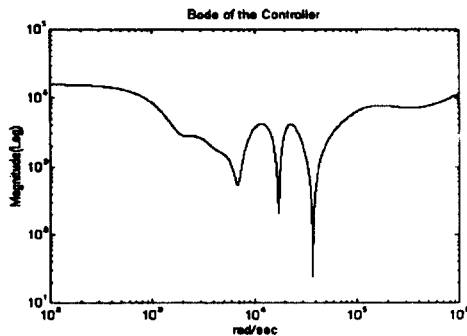


Figure 4.4 Frequency characteristics of the designed controller.

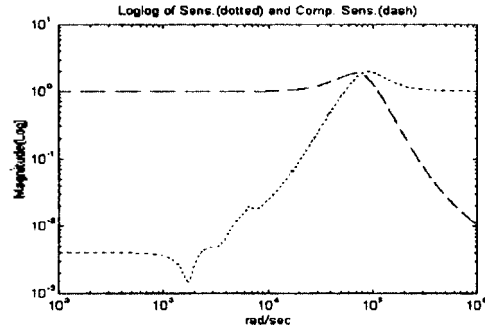


Figure 4.5 Bode plot of sensitivity function and its complementary sensitivity function.

V. Experimental Results

For experiment, the designed controller expressed in Eq.(4.3) was realized with operational amplifiers and the external noise source was simulated using another loud speaker. Figure 5.1 shows the experimental system diagram. We simulated noisy environment using noise signals generated when vehicles runs and a crane works in constructional field. The output signals of the microphone were measured in both cases when the controller was on and off to see the controller's performance.

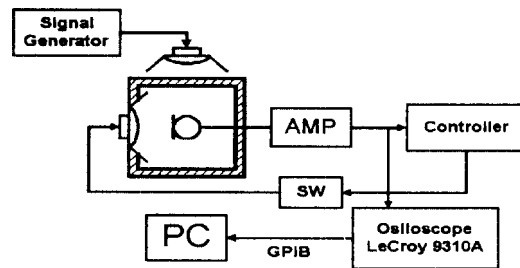


Figure 5.1 Block Diagram of the controller.

Figure 5.2 and 5.3 show noise signals and their spectrums. In these Figures, the dotted signal represent the measured noise signal when the controller was off and the solid signal when the controller was on. As one can see, in the range between 100Hz and 700Hz, the noise signals were reduced by more than 20dB. Therefore, the designed controller could be used as the active noise controller for the small cavity.

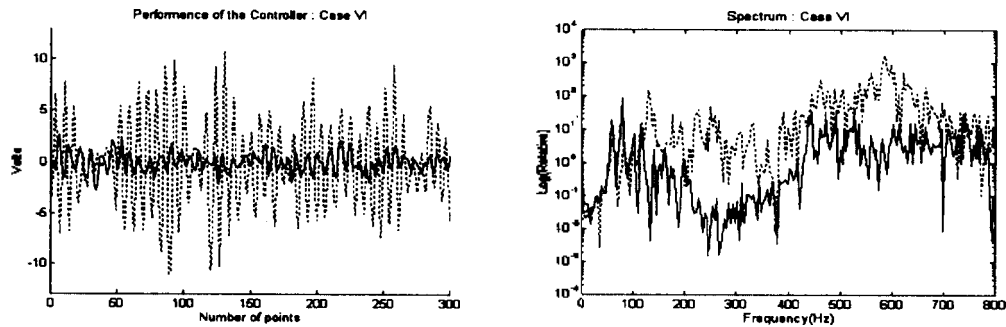


Figure 5.2 Noise control of Car Traffic Noise and its spectrum.

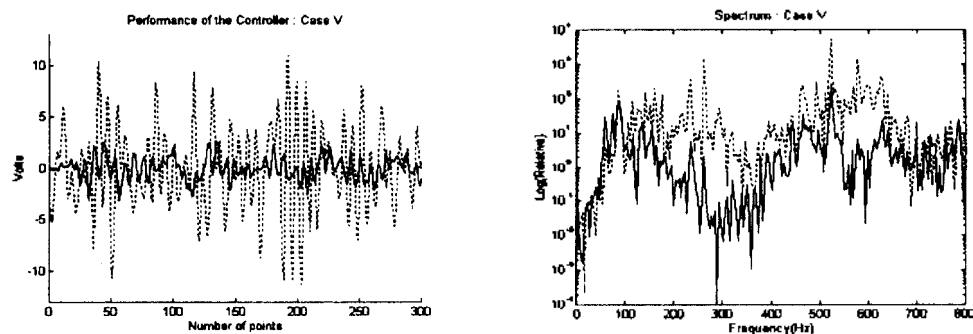


Figure 5.3 Noise control of Crane Noise and its spectrum.

VI. Conclusion

This paper studied on the robust active noise controller for a small cavity to reduce noise signal induced in the small cavity. In the small cavity, there are nonlinear characteristics and modeling errors, which we supposed as uncertainties. By solving the H_∞ robust control problem under these uncertainties, a robust controller could be designed to minimize the H_∞ norm of the mixed sensitivity function, which implied that the noise controller was designed. The designed controller was implemented with analog Op-Amps and it showed good performances in reducing noise signals in the small cavity within the 100Hz-700Hz ranges.

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▲Tae-Jin Chung



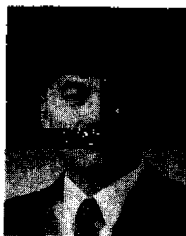
He received the B.S. and M.S. degree in Electrical Engineering from Soongsil University in 1994 and 1996, respectively. Since March 1996, he has been a student in Ph.D. course in Soongsil University. His current research interests include Active Noise Control, Robust Control, and System Identification.

▲Hak-Joon Oh



He received the B.S. and M.S. degree in Electrical Engineering from Soongsil University in 1993 and 1995, respectively. From March 1996, he has been a student in Ph.D. course in Soongsil University. Also, he is a research engineer in Daewoo Motor Technical Center. His current research interests include Active Noise Control, Digital Signal Processing, and System Identification.

▲Chi-Hyung You



He received the B.S. and M.S. degree in Electrical Engineering from Soongsil University in 1988 and 1991, respectively. From March 1995, he has been a student in Ph.D. course in Soongsil University. Also, he is a Professor in Soongsil Computer Academy. His current research interests include Digital Signal Processing, Fault Detection and Diagnosis, and Active Noise Control.

▲Chan-Soo Chung



He received B.S. degree in Electrical Engineering at Seoul National University in 1972, and M.S. and Ph.D degrees both in electrical engineering at Seoul National University in 1980 and 1987, respectively.

From March 1972 to July 1975 he had been a member of the teaching staff in Naval Academy. From August 1975 to February 1981 he had been an Associate Professor in Ulsan Junior College of Technology and Dongyang Junior College of Technology. Since March 1981, he has been a Professor of Electrical Engineering at Soongsil University, Seoul, Korea. His current research interests include Active Noise Control, Digital Signal Processing, Fault Detection and Diagnosis.