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옷걸이형 다이의 설계에서 매개 변수의 영향

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Parametric Study in Design of Coat-Hanger Die

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요 약

매니폴드 부분이 선형 경사진 옷걸이형 다이의 최적 설계를 수행하였다. 다이 흐름을 3차원 모델을 이용하여 해석함으로써 정확한 최적 설계 결과를 얻을 수 있었으며, 형상 최적화 과정을 고찰함으로써 최적 설계 도구의 효율성을 조사하였다. 공정의 여러 매개 변수들이 매니폴드의 최적 형상에 미치는 영향을 조사하기 위하여 멱법칙 유체의 멱법칙 지수, 다이 슬롯의 두께 및 매니폴드 각도를 변화시키며 최적 설계를 수행하였다.

Abstract — Optimal design has been performed for a linearly tapered coat-hanger die. The accurate optimal design was achieved using three-dimensional model of die flow. The optimization history was investigated to demonstrate the efficiency of the optimal design algorithm. In order to examine the effect of process parameters on the optimal manifold profile, the optimal design is carried out for various values of power-law index of polymer melt, die thickness, and manifold angle.
Keywords: Coat-Hanger Die, Optimal Design, Three-Dimensional Model, FEM

1. Introduction

Extrusion dies are widely used in polymer processing to produce films or sheets. The primary function of extrusion die is to uniformly distribute polymer melt flow. The flow cross-section of polymer melt significantly varies from die inlet to die exit. The die exit plane is designed to be the same as the cross-sectional shape of films or sheets. The most distinct characteristics of films or sheets is the large aspect ratio of rectangular cross-section. The primary concern of product quality is the uniformity of thickness across the transverse direction. It can be accomplished by the uniform flow rate distribution at die exit. Therefore optimal design of extrusion die has been of major interest in industry since the beginning of polymer processing.

The various optimal design equations of coat-hanger dies have been suggested by many researchers using one-dimensional flow model. The main advantage of this model is that the design equation can be derived analytically in most cases. The basic procedure and design results are similar to each other. Matsubara[1] developed an optimal design equation to get uniform flow rate and residence time distributions. Winter and Fritz[2] used rectangular cross-section of manifold to get uniform flow rate distribution and uniform shear stress at die wall. Liu *et al.*[3] considered a linearly tapered coat-hanger die with the various cross-sectional shapes of manifold such as tear-drop, triangle, and circle. The general procedures and various design schemes of one-dimensional analysis were summarized by Michaeli[4]. In most of design works based on one-dimensional model, the polymer melt is assumed to be power-law fluid, and the optimal design equation is analytically derived. Liu *et al.*[5] suggested a numerical solution scheme of one-dimensional flow model with the various fluid models such as power-law, Ellis, and Bingham fluid.

The more complicated analysis of die flow is based on two- and three-dimensional flow models. Two-dimensional

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flow model is obtained by application of lubrication approximation to the die flow. The thickness of die is assumed to be very small compared to the width and length of die. The flow model is solved using various numerical techniques such as finite difference method (FDM)[6] or finite element method (FEM)[7]. FEM is more frequently used in most of three-dimensional die analyses because it can treat the complex geometry of extrusion die more conveniently than FDM. Most of the two- and three-dimensional analyses have been focused on a flow simulation study for a given die geometry. Vergnes *et al.*[6] simulated non-isothermal flow in a coat-hanger die using two-dimensional flow model. There are a few number of optimal design researches based on two- or three-dimensional model. Smith *et al.*[7] combined sensitivity analysis with two-dimensional flow model and developed a shape optimization problem. In their optimization scheme, the die thickness was optimized in the whole domain of extrusion die. The optimal design of extrusion die was performed through trial-and-error scheme with three-dimensional flow modeling [8-10]. The die geometry determined by one-dimensional optimal design was updated using the result of three-dimensional flow simulation[8,9]. From the result of three-dimensional flow simulation, Na and Kim[10] showed that one-dimensional design generates uniform flow rate distribution only in a limited range of process condition.

The flow within a coat-hanger die is by nature three-dimensional due to the complex geometry. The cross-sectional shape in the flow direction significantly varies from the manifold to the slot section. The die flow has to be analyzed using three-dimensional model to get accurate flow field. In consequence, it is necessary to perform the optimal design based on the three-dimensional flow modeling. Na and Lee[11] has developed an optimal design tool based on three-dimensional flow model for a linearly tapered coat-hanger die. They formulated the design problem as an inverse problem. Their design method is automatic and independent of flow modeling.

The original design problem formulated as an inverse problem is ill-posed in itself. In order to alleviate the ill-posed nature of the inverse problem, the concept of regularization has been introduced in this study. Regularization of a problem refers to solving a related problem, called the regularized problem, with a solution that is more "regular" in a certain sense than the solution of the original problem[12].

In this study, the effect of process parameters, such as power-law index and geometrical parameters, on the optimum manifold geometry is investigated through the optimal design method developed by Na and Lee[11]. It will be shown that the accurate design can be achieved as process parameters are changed.

2. Optimal Die Design

The optimal design method employed in this study is briefly outlined to help understand the result of optimal design. Details of the design method are given by Na and Lee[11].

The schematic geometry of a linearly tapered coat-hanger die considered in this study is shown in Fig. 1. The characteristics and performance of the coat-hanger die are determined by the distribution of manifold cross-sectional area in the transverse direction. The manifold geometry is represented by the profile function,

$$h = h(y) \qquad (1)$$

where h is the characteristic length of manifold cross-section, and y is the lateral coordinate as shown in Fig. 1. The manifold profile function is discretized into finite data points, (y_i, h_i) for i=1, ⋯, N_I. The points, y_1 and y_{N_I}, correspond to 0 and L, respectively. The complete manifold profile is obtained using the cubic spline interpolation of the finite data points as

$$h = \sum_{i=1}^{N_I} B_i(y) h_i \qquad (2)$$

where $B_i(y)$ is a cardinal cubic interpolation function such that $B_i(y_j)=\delta_{ij}$. $B_i(y)$ can be considered as a cubic spline function of a reference data set, (y_j, δ_{ij}) for j=1, ..., N_I. Hence the manifold geometry is determined by the finite number of geometrical variables, h_i's.

The design variables are defined using h_i's as

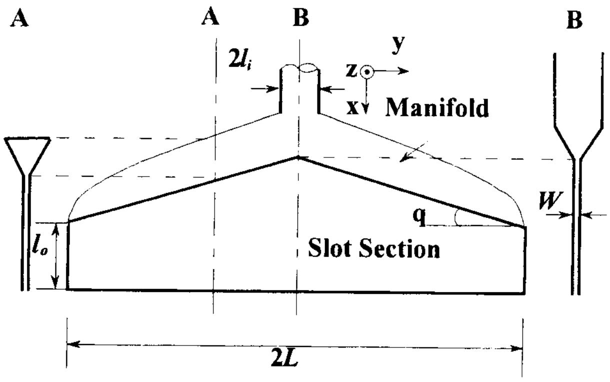

Fig. 1. Geometry of linearly tapered coat-hanger die.

$$h_i = \sum_{j=1}^{i} H_j \tag{3}$$

for i=1, ..., N_I. H_i's efficiently represent the effect of the manifold profile on the flow rate distribution at die exit. The sensitivity of flow rate distribution with respect to design variables is greatly improved by using H_i's.

The optimal solution of the design variables is obtained from the optimization problem which is developed through the inverse formulation. The design objective function is defined as the square sum of pressure gradient deviations at die exit,

$$\Theta = \frac{1}{N_{EY}}\left(\frac{\partial p}{\partial x}\right)_c^{-2} \sum_{i=1}^{N_{EY}} \left[\overline{\left(\frac{\partial p}{\partial x}\right)} - \left(\frac{\partial p}{\partial x}\right)_i\right]^2 \tag{4}$$

where $(\partial p/\partial x)_i$ is the pressure gradient in each finite element at die exit, and $\overline{(\partial p/\partial x)}$ is the average pressure gradient. N_{EY} is the number of finite elements in y-direction. The objective function is normalized with respect to N_{EY} and characteristic pressure gradient, $(\partial p/\partial x)_c$. The ill-conditioned nature of the inverse problem is eliminated using the penalty function defined as

$$\Omega = \frac{L^5}{h_c^2}\int_0^L \left(\frac{\partial^3 h}{\partial y^3}\right)^2 dy \tag{5}$$

and it is referred as stabilizer of third order. The total objective function of the optimization problem is defined as

$$M = \Theta + \alpha\Omega \tag{6}$$

where α is the regularization parameter which controls the effect of the penalty function on the optimal solution profile.

The problem to be solved in this study is the minimization of the augmented objective function M defined in Eq. (6) subject to the design variable relation described in Eq. (3) and the three-dimensional flow model equation.

3. Results and Discussion

In this study, manifold cross-section is assumed to be equilateral triangle as shown in Fig. 1. The characteristic length of manifold cross-section, h, is defined as the side length of equilateral triangle.

The dimensions of a reference die geometry are as follows. The thickness of slot section, W, is 0.01 m. Half the die width, L, is 0.5m. Half the width of die inlet, l_i, and the length of land region, l_o, are 0.04 m and 0.1m, respectively. The manifold angle, q, is 15°.

In the optimal design method used in this study, the die flow is analyzed using three-dimensional model and the numerical solution is obtained using FEM. The die flow is assumed to be incompressible, creeping and isothermal. The polymer melt is assumed to be power-law fluid as

$$\eta = K\dot{\gamma}^{n-1}. \tag{7}$$

n=0.5 is used as a reference fluid. Only a quadrant of the total flow domain is considered as numerical domain taking advantage of the symmetric die geometry. The bird-eye's view of three-dimensional mesh is shown in Fig. 2. The number of finite elements is 268 in the manifold and inlet region and 160 in the slot section.

Two parameters, N_I and α, have to be specified to apply the optimal design method. Manifold profile function is defined by the cubic interpolation of finite data points. N_I=9 is used in this study. The optimum manifold profile is successfully obtained using the number of interpolation points. Na and Lee[11] determined the optimal regularization parameter as 10^{-8} that can be used in various process condition.

3.1. Optimization history

The changes of manifold profile, pressure gradient, and flow rate distributions are shown in Fig. 3 when uniform manifold profile is used as an initial guess. The manifold profile rapidly converges to the optimum profile with the poor initial guess. It takes only four iterations to get the optimum solution. The manifold profile, pressure gradient, and flow rate distributions show the similar pattern of variation with iterations. The flow rate distribution exhibits non-uniformity at die side end due to no-slip boundary condition. In contrary to the non-uniformity of flow rate, the flat distribution of pressure gradients is obtained as an optimum solution which makes the minimization of design objective function more effective.

3.2. Effect of rheological and geometrical parameters

The optimal design is carried out as the rheological and

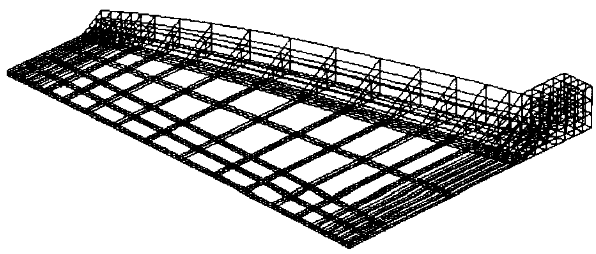

Fig. 2. Bird's-eye view of three-dimensional mesh.

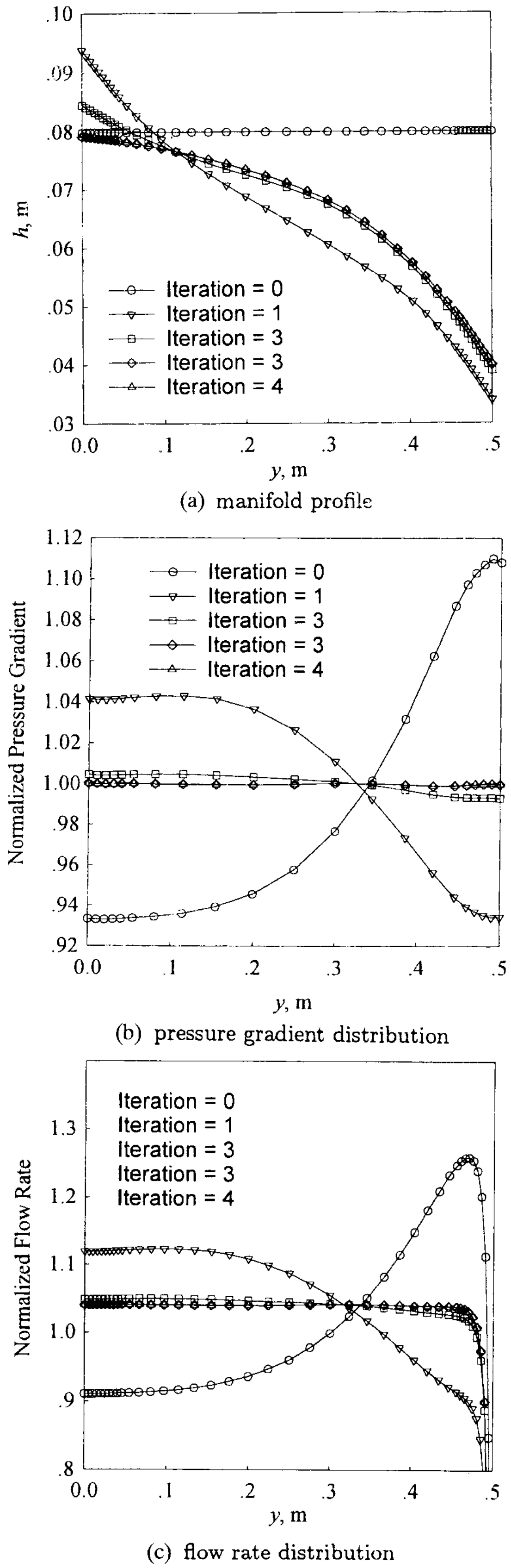

Fig. 3. Optimization history.

gemoetrical parameters vary. In order to show the superiority of three-dimensional design, the optimum manifold profile are compared with that of one-dimensional design developed by Liu *et al.*[3].

The rheological behavior of power-law fluid is determined by power-law index, n. The shear thinning effect increases as n decreases. The optimum manifold profile will be significantly affected by power-law index. The change of manifold profile is shown in Fig. 4 as n varies. The characteristic length of manifold cross-section increases in the whole range as n decreases. The global shape of manifold profile does not change with power-law index. The difference of manifold profiles between optimal design and one-dimensional design can be noted in all cases.

Slot thickness, W, and manifold angle, q, are the governing geometrical parameters of a linearly tapered coat-hanger die. The optimum manifold geometry is closely related to the values of the two parameters. Figs. 5 and 6 show the effects of slot thickness and manifold angle on the optimum manifold geometry. The optimum manifold profile is significantly affected by the slot thickness and manifold angle. The change of manifold profile is larger at die center than at die side end in both cases. From the result of Fig. 5, it can be noted that the manifold profile is very sensitive to the change of slot thickness. Fig. 6 shows that the difference of manifold profile between optimal design and one-dimensional design increases with manifold angle.

Figs. 4~6 show that the manifold profiles obtained by one-dimensional design method generally exhibit smaller

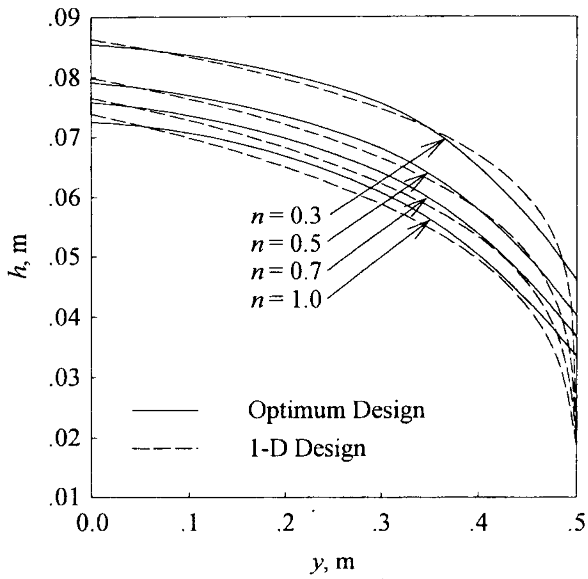

Fig. 4. Effect of power-law index on manifold profile.

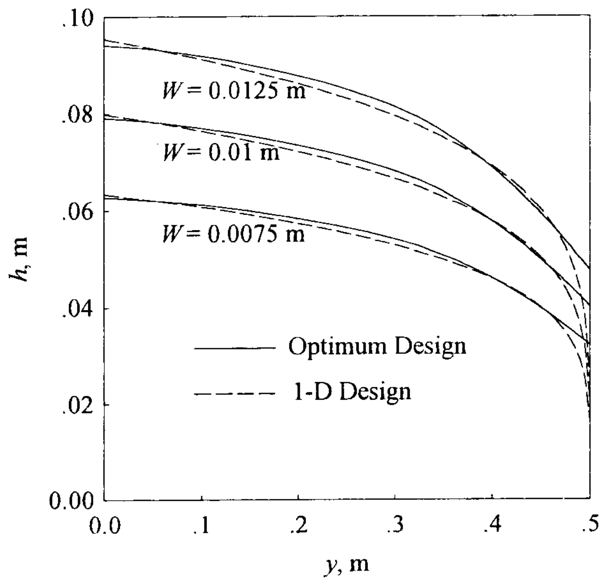

Fig. 5. Effect of slot thickness on manifold profile.

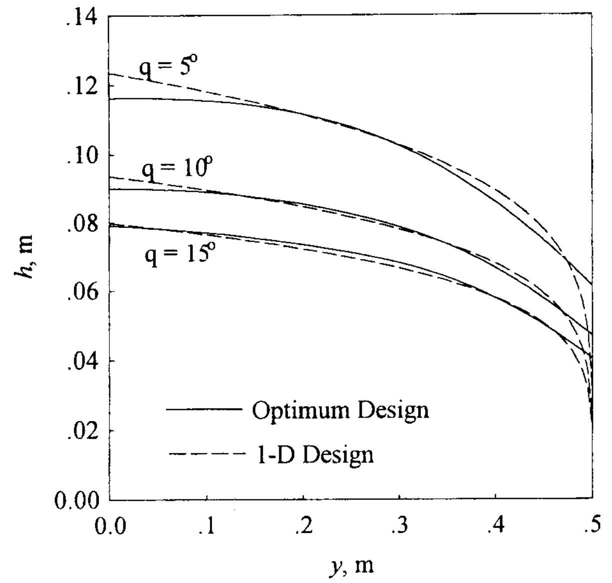

Fig. 6. Effect of manifold angle on manifold profile.

curvature for the most part of the manifold than that obtained by optimal design method over the entire rheological and geometrcal parameter ranges of tests. Hence the one-dimensional design results enforce the polymer melt flow concentrated on the die center yielding non-uniform flow distribution[11].

4. Conclusion

Three-dimensional optimal design is successfully accomplished in the various process conditions. The manifold profile rapidly converges to the optimum profile that generates uniform pressure gradient and flow rate distribution. The manifold profile increases in the whole range as power-law index decreases. The manifold profile is found to be very sensitive to the slot thickness. The manifold angle also affect the manifold profile. The effect of slot thickness and manifold angle on manifold profile increases from die side end to die center.

Acknowledgement

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Nomenclature

B : cardinal cubic interpolation function
H : design variable, m
h : characteristic length of manifold cross-section, Kg m^{-1} sec^{n-2}
K : power-law constant, Kg m^{-1} sec^{n-2}
L : half the die width, m
l_i : half the die inlet width, m
l_o : the land length, m
M : total objective function,
N_{EY} : number of finite element in y-direction
N_I : number of interpolation points
n : power-law index
q : manifold angle
W : the slot thickness, m
x, y, z : global coordinates

Greek Letters

α : regularization parameter
$\dot{\gamma}$: magnitude of rate of strain tensor
η : viscosity
Θ : design objective function
Ω : penalty function

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