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DEVELOPMENT OF TERRAIN CONTOUR MATCHING ALGORITHM FOR THE AIDED INERTIAL NAVIGATION USING RADIAL BASIS FUNCTIONS

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ABSTRACT

We study on a terrain contour matching algorithm using Radial Basis Functions (RBFs) for aided inertial navigation system for position fixing aircraft, cruise missiles or re-entry vehicles. The parameter optimization technique is used for updating the parameters describing the characteristics of an area with modified Gaussian least square differential correction algorithm and the step size limitation filter according to the amount of updates. We have applied the algorithm for matching a sampled area with a target area supposed that the area data are available from Radar Terrain Sensor (RTS) and Reference Altitude Sensor (RAS).

1. INTRODUCTION

When we are to find a pre-determined area from an aircarft, a flying missile, or an airborne vehicle we need to develop a terrain contour matching algorithm for updating parameters which describe the shape and charateristics of the area. The parameters are updated with modified Gaussian least square differential correction algorithm (Junkins 1978) and a step size limitation filter (Gong 1995) according to the amount of parameter updates. Terrain contour matching algorithm is a technique for position fixing airbone vehicle by matching terrain contours passing under an airborne vehicle with stored digital contour map data or digital stereo photo data. Terrain contour matching algorithm is based on the fact that terrain elevation contours over a region of the earth are unique to that region. As fingerprints provide identification of a person, terrain contours provide identification of a location on the earth. The system is self-contained and provides precision guidance/navigation for an aircraft, drones, cruise missiles or re-entry vehicles.

In this study we develop a terrain contour matching algorithm for the aided inertial navigation using RBFs. RBFs have been studied and used historically for approximation in a curve-fitting setting and for fitting irregularly positioned data points (Powell 1981) and have recently been used as the basis functions for neural and related input/output approximation and learning networks (Girosi

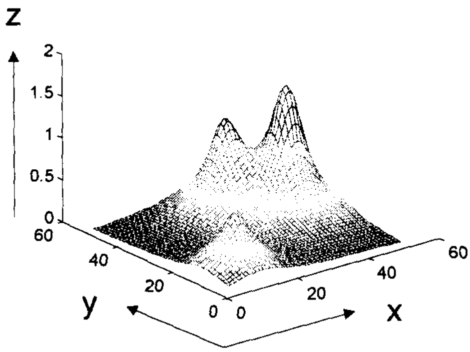

Figure 1. Terrain contour of an area.

→ y

↓ x

0.266	0.389	0.279	0.170	0.126	0.108	0.101	0.094	0.086	0.075
0.388	0.747	0.407	0.209	0.154	0.139	0.136	0.131	0.119	0.101
0.278	0.406	0.304	0.207	0.179	0.184	0.199	0.201	0.178	0.141
0.168	0.206	0.205	0.196	0.212	0.258	0.328	0.364	0.301	0.207
0.121	0.147	0.171	0.204	0.264	0.376	0.610	0.836	0.564	0.297
0.102	0.129	0.168	0.232	0.343	0.521	0.933	1.624	0.832	0.356
0.092	0.121	0.172	0.273	0.489	0.766	0.847	0.927	0.600	0.313
0.084	0.114	0.168	0.288	0.622	1.179	0.761	0.509	0.358	0.232
0.076	0.104	0.147	0.237	0.424	0.617	0.479	0.324	0.234	0.168
0.066	0.086	0.117	0.166	0.233	0.280	0.257	0.206	0.160	0.124

Figure 2. Numrical representation of an area.

& Poggio 1989). The algorithm is presented in Section 2. The results of applying the algorithm for a sampled area to compare with a target area are explained in Section 3. The conclusions are presented in Section 4.

2. ALGORITHM

We have a pre-determined area whose terrain contour is presented in Figure 1 and whose numerical representation is shown in Figure 2.

The principle of the numerical representation is shown in Figure 3, where the RTS is a radar altimeter system, usually pulsed, which measures aircraft or airborne vehicle clearance above the terrain and the RAS is a barometric altimeter, a vertical accelerometer, a combination of both or the vertical channel of an inertial navigation system. The numerical representation is obtained by subtracting the RTS measurement from the RAS measurement in grids in Figure 3. The subtraction removes the effects of any vertical motion of the airborne vehicle (Siouris 1993).

Let's consider the area which can be expressed by the RBFs as follows (Gong 1995);

$$z = \sum_{i=1}^{N} a_i f_i(\mathbf{x}, \mathbf{p}) \tag{1}$$

where a_i are the height parameters for each RBF, $f_i(\mathbf{x}, \mathbf{p})$ which is described as follows;

$$f_i(\mathbf{x}, \mathbf{p}) = \frac{\mathbf{1}}{\mathbf{1} + (\mathbf{x} - \mathbf{x_i})^{\mathbf{T}} \mathbf{W_i} (\mathbf{x} - \mathbf{x_i})} \tag{2}$$

where $\mathbf{x_i}$ are the two dimensional vectors which represent the positions of the peaks of RBFs

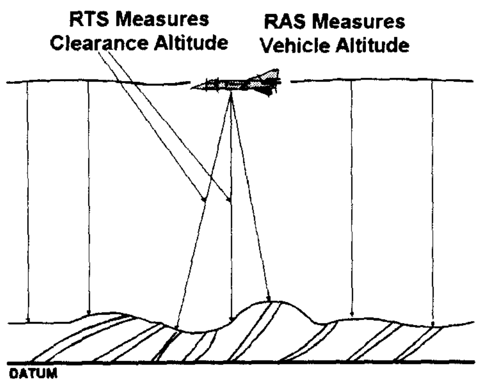

Figure 3. RTS and RAS measurements.

and $\mathbf{W_i}$ are the weight matrix for $\mathbf{x_i}$ and are described as follows;

$$\mathbf{W_i} = \begin{bmatrix} w_i^2 & \cdots & \mu_{kj}w_iw_j \\ \cdots & \ddots & \cdots \\ \mu_{kj}w_iw_j & \cdots & w_n^2 \end{bmatrix} = \mathbf{W_i}(w_1, \cdots, w_n, \mu_{12}, \cdots, \mu_{ij}, \cdots) \tag{3}$$

For this particular implementation, the states are geometrically rectangular and regularly spaced grids in two dimensions. The parameters herein are the heights, centers of peaks, and the weights of the RBFs and are expressed for each RBF as follows;

$$\mathbf{p_i} = [\mathbf{a_i} \ \mathbf{x_{ci}} \ \mathbf{w_{1i}} \ \mathbf{w_{2i}} \ \mu_{\mathbf{12i}}]^{\mathbf{T}} \tag{4}$$

then the total parameters are composed of $\mathbf{p_i}$ as follows ;

$$\mathbf{P} = [\mathbf{p_1} \ \cdots \ \mathbf{p_N}]^{\mathbf{T}} \tag{5}$$

The Gaussian least square differential correction algorithm to "learn" $\mathbf{P_r}$ with current/apriori estimates given a diminishing weight is given by;

$$\mathbf{P_{r+1}} = \mathbf{P_r} + \left[\mathbf{A_r^T W_r A_r} + \mathbf{Q_r^{-1}}\right]^{-1} \mathbf{A_r^T W_r \Delta Y_r}, \ \ \mathbf{Q_0} = \mathbf{I}, \ \ \mathbf{Q_r} = \mathbf{r Q_{r-1}}, \ \ \mathbf{r} = \mathbf{1}, \mathbf{2}, \cdots \tag{6}$$

where $\mathbf{P_r}$ is the current parameters as in Equation (5) and $\mathbf{W_r}$ is the weight matrix as in Equation (3) and $\mathbf{Q_r}$ is the additional diminishing weight which puts less weight as the iteration goes on and addressed in $\mathbf{Q_r} = \mathbf{rQ_{r-1}}$ and $\mathbf{A_r}$ can be described as follows;

$$\mathbf{A_r} = \left[\frac{\partial \mathbf{Y}(\mathbf{P_r})}{\partial \mathbf{P_r}}\right] \tag{7}$$

and $\mathbf{Y}(\mathbf{P_r})$ is the current value of the area which is described in Equation (8) and $\widetilde{Y}$ is the target value of the area which is described in Equation (9).

$$\mathbf{Y}(\mathbf{P_r}) = \left\{ \begin{array}{c} \sum_{i=1}^{N} a_i f_i(x_1, p_{ir}) \\ \vdots \\ \sum_{i=1}^{N} a_i f_i(x_{TOT}, p_{ir}) \end{array} \right\} \tag{8}$$

$$\widetilde{\mathbf{Y}} = \left\{ \begin{array}{c} z_1 \\ \vdots \\ z_{TOT} \end{array} \right\} \tag{9}$$

where N and TOT are the number of RBFs and the total number of grids in the area respectively. The errors between the target values and the current values of the grids which should be within a tolerance can be described as follows;

$$\mathbf{\Delta Y} = \widetilde{\mathbf{Y}} - \mathbf{Y}(\mathbf{P_r}) \tag{10}$$

Gauss's celebrated principle of least squares selects, as an optimum choice for the updated parameters, the particular $\mathbf{P_r}$ which minimize the residuals (Junkins & Kim 1993).

$$\mathbf{\Phi_r} = (\mathbf{\Delta Y} - \mathbf{A \Delta P_r})^T \mathbf{W_r} (\mathbf{\Delta Y} - \mathbf{A \Delta P_r}) \tag{11}$$

In order that $\Delta \mathbf{P_r}$ yield a minimum of Equation (11) (Junkins 1978), we have the requirements

necessary condition

$$\nabla_{\Delta P_r} \mathbf{\Phi_r} = \left\{ \begin{array}{c} \frac{\partial \Phi_r}{\partial p_{1r}} \\ \vdots \\ \frac{\partial \Phi_r}{\partial p_{Nr}} \end{array} \right\} = -2\mathbf{A}^T \mathbf{W_r} \Delta \mathbf{Y} + 2\mathbf{A}^T \mathbf{W_r} \mathbf{A} \Delta \mathbf{P_r} = 0 \tag{12}$$

sufficient condition

$$\nabla^2_{\Delta P_r} \Phi_r = 2(\mathbf{A}^T \mathbf{W_r} \mathbf{A}) \tag{13}$$

must be positive definite.
From the necessary condition, we obtain the weighted least square normal equations

$$(\mathbf{A}^T \mathbf{W_r} \mathbf{A}) \Delta \mathbf{P_r} = \mathbf{A}^T \mathbf{W_r} \Delta \mathbf{Y} \tag{14}$$

which, in principle, are invertible to obtain

$$\Delta \mathbf{P_r} = (\mathbf{A}^T \mathbf{W_r} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W_r} \Delta \mathbf{Y} \tag{15}$$

Here we consider the diminishing weight, $\mathbf{Q_r^{-1}}$ in Equation (6), which puts more weights in the first several iterations and the weight becomes smaller as the iteration goes on.

$$\Delta\mathbf{P_r} = \left[\mathbf{A_r^T W_r A_r} + \mathbf{Q_r^{-1}}\right]^{-1} \mathbf{A_r^T W_r \Delta Y_r} \tag{16}$$

We find that the convergence is enhanced if we use a step size limitation filter (Gong 1995) according to the value of $\Delta\mathbf{P_r}$ as follows;

$$\mathbf{P_{r+1}} = \mathbf{P_r} + \mathbf{\Delta P_r} \tag{17}$$

where

$$|\Delta\mathbf{P_r}| = \sqrt{\Delta\mathbf{P_r}^T \Delta\mathbf{P_r}} \tag{18}$$

If $\Delta\mathbf{P_r} \leq \epsilon$ for acceptably small ϵ, then we use the full correction

$$\Delta\mathbf{P_r} = \left[\mathbf{A_r^T W_r A_r} + \mathbf{Q_r^{-1}}\right]^{-1} \mathbf{A_r^T W_r \Delta Y_r} \tag{19}$$

else if $\Delta\mathbf{P_r} \geq \epsilon$ for acceptably small ϵ, then we use the scaled correction

$$\Delta\mathbf{P_r} = \left[\frac{\epsilon}{|\Delta\mathbf{P}|}\right] \left[\mathbf{A_r^T W_r A_r} + \mathbf{Q_r^{-1}}\right]^{-1} \mathbf{A_r^T W_r \Delta Y_r} \tag{20}$$

When a small increase ($\Delta\mathbf{P_r}$) cannot be achieved, while matching the target area within a tolerance, the previously converged solution is adopted as the optimum.

3. RESULTS

In Figure 4 we have a sampled area in ITER.1 which would be the area to be compared with the target area. As we update the parameters for the heights and centers of RBFs according to the modified Gaussian least square differential correction algorithm with the step size limitation filter the sampled area converge to the target area in a couple of iterations as in ITER.2 and ITER.3 in Figure 4. As the iteration continues the parameter update becomes smaller with the effect of the diminishing weight in the parameter update algorithm as in ITER.4 and ITER.5 in Figure 4.

From ITER.6 which is not shown in Figure 4 we can not improve the parameter update, $\mathbf{\Delta P_r}$ in Equation (17) since the sampled area data is already within the predetermined tolerance of $10^{-8.0}$ for the heights and peaks of RBFs. Therefore, we choose the parameters of ITER.5 as the optimum parameters describing the target area.

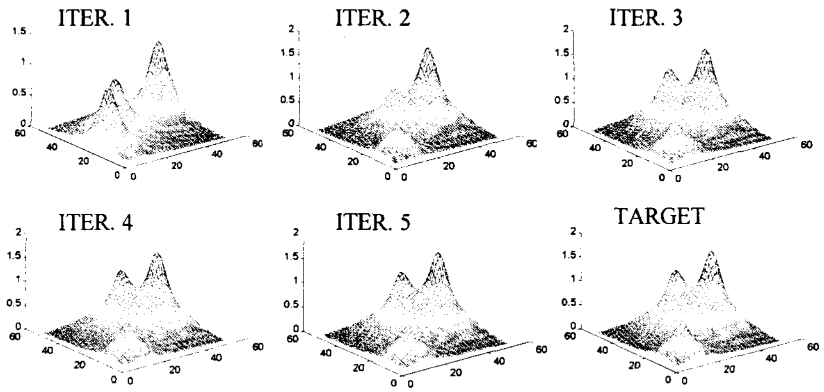

Figure 4. Target following iterations.

4. CONCLUSIONS

We have developed a new terrain contour matching algorithm for the aided inertial naviagtion using Radial Basis Functions with parameter optimization technique of modified Gaussian least square differential correction algorithm and the step size limitation filter which can be applied to fixing position for an aircraft, drones, cruise missiles or re-entry vehicles. We need only several parameter update iterations to check if the sampled area match the target area according to the algorithm. In future study we need to develop a relative floating terrain contour matching algorithm to check if several sampled points of an area match the target area.

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