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THE ATTITUDE STABILITY ANALYSIS OF A RIGID BODY WITH MULTI-ELASTIC APPENDAGES AND MULTI-LIQUID-FILLED CAVITIES USING THE CHETAEV METHOD

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ABSTRACT

The stability problem of steady motion of a rigid body with multi-elastic appendages and multi-liquid- filled cavities, in the presence of no external forces or torque, is considered in this paper. The flexible appendages are modeled as the clamped-free-free-free rectangular plates, or/and as the discrete mass- spring sub-system. The motion of liquid in every single ellipsoidal cavity is modeled as the uniform vortex motion with a finite number of degrees of freedom. Assuming that stationary holonomic constraints imposed on the body allow its rotation about a spatially fixed axis, the equation of motion for such a systematic configuration can be very complex. It consists of a set of ordinary differential equations for the motion of the rigid body, the uniform rotation of the contained liquids, the motion of discrete elastic parts, and a set of partial differential equations for the elastic appendages supplemented by appropriate initial and boundary conditions. In addition, for such a hybrid system, under suitable assumptions, their equations of motion have four types of first integrals, *i.e.*, energy and area, Helmholtz' constancy of liquid- vortexes, and the constant of the Poisson equation of motion. Chetaev's effective method for constructing Liapunov functions in the form of a set of first integrals of the equations of the perturbed motion is employed to investigate the sufficient stability conditions of steady motions of the complete system in the sense of Liapunov, *i.e.*, with respect to the variables determining the motion of the solid body and to some quantities which define integrally the motion of flexible appendages. Thesė sufficient conditions take into account the vortexes of the contained liquids, the vibration of the flexible components, and the coupling among the liquid-elasticity-solid.

1. INTRODUCTION

It has been well-known that some of the modern spacecraft is composed of a rigid body, liquid and elastic components. Broadly speaking, the coupling among them makes the spacecraft attitude dynamics very complex. The equations governing such a systematic configuration, under suitable assumptions, consist of a set of ordinary differential equations for the rigid body and a set of partial differential equations for the contained liquid and elastic appendages supplemented by appropriate initial and boundary conditions., This kind of coupling system has an infinite number of degrees of freedom.

In 1958 the U.S. Satellite Explore I tumbled after only a few hours of flight. It was concluded that the four turnstile wire antennae were dissipating energy, causing a transfer of body spin axis from the axis of minimum inertia to a transverse axis of maximum inertia. Recently the anomalies in spinning rockets and spacecraft carrying liquid loads have been reported. The main features of these anomalies are the instability of the spacecraft attitude dynamics which manifests as exponential growths in nutation, liquid oscillations or even the control torque.

The dynamics of the liquid-filled rigid body with flexible appendages is one of the pressing difficult problems in the field of space sophisticated technology. There has been a lot of literature which is devoted to these difficult problems concerning the control and stability of the rigid spacecraft with flexible appendages. The papers or books written by Meirovitch (1970), Morozov *et al.* (1973), Kane *et al.* (1983), Posbergh *et al.* (1987), Junkins *et al.* (1991, 1993), Matsuno *et al.* (1996), should provide copious useful information. There has also been lots of literature on the problems concerning the dynamics of contained rotating liquids, such as the papers written by Rumyantsev (1964), Pfeiffer (1977), Agrawal (1982), Or *et. al.* (1994), Kuang (1992), Kuang *et al.* (1994a,b, 1997). Rubanovskii (1982) studied the stability of steady motions of a rigid body with an elastic shell partially filled liquid. The sufficient conditions of stability are derived from the solution of the problem dealing with the minimum problem of the changed potential energy of the system by studying the second variation. Wang & Kuang (1993) investigated the nonlinear stability conditions of steady motions of liquid-filled rigid body with linear flexible shear beams using "the Energy Casimir" method. Rumyantsev (1995) compared the three methods of constructing Lyapunov functions for the system with a finite number of degrees of freedom. The three methods are: 1) Chetaev's method developed since the 1950's in Russia: 2) the Energy-Casimir method developed in the 1980's in U. S. A.: 3) the Energy-Momentum method employed for Hamiltonian system (Holm *et al.* 1985). The research showed that Chetaev's method is the most generalized method which can theoretically be used to investigate the stability problem of the perturbed motion of the nonlinear dynamical system including the system defined by the distributed parameters.

In this paper Chetaev's effective method for constructing Lyapunov functions in the form of a set of first integrals of the equations of the perturbed motion is employed to investigate the sufficient stability conditions of steady motions of the complete system in the sense of Lyapunov, *i.e.*, with respect to the variables determining the motion of the solid body and to some quantities which define integrally the motion of flexible appendages.

Assuming that the shape of liquid tanks is ellipsoidal and that the completely filled liquids are inviscid and incompressible, the motion of contained liquids can be simplified as the uniform vortex motion, which makes the problem of infinite degrees of freedom of the contained liquids changed into the problem of finite degrees of freedom. In addition, the elastic solar array paddle is simplified as a clamped-free-free-free rectangular plate (Matsuno *et al.* 1996), the nutational dampers are modeled as the mass-spring oscillators, and the multi-liquid-filled cavities are located symmetrically and off-centrally.

2. PROBLEM FORMULATION AND FIRST INTEGRALS

Let $O_{\xi\eta\zeta}$ be the space-fixed system of coordinates; O_{xyz} be the body-fixed system of coordinates and be along the principal axes of the ellipsoidal tank; Let a_j, b_j, c_j be the three semi-axis length of the ellipsoid; the point O be the center of mass of the system; $\gamma_1, \gamma_2, \gamma_3$ be cosines of angles formed by axis η with axes x, y, z; $\vec{\omega}$ $(\omega_1, \omega_2, \omega_3)$ be the angular velocity vector of the rigid body; Ω_j $(\Omega_{j1}, \Omega_{j2}, \Omega_{j3})$ be the uniform vortex vector of the contained liquid in the cavity-j $(j = 1, 2, ..., n)$; I_1, I_2, I_3 denote the sums of the moments of inertia of the centered rigid body and of the Zhukovsky equivalent solid bodies of the contained liquids in all ellipsoidal tanks; I_{j1}, I_{j2}, I_{j3} denote the differences between the moments of inertia of the "consolidated" liquid and of Zhukovsky equivalent solid body of the contained liquid in the ellipsoidal tank-j. Let x_j, y_j, z_j be the coordinates of the center of mass of liquid contained in the tank-j in the body-fixed system of coordinates. Let A_1, B_1, C_1 be the moments of inertia of the centered rigid body with respect to the body axis, respectively. Let A_j^*, B_j^*, C_j^* be the moments of inertia of the Zhukovsky equivalent solid body of the liquid contained in the ellipsoidal tank-j. Then

$$
\begin{aligned}
I_1 &= A_1 + \sum_{j=1}^{n} A_j^* \\
I_2 &= B_1 + \sum_{j=1}^{n} B_j^* \\
I_3 &= C_1 + \sum_{j=1}^{n} C_j^* \\
A_j^* &= \frac{M_a}{5}\frac{(b_j^2 - c_j^2)^2}{b_j^2 + c_j^2} + M_a(y_j^2 + z_j^2) \\
A_j^* &= \frac{M_a}{5}\frac{(c_j^2 - a_j^2)^2}{c_j^2 + a_j^2} + M_a(z_j^2 + x_j^2) \\
A_j^* &= \frac{M_a}{5}\frac{(a_j^2 - b_j^2)^2}{a_j^2 + b_j^2} + M_a(x_j^2 + y_j^2) \\
I_{j1} &= 0.8M_a\frac{b_j^2 c_j^2}{b_j^2 + c_j^2}
\end{aligned}
$$

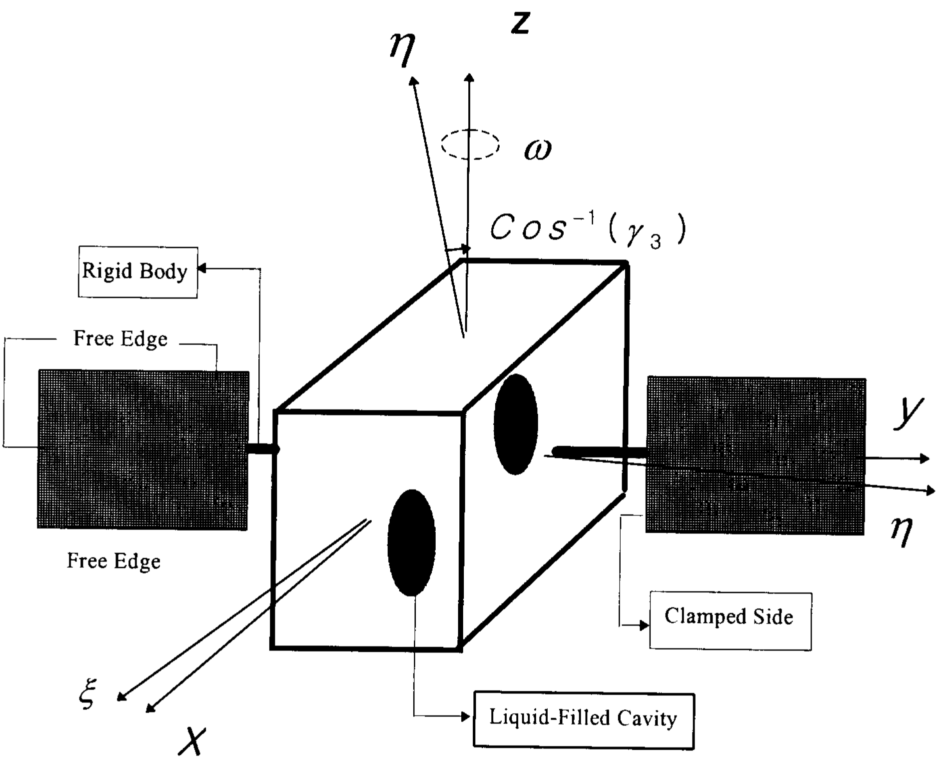

Figure 1. The schematic configuration of the liquid-filled rigid body with flexible appendages. (The discrete oscillators are not shown in here).

$$
\begin{aligned}
I_{j2} &= 0.8M_a\frac{a_j^2c_j^2}{a_j^2+c_j^2} \\
I_{j3} &= 0.8M_a\frac{a_j^2b_j^2}{a_j^2+b_j^2},
\end{aligned}
$$

where $M_a = 4\pi\rho a_j b_j c_j/3$, in here ρ is the density of the liquid contained in the ellipsoidal tank-j.

Let the length of solar array paddle be L_1, the width be L_2, the thickness be C, the distance between the axis O_z and the clamped side of the solar array paddle be d; the components of the elastic deformation vector of the plate be $u(y,z,t), v(y,z,t), w(y,z,t)$, where $-0.5L_2 \leq z \leq 0.5L_2$ and $d \leq |y| \leq d + L_2$; the mass density per unit volume be ρ; the Poisson's ratio be ν; the modulus of elasticity be E, in here, it is also assumed that the nominal positions of the neutral surface of solar arrays under the undeformable states are along O_{xy} plane of coordinates. Let the mass of the discrete oscillator-q be $M_q(q = 1, 2, ..., N)$; the relative equilibrium point of oscillator-q be r_{q1}, r_{q2}, r_{q3}; the stiffness coefficients of the oscillator-q be K_{q1}, K_{q2} and K_{q3} along x, y and z-axis, respectively; the components of the relative displacement vector of the oscillator be u_{q1}, u_{q2}, u_{q3} away from the relative equilibrium point of the oscillator-q along x, y and z, respectively. The coordinates are shown in Figure 1.

Throughout the paper we assume that the complete assemblage is moving freely in space, in the absence of external forces or torque. It is also assumed that the center of mass of the system is inertially fixed at the origin, and the deflections of the appendages away from the nominal are sufficiently small that the center of mass of the system has negligible motion.

We denote the absolute position vector, $\vec{R}$, relative to axes $\xi\eta\zeta$ of an element $\rho dxdydz$ in the elastic plate by

$$\vec{R} = [u(y, z, t) + x]\vec{e_1} + [v(y, z, t) + y]\vec{e_2} + [w(y, z, t) + z]\vec{e_3} \tag{1}$$

where $\vec{e_1}, \vec{e_2}, \vec{e_3}$ are unit vectors along axes x, y, z, respectively. In the same way the absolute position of oscillators are given by the vectors:

$$\vec{R} = [u_{q1} + r_{q1}]\vec{e_1} + [u_{q2} + r_{q2}]\vec{e_2} + [u_{q3} + r_{q3}]\vec{e_3} \tag{2}$$

By the above-mentioned definition, the axes xyz rotate with angular velocity $\vec{\omega}$ relative to inertial space $O_{\xi\eta\zeta}$, so we have:

$$\vec{\omega} = \omega_1\vec{e_1} + \omega_2\vec{e_2} + \omega_3\vec{e_3} \tag{3}$$

and the vector of cosines of angles formed by axis ζ with axes xyz can be described as

$$\vec{\gamma} = \gamma_1\vec{e_1} + \gamma_2\vec{e_2} + \gamma_3\vec{e_3} \tag{4}$$

Assuming that stationary holonomic constraints imposed on the body allow its rotation about the spatially fixed axis O_ζ, the equations of motion for such a system can be derived from the Hamilton-Ostrogradskii principle. Neglecting the damping of the oscillators, the equations of motion of the system have integrals of energy and areas, and the first integral of equations of motion of the contained liquids as well as the first integral of the Poisson equation. The existence of the first integrals is very important in using the Chetaev method under consideration.

The first integral of energy:

$$V_1 = \frac{1}{2}\left[\sum_{i=1}^{3}\left(\frac{m_i^2}{l_i} + \sum_{j=1}^{n}\frac{G_{ji}^2}{l_{ji}}\right)\right] + K_d + V_e + \iiint_{V_0} K_p\rho dxdydz = \text{constant}, \tag{5}$$

where the domain V_0 : $-0.5c \le x \le 0.5c$, $d \le |y| \le d + L_1$, $-0.5L_2 \le z \le 0.5L_2$, and K_d is the kinetic energy of the oscillators:

$$K_d = \frac{1}{2}\sum_{q=1}^{2} M_q \left[\frac{\partial \vec{R}_q}{\partial t} + \vec{\omega} \times \vec{R}_q\right]^2 \tag{6}$$

and K_p is the density function of kinetic energy of the elastic solar array paddle,

$$K_p = \frac{1}{2}\left[\frac{\partial \vec{R}}{\partial t} + \vec{\omega} \times \vec{R}\right]^2 \tag{7}$$

and V_e is equal to the potential energy of the elastic solar array paddle plus the potential energy of the oscillators, *i.e.*,

$$\begin{aligned} V_e &= \frac{Ec}{e(1-\nu^2)} \iint_S F_1(y,z)dydz + \frac{Ec^3}{24(1-\nu^2)} \iint_S F_2(y,z)dydz \\ &+ \frac{1}{2}\sum_{q=1}^{N}(K_{q1}u_{q1}^2 + K_{q2}u_{q2}^2 + K_{q3}u_{q3}^2) = \text{constant}, \end{aligned} \tag{8}$$

where

$$F_1(y,z) = \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + 2\nu\frac{\partial v}{\partial y}\frac{\partial w}{\partial z} + 2(1-\nu)\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

$$F_2(y,z) = \left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \left(\frac{\partial^2 u}{\partial z^2}\right)^2 + 2\nu\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial z^2} + 2(1-\nu)\left(\frac{\partial^2 u}{\partial y \partial z}\right)^2$$

and $S : d \le |y| \le d + L_1$, and $-0.5L_2 \le z \le 0.5L_2$. In here the solar arrays are assumed as a clamped-free-free-free flexible plate whose boundary conditions can be found in Matsuno *et al.* (1996).

The first integral of area:

$$V_2 = \sum_{i=1}^{3}\left[m_i + \sum_{j=1}^{N} G_{ji} + \frac{\partial K_d}{\partial \omega_i} + \iiint_{V_0}\left(\frac{\partial K_p}{\partial \omega_i}\right)\rho dxdydz\right]\gamma_i = \text{constant}, \tag{9}$$

where

$$\begin{aligned} m_1 &= l_1\omega_1 \quad & m_2 &= l_2\omega_2 \quad & m_3 &= l_3\omega_3 \\ G_{j1} &= l_{j1}\Omega_{j1} \quad & G_{j2} &= l_{j2}\Omega_{j2} \quad & G_{j3} &= l_{j3}\Omega_{j3} \end{aligned}$$

The first integral of the Poisson equation:

$$V_3 = \gamma_1^2 + \gamma_2^2 + \gamma_3^2 - 1 = \text{constant} \tag{10}$$

The first integral of the uniform vortex of the contained liquid in the ellipsoidal cavity-j:

$$V_{4j} = A_{j1}G_{j1}^2 + A_{j2}G_{j2}^2 + A_{j3}G_{j3}^2 = \text{constant} \qquad (j = 1, 2, ..., n), \tag{11}$$

where

$$A_{j1} = \left(\frac{b_j c_j}{I_{j1}}\right)^2 \qquad A_{j2} = \left(\frac{a_j c_j}{I_{j2}}\right)^2 \qquad A_{j3} = \left(\frac{a_j b_j}{I_{j3}}\right)^2 \tag{12}$$

3. STABILITY ANALYSIS

The equations of motion of the system have the particular solution:

$$\begin{aligned} \vec{m}_e &= (0, 0, I_3\omega)^T \\ \vec{\gamma}_e &= (0, 0, 1)^T \\ \vec{G}_{je} &= (0, 0, I_{j3}\Omega_j)^T \qquad (j = 1, 2, ..., n) \\ u_{q1} &= u_{q2} = u_{q3} = 0 \qquad (q = 1, 2, ..., n) \\ u &= v = w = 0 \end{aligned} \tag{13}$$

describing the uniform rotation of the solid-liquid-elasticity system about the z-axis coinciding with the fixed ζ axis, and a relative elliptical rotation around the same axis, where the magnitudes ω and Ω_j may have arbitrary values. In what follows we shall restrict ourselves precisely to the case $\varepsilon \leq \Omega_j \leq \omega$, here ε is infinitesimally small.

Let us assume that the particular solution (13) is the unperturbed motion and analyze its stability with respect to variables $m_i, G_{ji}, \gamma_i, u_{qi}, u, v, w (i = 1, 2, 3; j = 1, 2, ..., n; q = 1, 2, .., N)$.

Assuming that in the perturbed motion

$$\begin{aligned} m_3 &= m_{3p} + I_3\omega \\ \gamma_3 &= \gamma_{3p} + 1 \\ G_{j3} &= G_{j3p} + I_{j3}\Omega_3 \qquad (j = 1, 2, ..., n) \end{aligned}$$

and retaining the previous designations for the rest of variables. It is obvious that in the general case the equations of the perturbed motion will possess the first integrals (8)-(9).

In order to construct Liapunov's functions, Chetaev's method is applied. Let us analyze the function:

$$V = 2V_1 + \alpha V_2 + \mu V_3 + \sum_{j=1}^{n} \lambda_j V_{4j} + 6J_3\omega^2 \tag{14}$$

where $\alpha, \mu, \lambda_j (j = 1, 2, ..., n)$ are constants to be determined, J_3 is the moment of inertia of flexible appendages about the axis O_z under the undeformed state.

Substituting the particular solution (13) into equation (14), the V-functions are reduced to:

$$\begin{aligned} V(\text{in the equilibrium state}) &= 6J_3\omega^2 + \left[\frac{\alpha}{2}+\omega\right]^2 \\ &+ \sum_{j=1}^{n} \frac{1}{1/I_{j3}+\lambda_j A_{j3}} \left[I_{j3}\Omega_j + \frac{\alpha}{2(1/I_{j3}+\lambda_j A_{j3})}\right]^2 \\ &+ \left[\mu - (I_3+J_3)\frac{\alpha^2}{4} - \sum_{j=1} n \frac{\alpha^2}{4(1/I_{j3}+\lambda_j A_{j3})}\right] \end{aligned} \tag{15}$$

Setting the terms inside each of the square brackets in equation (15) to be zero, we can obtain the constants $\alpha, \mu, \lambda_j (j = 1, 2, ..., n)$ as follows:

$$\alpha = -2\omega; \quad \mu = \left[(I_3+J_3)\omega + \sum_{j=1}^{n} I_{j3}\Omega_j\right]\omega; \quad \lambda_j = \frac{(\omega-\Omega_j)I_{j3}}{\Omega_j a_j^2 b_j^2} \tag{16}$$

Through analyzing the V-function (14), it can be changed into the form:

$$\begin{aligned} V &= \frac{m_1^2}{I_1} + \alpha m_1\gamma_1 + \sum_{j=1}^{n}\left[(\frac{1}{I_{j1}} + \lambda_j A_{j1})G_{j1}^2 + \alpha G_{j1}\gamma_1\right] + \kappa_1\gamma_1^2 \\ &+ \frac{m_2^2}{I_2} + \alpha m_2\gamma_2 + \sum_{j=1}^{n}\left[(\frac{1}{I_{j2}} + \lambda_j A_{j2})G_{j2}^2 + \alpha G_{j2}\gamma_2\right] + \kappa_2\gamma_2^2 \\ &+ \frac{m_3^2}{I_3} + \alpha m_3\gamma_3 + \sum_{j=1}^{n}\left[(\frac{1}{I_{j3}} + \lambda_j A_{j3})G_{j3}^2 + \alpha G_{j3}\gamma_3\right] + \kappa_3\gamma_3^2 \\ &+ \iiint_{V_0}\left[\frac{\partial\vec{R}}{\partial t} + \vec{\omega}\times\vec{R} - \frac{\alpha}{2}\left(\vec{\gamma}\times\vec{R}\right)\right]^2 \rho dxdydz \\ &+ \sum_{q=1}^{N} M_q\left[\frac{\partial\vec{R}_q}{\partial t} + \vec{\omega}\times\vec{R}_q - \frac{\alpha}{2}\left(\vec{\gamma}\times\vec{R}_q\right)\right]^2 + F \end{aligned} \tag{17}$$

Here the function F is:

$$\begin{aligned} F &= (\mu-\kappa_1)\gamma_1^2 + (\mu-\kappa_2)\gamma_2^2 + V_e + 6J_3\omega^2 \\ &- \iiint_{V_0}\left[\frac{\alpha}{2}\left(\vec{\gamma}\times\vec{R}\right)\right]^2 \rho dxdydz - \sum_{q=1}^{N} M_q\left[\frac{\alpha}{2}\left(\vec{\gamma}\times\vec{R}_q\right)\right]^2 \\ \kappa_1 &= I_1\omega^2 + \sum_{j=1}^{n}\frac{I_{j1}^2\omega^2}{I_{j1}+\lambda_j c_j^2 b_j^2} \end{aligned} \tag{18}$$

$$\kappa_2 = I_2\omega^2 + \sum_{j=1}^{n} \frac{I_{j2}^2\omega^2}{I_{j2} + \lambda_j c_j^2 a_j^2}$$

According to the Rayleigh quotient theory the following inequality holds:

$$V_e \geq \iiint_{V_0} \left[\Lambda_p^2(v^2 + w^2) + \Lambda_b^2 u^2\right] \rho dxdydz + \sum_{q=1}^{N} M_q(\Lambda_{q1}^2 u_{q1}^2 + \Lambda_{q2}^2 u_{q2}^2 + \Lambda_{q3}^2 u_{q3}^2), \quad (19)$$

where Λ_{qi} denotes the eigenvalue associated with the vibration u_{qi}, and Λ_q denotes the lowest eigenvalue associated with in-plane vibrations v and w of the thin-plate, and Λ_b denotes the lowest eigenvalue associated with the out-of-plane vibration u of the thin-plate.

By appropriately analyzing, V-function stands for the sum of five quadratic forms except function F. Using the energy inequality (19) and the inequality $(A - B)^2 \leq 2(A^2 + B^2)$, and omitting terms of the third and higher with respect to the variables $\frac{\partial \vec{R}}{\partial t}, \vec{\gamma}, \vec{R}, \frac{\partial \vec{R}_q}{\partial t}, \vec{\omega}, \vec{R}_q$, the following inequality can be obtained:

$$\begin{aligned} F &\geq (\mu + 2J_3\omega^2 - \kappa_1 - 2J_1\omega^2)\gamma_1^2 + (\mu + 2J_3\omega^2 - \kappa_2 - 2J_2\omega^2)\gamma_2^2 \\ &+ \iiint_{V_0} \left[(\Lambda_b^2 - 3\omega^2)u^2 + (\Lambda_p^2 - 3\omega^2)v^2 + \Lambda_p^2 w^2\right] \rho dxdydz \\ &+ \sum_{q=1}^{N} M_q \left[(\Lambda_{q1}^2 - 3\omega^2)u_{q1}^2 + (\Lambda_{q2}^2 - 3\omega^2)u_{q1}^2 + \Lambda_{q3}^2 u_{q3}^2\right] \\ &+ \iiint_{V_0} \omega^2 \left[(u - 2x)^2 + (v - 2y)^2\right] \rho dxdydz \\ &+ \sum_{q=1}^{N} \omega^2 M_q \left[(u_{q1} - 2r_{q1})^2 + (u_{q2} - 2r_{q2})^2\right] + \ldots \end{aligned} \quad (20)$$

Here J_1, J_2, J_3 are the moments of inertia of the undeformed flexible appendages about O_x, O_y, O_z axes, respectively, *i.e.*,

$$\begin{aligned} J_1 &= \iiint_{V_0} (z^2 + y^2)\rho dxdydz + \sum_{q=1}^{N}(r_{q3}^2 + r_{q2}^2) \\ J_2 &= \iiint_{V_0} (z^2 + x^2)\rho dxdydz + \sum_{q=1}^{N}(r_{q3}^2 + r_{q1}^2) \\ J_3 &= \iiint_{V_0} (x^2 + y^2)\rho dxdydz + \sum_{q=1}^{N}(r_{q1}^2 + r_{q2}^2) \end{aligned}$$

By using the principles of Liapunov stability theory with respect to a part of the variables:

$$n_j, \gamma_j, G_{ji}, \quad \iiint_{V_0} \left[\frac{\partial \vec{R}}{\partial t} + \vec{\omega} \times \vec{R} - \frac{\alpha}{2}\left(\vec{\gamma} \times \vec{\omega}\right)\right] \cdot \vec{e}_j \rho dx dy dz,$$
$$M_q \left[\frac{\partial \vec{R}_q}{\partial t} + \vec{\omega} \times \vec{R}_q - \frac{1}{2}\left(\vec{\gamma} \times \vec{R}_q\right)\right] \cdot \vec{e}_j, \quad \iiint_{V_0} (u - 2x) \rho dx dy dz,$$
$$\iiint_{V_0} (v - 2y) \rho dx dy dz, \quad \iiint_{V_0} w \rho dx dy dz, \quad M_q(u_{q1} - 2r_{q1}),$$
$$M_q(u_{q2} - 2r_{q2}), \quad M_q u_{q3} \quad (j = 1, 2, 3; q = 1, 2, ..., N)$$

the sufficient conditions of stability of the particular solution (13) are the following inequalities:

$$\begin{aligned} \mu + 2J_3\omega^2 - \kappa_1 - 2J_1\omega^2 > 0 \\ \mu + 2J_3\omega^2 - \kappa_2 - 2J_2\omega^2 > 0 \end{aligned} \tag{21}$$

$$\begin{aligned} \Lambda_b > \sqrt{3}\omega; \quad \Lambda_p > \sqrt{3}\omega \\ \Lambda_{q1} > \sqrt{3}\omega; \quad \Lambda_{q2} > \sqrt{3}\omega \end{aligned} \tag{22}$$

Note that (21) and (22) are similar to the stability criteria of the rigid body with multi-liquid-filled cavities:

$$\mu - \kappa_1 > 0; \quad \mu - \kappa_2 > 0, \tag{23}$$

therefore, it is concluded that the self-spinning stability criteria of the rigid body with multi-liquid-filled cavities are the extension of the self-spinning stability criteria of the rigid body with single-liquid-filled cavities (Rumiantsev 1995, Kuang 1992,1994). The equations (23) and (24) state that the angular frequency of the body should not exceed the characteristic oscillating frequencies of the flexible appendages, *i.e.*:

$$\omega \leq \min\{\frac{1}{\sqrt{3}}\Lambda_b, \frac{1}{\sqrt{3}}\Lambda_p, \frac{1}{\sqrt{3}}\Lambda_{q1}, \frac{1}{\sqrt{3}}\Lambda_{q2}\} \tag{24}$$

4. CONCLUSIONS

This paper presents a Liapunov stability theory applicable to dynamical systems characterized by simultaneous discrete and distributed parameters. The Chetaev's effective method is introduced to investigate the stability criteria of the perturbed motion. By invoking the constants of the first integrals of the researched system and making use of certain properties of Rayleigh's quotient with respect to the oscillation theory of flexible appendages, the appropriate quadratic forms in V-function with respect to the perturbed attitude components are constructed under certain circumstances. The constructed quadratic forms can be used in conjunction with the newly formulated stability theory about a part of the variables to predict the self-spinning stability of the multi-liquid-filled rigid body with multi-elastic appendages.

The results worked out here using Chetaev's method can also be obtained by means of "Energy-Casimir" method developed by Arnold (1978), Marsden (1992), Aeyels (1992), and *et al.* In order to establish rigorous nonlinear stability criteria using "Energy-Casimir" method, certain convexity estimates must be carried out. Chetaev method is easier to understand and master than the "Energy-Casimir" method.

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