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암반절리에 대한 교란상태 모델링 (이론과 응용)

박인준[1)] · 전석원[2)]

Disturbed State Modeling for Joints of Rock (Theory and Implementation)

Inn-Joon Park and Seok-Won Jeon

ABSTRACT This research is intended to investigate the behavior of the jointed rock under various loading conditions: static or dynamic load. The distributed state concept (DSC) is based on the idea that the response of the joint can be related to and expressed as the response of the reference states: relative intact (RI) and fully adjusted (FA) states. In the DSC, an initially RI joint modifies continuously through a process of natural self-adjustment, and a part of it approaches the FA state at randomly disturbed locations in the joint areas. In this study, based on the DSC concept, RI state, FA state, and disturbance function (D) are defined for characterizing the behavior of rock joint. From the results of this research, it can be stated that DSC model is capable of capturing the physical behavior of jointed rock such as softening and hardening and considering the size of joint and roughness of joint surface.

Key words : back analysis, constitutive relations, dynamics, joint, numerical modeling

초 록 외부로부터 정적 혹은 동적 하중을 받는 암반절리의 거동특성을 규명하기 위해서 교란상태 개념(Disturbed State Concept, DSC)을 이용한 구성방정식 이론과 이 이론을 수치해석에 적용하기 위한 응력-변형률 관계식을 소개한다. 본 연구에서 제안한 DSC 이론은 변형중인 암반절리가 상대적으로 손상되지 않은 상태(Relative Intact; RI)와 완전 파괴된 상태(Fully Adjusted; FA)의 혼합으로 표현될 수 있다는 가정에 기초를 두고 있다. 여기서 사용된 두가지 상태, 즉 RI 상태와 FA 상태는 암반절리의 파괴정도를 나타내는 지표가 된다. 이러한 가정을 기초로 임의의 하중을 받는 절리는 초기 RI 상태에서 점진적으로 재료 내부의 미세구조 조정기능을 거치면서 최종적으로 파괴가 발생하는 FA 상태로 진행한다. 본 연구에서는 RI 상태, FA 상태 그리고 재료의 파괴정도를 나타내는 교란도 함수(D)를 해석적으로 정의하여 암반절리의 역학적 거동특성을 표현하기 위한 응용화를 시도하였다. DSC 모델은 암반절리의 경화 및 연화특성을 표현할 수 있으며, 절리의 크기 및 표면의 거칠도 등을 고려할 수 있다.

핵심어 : 역해석, 구성방정식, 동력학, 절리, 수치모델링

1. Introduction

Joint of rock has been the topic of extensive laboratory researches for a long time (Amonton, 1699; Coulomb, 1785). As results, several models have been developed for defining the behavior of joint. There are two types of joint models : failure model and elasto-plastic model.

1.1 Failure models

Failure models describe the shear stress in relation to the normal stress and other parameters. This relation is generally nonlinear as long as the range of the normal stress is wide enough. Usually, the shear stress reaches a peak value and then decreases to a residual value. This phenomenon is termed softening as found in most rocks (Goodman, 1974; Hoek and Bray, 1974). In failure models, emphasis is focused mainly on the modeling of peak and residual shear stresses.

1) 정회원, 연세대학교 공과대학 토목공학과 박사후 연수과정
2) 정회원, 서울대학교 공과대학 지구환경시스템공학부 전임강사

Amonton(1699) proposed the earliest friction law, that is, the friction stress, τ, is proportional to the normal stress, σ_n, applied;

$$\tau = \mu \sigma_n \quad (1)$$

where μ is the proportionality constant termed friction coefficient, which is also expressed as :

$$\mu = \tan \phi \quad (2)$$

where ϕ is called the friction angle of the surface. Coulomb's law(1785) gives a better description of friction by introducing a cohesion interception c. Patton(1966) proposed a model based on a series of tests on the "saw-tooth" shaped artificial joints. The peak shear strength is assumed bilinear. Barton and Chouby(1977) proposed a model for peak shear strength of rock joints after summarizing extensive tests upon specially prepared artificial rock joints. The peak shear strength was expressed as

$$\tau_p = \sigma_n \tan\left[\mathrm{JRC}\log\left(\frac{\mathrm{JCS}}{\sigma_n}\right) + \phi_b\right] \quad (3)$$

where JRC and JCS are the Joint Roughness Coefficient and Joint Compression Strength respectively, and ϕ_b is the residual friction angle. Schneider(1975) modified Patton's bilinear model by combining the angle of asperities of natural rock joints.

Most failure models described above attempt to relate the shear strength of a rough joint with the slope angle or the shear strength with the strength of the asperities. However, failure models do not give the description of the stress-strain relationship which is necessary for calculations involved with displacements other than the strength of the joints. The elastic and elasto-plastic models are capable of providing the stress-strain relationship as discussed below.

1.2 Elastic and elasto-plastic models

Goodman, Taylor, and Brekke(1971) proposed a nonlinear model for rock joints. This model has the off-diagonal terms of the stiffness matrix which are considered as the coupling terms between the shear and normal behavior. Ghaboussi and Wilson (1973) proposed the possible application of the plasticity theory in joint modeling by assuming the associative flow rule. The yield functions used are the Mohr-Coulomb failure law for non-dilatant joint, and the Cap (DiMaggio and Sandler, 1971) model of yield functions for dilatant joint. Zienkiwicz *et al.* (1977) proposed an elastic-viscoplastic model for joint. The yield function F used is the Mohr-Coulomb failure law. Both associative and non-associative potential function Q has a similar type as the yield function. Plesha (1987) proposed a non-associative plasticity joint model. The main feature of this model is to use a parameter called the asperity angle to characterize the strength and deformation behavior of the joint. At the same time, Desai and Fishman(1987) developed a non-associative plasticity model by specializing a general 3-D Hierarchical Single Surface model (HiSS Model) (Desai *et al.*, 1984, 1986). The yield function F and the displacement potential function Q are expressed as

$$F = \tau^2 + \alpha \sigma_n^n - \gamma \sigma_n^2 = 0 \quad (4)$$

$$Q = \tau^2 + \alpha_Q \sigma_n^n - \gamma \sigma_n^2 = 0 \quad (5)$$

where n and γ are material constants, α is the hardening function, and α_Q is the non-associative hardening function. This model can be used for both quasi-static and cyclic loading conditions. However, softening can not be captured.

In this research, a modified version of Disturbed State Concept (DSC) model (Desai, 1992; Desai, 1995; Rigby and Desai, 1995; Park, 1997) is proposed to model both hardening and softening behavior with a framework that can include a number of important characteristics of joints.

2. Disturbed sate concept modeling for joint

The disturbed state concept (DSC) extends continuum theory representations of material behavior to include observed nonhomogeneous and discontinuous behavior such as microcracking, damage, and softening. It is based on the DSC that allows incorporation of microstructural changes due to the applied forces, that cause transitions in the material from relative intact (RI) state, through a process of natural self adjustment, to the fully adjusted or critical (FA) state. The process of transition from the RI to FA state involves changes in the microstructural properties of the joint material, affected by factors such as roughness, asperities, particle size and shape, and inter-particle characteristics. The observed material behavior is thus defined as a combination of the two material reference states, RI state and FA state, which are related through the disturbance function, D (Fig. 1). The concept of the disturbed state of a joint can be expressed by the two reference states (RI and FA) and D.

The disturbed state for a joint is the intermediate state from the original state until the critical state is researched. During the disturbed state, the damageable material and non-damageable material co-exist. From the DSC theory for a joint material (Desai, 1995), the damageable material represents those asperities that are broken or lose contact during shearing, and those contacts that are separated by the debris. The non-damageable material represents those asperities that are not breakable for the given normal stress, and would include plateaux formed and compacted gouge material formed during shearing. Consequently, strain softening may result if the joint becomes smoother.

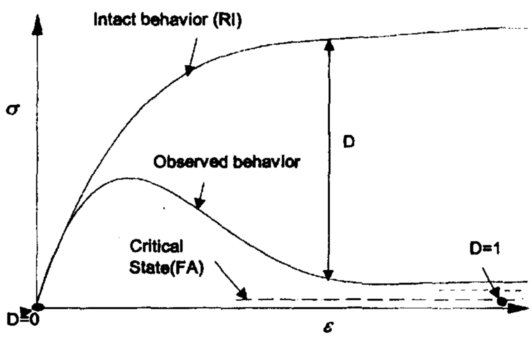

Fig. 1. Schematic of Stress-Strain Behavior (after Park, 1997)

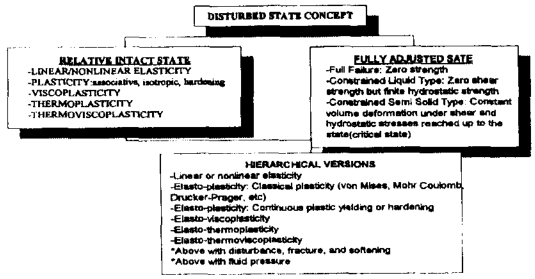

Fig. 2. A Variety of Versions of DSC Models (after Desai, 1992)

The RI state material of a joint can be defined as a contact that keeps its original contact property before being damaged or losing contact. The RI material is described using continuum models that may be based on linear or nonlinear elasticity, elastoplasticity, viscoplasticity, and thermovisco-plasticity (Fig. 2).

The FA state can be modelled assuming the material that carries hydrostatic stress and shear stress like a constrained viscous liquid. The joint is said to be at the FA state when, at large shear displacements, both the dilation and shear resistance of a joint reach their stabilized values under a certain normal load.

Incorporating the RI and FA states, the coupled (observed) response can model softening behavior, anisotropic effects, and progression of damage.

To best describe the various references states for jointed rock, a simple example is presented herein. Consider a bucket filled with ice. When heated, the ice will thaw into water. The ice represents the material in its original state or RI state, while the water represents the material in FA state, and heating is the factor that causes the disturbance, D. During the period starting from ice (RI state) and ending with water (FA state), there are many intermediate states where the container includes both ice and water. These intermediate states are said to be in the disturbed state. During the disturbed state, the ice gradually changes to water and there exists a mixture of ice and water.

2.1 Idealization of a joint

A joint is the region of two opposing surfaces of two contacting solids. The physical properties of a joint are determined by these two surfaces and their contact conditions. To mathematically model a joint or interface, the joint is usually considered as a planar surface with two joint walls and a contact space (Fig. 3). Here, the contact space is the contact zone of the opposing asperities and it can be assigned as averaged thickness t. A coordinate system can be established where the planar surface is considered; the tangential direction with shear stresses τ, and relative shear displacements u^r, and the direction orthogonal to the planar surface is the normal direction with normal stresses σ_n, and relative normal displacement v^r. If T and N are the tangential and normal forces applied (Fig. 3), and A_0 is the nominal joint area, then the normal and shear stresses are :

$$\tau = \frac{T}{A_0} \tag{6}$$

$$\sigma = \frac{N}{A_0} \tag{7}$$

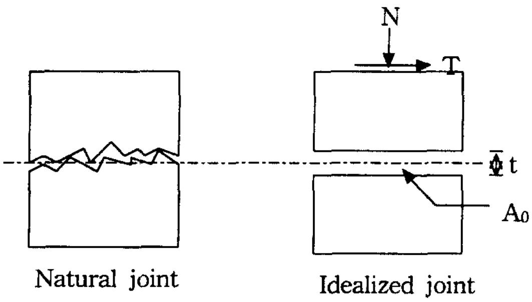

Fig. 3. Idealization of a Joint

The relative shear displacement u^r, in contact zone is composed of elastic shear deformation of the contact asperities u^e, plastic shear deformation of the contact asperities u^p, and slip displacement between the contact asperities of the joint, u^s

$$u^r = u^e + u^p + u^s \tag{8}$$

In an analogous manner, the relative normal displacement, v^r, can be defined as

$$v^r = v^e + v^p + v^s \tag{9}$$

If small strains are assumed, the joint thickness, t, can be used to convert relative displacements into equivalent strains. As $t \rightarrow 0$, the in-plane strain $\varepsilon_x \rightarrow 0$ and can be negligible (Sharma and Desai, 1992). In view of this, the in-plane stress, σ_x, will also be small and can be negligible, particularly when the Poisson's ratio, ν, is small. In terms of two-dimensional idealization, the strain-displacement relations are

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_n \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ v_r/t \\ u_r/t \end{pmatrix} \tag{10}$$

and the related stress components are

$$\begin{pmatrix} \sigma_x \\ \sigma_n \\ \tau \end{pmatrix} = \begin{pmatrix} 0 \\ N/A_0 \\ T/A_0 \end{pmatrix} \tag{11}$$

2.2 Description of the RI state

The relative intact state is described using the modified HiSS δ_0 model (Desai and Wathugala, 1987). The δ_0 model is based on the associative plasticity and isotropic hardening (potential function Q=yield function F) rule. In this model, the yield function, F, is given as :

$$F = \tau^2 + \alpha\sigma_n^n - \gamma\sigma_n^2 \tag{12}$$

where τ is the shear stress, σ_n is the normal stress, and n and γ are material parameters. α is the hardening function and it is expressed as

$$\alpha = \frac{a}{\xi_D^b} \tag{13}$$

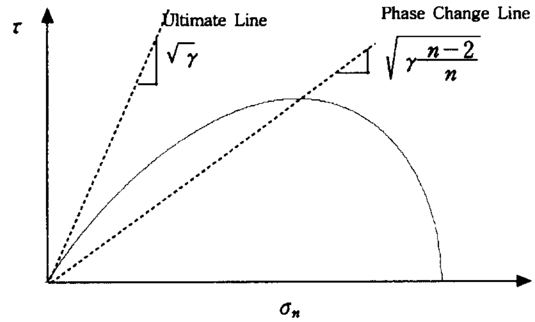

Fig. 4. The Yield Surface of HiSS δ_0 Model in $\sqrt{J_{2D}} - J_1$ Space

where a and b are material parameters and trajectory of plastic shear strain, ξ_D, is given as :

$$\xi_D = \int | d\gamma^p | \tag{14}$$

The yield surface F is a continuous set of convex surface which expands toward an ultimate yield surface during plastic shear deformation. The ultimate surface, τ_{ult}, which represents the asymptotic failure stress, is found by setting α equal to zero :

$$\tau_{ult} = \sqrt{\gamma}\sigma_n \tag{15}$$

This plots a straight line with slope $\sqrt{\gamma}$ as shown in Fig. 4. The locus of points expanding yield surface (where the tangent to the yield surface is parallel to the σ_n axis) is a line called the "Phase Change Line". By taking F=0 and $\frac{\partial F}{\partial \sigma_n} = 0$, Eq. (12) reduced to

$$\frac{\tau}{\sigma_n} = \sqrt{\gamma\left(\frac{n-2}{n}\right)} \tag{16}$$

The phase change line also plots as a straight line with slope $\sqrt{\gamma\frac{n-2}{n}}$ in τ vs. σ_n space (Fig. 4).

2.3 Description of the FA state

The critical state is a steady state where the shear stresses and normal displacement are stabilized. The joint model at the critical state consists of two parts : the modeling of the critical shear stress and the modeling of the critical dilation. The failure model proposed by Archard (1958) is a simple one yet it gives a very good description of the shear stress at the critical state. Archard's non-linear power law model can be expressed as follows :

$$\tau^c = C_0 \sigma_n^m \tag{17}$$

where C_0 and m are material parameters and the superscript c refers to the critical condition. And the final dilation at the critical state, v^c, is found to have a relation with the normal stress (Schneider, 1975), as

$$v^c = v^0 \exp(-k\sigma_n) \tag{18}$$

where v^0 is the maximum dilation when σ_n is equal to zero and k is a material parameter.

2.4 Description of DSC function

The disturbance function can be defined as

$$D = \frac{M_s^c}{M_s} \tag{19}$$

where M_s^c is the mass of solids in the FA state and M_s is the total mass of solids present. Initially with no disturbance the material is assumed to be entirely in the RI state, so D is zero. With full disturbance the material is assumed to be fully in FA state, and at an ultimate state, D_u. Theoretically, the disturbance, D, varies between 0 and 1, but many materials fail in an engineering sense before D reaches unity.

The proposed function for D (scalar) employed in this research was used by Armaleh and Desai (1990) :

$$D = D_u[1 - \exp(-A\xi_D^Z)] \tag{20}$$

where D_u is the ultimate disturbance and will be assumed to be unity for rock joint, A and Z are material parameters. This disturbance function will

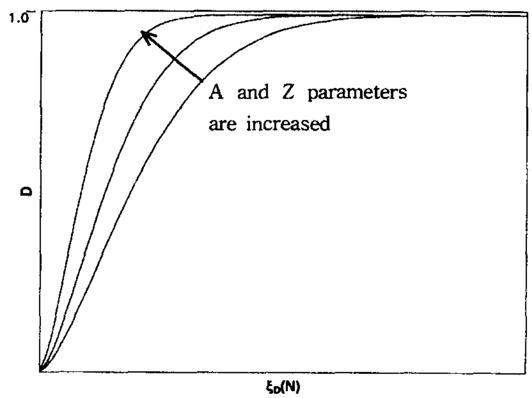

Fig. 5. Schematic of DSC Function (after Park, 1997)

be used twice to define a shear stress relationship using disturbance, D_τ, and an effective normal stress relationship using disturbance, D_n. Each curve in Fig. 5 is a representation of Eq. (20).

3. Incremental formulation for back prediction program

3.1 Derivation of the intact incremental stress-strain relation

Derivation of the intact incremental stress-strain relations follows the traditional elasto-plasticity formulation procedure (Desai and Wathugala, 1987). Let the following vectors be defined

$$\{d\sigma\} = \begin{Bmatrix} d\sigma_n \\ d\tau \end{Bmatrix} \tag{21}$$

$$\{d\varepsilon\} = \begin{Bmatrix} d\varepsilon_n \\ d\gamma \end{Bmatrix} \tag{22}$$

From the elastic stress-strain relationship and the flow rule for plasticity, an incremental form of the intact stress vector can be found as

$$\{d\sigma^i\} = [C^e](\{d\varepsilon\} - \lambda\{n^F\}) \tag{23}$$

where $[C^e]$ is the elastic constitutive matrix and is given by,

$$[C^e] = \begin{bmatrix} tk_n & 0 \\ 0 & tk_s \end{bmatrix} \tag{24}$$

where k_n and k_s are the normal and shear stiffness of the interface material. And employing the consistency condition of the yield function (dF =0), λ can be found as

$$\lambda = \frac{\left\{\frac{\partial F}{\partial \sigma^i}\right\}^T [C^e]\{d\varepsilon\}}{\left\{\frac{\partial F}{\partial \sigma^i}\right\}^T [C^e]\{n^F\} - \frac{\partial F}{\partial \alpha}} \tag{25}$$

and substituted into Eq. (23) to yield the constitutive relationship desired,

$$\{d\sigma^i\} = [C^{ep}]\{d\varepsilon\} \tag{26}$$

where the elasto-plastic matrix takes the form

$$[C^{ep}] = \left[[C^e] - \frac{[C^e]\left\{\frac{\partial F}{\partial \sigma^i}\right\}\{n^F\}^T[C^e]}{\left\{\frac{\partial F}{\partial \sigma^i}\right\}^T [C^e]\{n^F\} - \frac{\partial F}{\partial \alpha}L} \right] = \begin{bmatrix} C^{ep}_{nn} & C^{ep}_{ns} \\ C^{ep}_{sn} & C^{ep}_{ss} \end{bmatrix} \tag{27}$$

where L is defined as

$$L = \frac{\partial \alpha}{\partial \xi_D} < n_s^F > = -ab\,\xi_D^{-b-1} < n_s^F > \tag{28}$$

for the hardening function defined in Eq. (13) where < > are McAuley's brackets.

$$\begin{cases} < n_i^F > = 0 & \text{for } n_i^F \le 0 \\ < n_i^F > = n_i^F & \text{for } n_i^F > 0 \end{cases} \tag{29}$$

3.2 Derivation of the DSC incremental stress equation

Assuming the thickness of joint is the same for all three material phases, equilibrium of forces in the disturbed material, and the definition of disturbance in Eq. (19), the following relationship between phase stresses can be derived

$$\begin{Bmatrix} \sigma_n^a \\ \tau^a \end{Bmatrix} = \begin{Bmatrix} (1-D_n)\sigma_n^i \\ (1-D_\tau)\tau^i \end{Bmatrix} + \begin{Bmatrix} D_n\sigma_n^c \\ D_\tau\tau^c \end{Bmatrix} \tag{30}$$

where D_n and D_τ are defined using Eq. (20). The normal disturbance function, D_n, can be used to model the relative normal displacements and D_τ, is the disturbance function for relative shear displacements.

Differentiating Eq. (30),

$$\begin{Bmatrix} d\sigma_n^a \\ d\tau^a \end{Bmatrix} = \begin{Bmatrix} (1-D_n)\,d\sigma_n^i \\ (1-D_\tau)\,d\tau^i \end{Bmatrix} + \begin{Bmatrix} D_n\,d\sigma_n^c \\ D_\tau d\tau^c \end{Bmatrix} + \begin{Bmatrix} (\sigma_n^c-\sigma_n^i)\,dD_n \\ (\tau^c-\tau^i)\,dD_\tau \end{Bmatrix} \quad (31)$$

If there is no change in stresses at FA state, $d\sigma_n^c$ and $d\tau^c$ are zero. Substituting Eqs. (17), (26), and (27) into Eq. (31) gives the DSC incremental stress-strain equations,

$$\begin{Bmatrix} d\sigma_n \\ d\tau^a \end{Bmatrix} = \begin{Bmatrix} (1-D_n)(C_{nn}^{ep}d\varepsilon_n^i+C_{ns}^{ep}d\gamma) \\ (1-D_\tau)(C_{sn}^{ep}d\varepsilon_n^i+C_{ss}^{ep}d\gamma) \end{Bmatrix} + \begin{Bmatrix} (\sigma_n^c-\sigma_n^i)\,dD_n \\ (\tau^c-\tau^i)\,dD_\tau \end{Bmatrix} \quad (32)$$

and

$$\{d\sigma^a\}=[C^{DSC}]\{d\varepsilon\}+\{dD(\sigma^c-\sigma^i)\} \quad (33)$$

where the term $\{dD(\sigma^c-\sigma^i)\}$ contributes negative values during softening and $[C^{DSC}]$ is DSC constitutive matrix.

4. Implementation of the DSC joint model

With the advancement of numerical procedures such as the finite element method, rock joints have been represented by various types of two- and three-dimensional elements. In this research, the two-dimensional joint element is represented by thin-layer interface element (Desai *et al*, 1984) with four noded isoparametric solid element.

A constitutive model, such as the DSC joint model proposed above, can be adapted from Eq. (33) where $[C^{DSC*}]$ is the element DSC joint constitutive matrix at local coordinate level. Eq. (33) becomes

$$\{d\sigma^a\}=[C^{DSC*}]\{d\varepsilon\}+\{dD(\sigma^c-\sigma^i)\} \quad (34)$$

Formulation at the local coordinate system involves definition displacements, $\{u\}$, at any point within the element, as

$$\{\bar{u}\}=[N]\{\bar{q}\} \quad (35)$$

where $[N]$ is the shape function matrix and $\{\bar{q}\}$ is the vector of nodal displacements for the element. Using the isoparametric concept, the incremental strain vector can be written as

$$\{d\varepsilon\}=[B]\{dq\} \quad (36)$$

where $[B]$ is the incremental strain-nodal displacement transformation matrix for an element. It is obtained by differentiating and combining rows of the matrix $[N]$.

In many joint boundary problems, the local coordinate system of the joint element is inclined at an angle, ϕ, with respect to horizontal in the global system. The transformation of the element strain and stress vectors to global vectors (indicated by using an overbar) are given by,

$$\{d\bar{\varepsilon}\}=[\beta]\{d\varepsilon\} \quad (37)$$

$$\{d\bar{\sigma}\}=[\beta']\{d\sigma^a\} \quad (38)$$

$$[\beta]=\begin{bmatrix} 0 & \cos\phi\sin\phi & -\cos\phi\sin\phi \\ 0 & (\cos\phi)^2 & \cos\phi\sin\phi \\ 0 & -2\cos\phi\sin\phi & (\cos\phi)^2-(\sin\phi)^2 \end{bmatrix} \quad (39)$$

$$[\beta']=\begin{bmatrix} 0 & \cos\phi\sin\phi & -2\cos\phi\sin\phi \\ 0 & (\cos\phi)^2 & 2\cos\phi\sin\phi \\ 0 & -\cos\phi\sin\phi & (\cos\phi)^2-(\sin\phi)^2 \end{bmatrix} \quad (40)$$

Assume the continuum is discretized into an assemblage of "m" distinct finite elements. The basic finite element equilibrium equation is written as a sum of the integrations over each element and can be written as

Table 1. DSC Joint Parameters

Category	Parameter	Comments
RI Material	E	Young's Modulus
	ν	Poisson's Ratio
	n	Phase Change Parameter
	γ	Ultimate Parameter
	a	Hardening Parameter
	b	$\alpha = \frac{a}{\xi_D^b}$
FA Material	C_0	Critical Parameter
DSC Function	A_n, Z_n	$D_n = [1 - \exp(-A_n \xi_D^{Z_n})]$
	A_τ, Z_τ	$D_n = [1 - \exp(-A_\tau \xi_D^{Z_\tau})]$

$$\sum_m \left(\int_{V(m)} [B(m)]^T \{ d\overline{\sigma}^a_{(m)} \} dV_{(m)} \right) = \sum \{Q\} \quad (41)$$

where $\{d\overline{\sigma}^a\}$ is given by

$$\{d\overline{\sigma}^a\} = [\beta][C^{DSC*}][\beta']^T \{d\overline{\varepsilon}\} \quad (42)$$

and $\{Q\}$ is a generalized load vector.

5. Model parameters

The proposed joint model involves a number of material constants which can be determined from a series of shear tests on joints or interfaces. The material constants can be divided into three categories : parameters for RI material, the constants for FA material, and the disturbance function parameters. The material constants are all listed in Table 1. There are eleven parameters needed for the DSC joint model.

6. Conclusions

The disturbed state modeling provides a powerful way of describing the behavior of joints and interfaces. It is based on the assumption that the behavior of a joint, or the behavior at the disturbed state can be expressed by the joint behavior at its reference states.

The reference states include the RI state and FA state. Basic models can be used to describe the simple behaviors at the reference states and the complex behavior at the disturbed state can be described by using the DSC joint model. Different models can be developed to describe various behaviors if different models are used for the material behaviors at the RI and FA states.

In this study, the behavior of RI state is modeled by using a HiSS δ_0 model with a small modification. The FA state is modeled according to the observations from the shear tests of joints. The DSC joint model based on two reference states thus developed is capable of describing the hardening and softening behavior of a joint under various stress paths.

The model can be easily implemented in finite element procedures and requires a realistic number of parameters for general use. Finally, this model is capable of capturing essential rock joint behavior including strain softening and hardening, relative motion, and dependence on the initial stress condition and loading history.

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