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〈Original Paper〉

Experimental Evaluation of Levitation and Imbalance Compensation for the Magnetic Bearing System Using Discrete Time Q-Parameterization Control

이산시간 Q 매개변수화 제어를 이용한 자기축수 시스템에 대한 부상과 불평형보정의 실험적 평가

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ABSTRACT

In this paper we propose a levitation and imbalance compensation controller design methodology of magnetic bearing system. In order to achieve levitation and elimination of unbalance vibration in some operating speed we use the discrete-time Q-parameterization control. When rotor speed $p=0$ there are no rotor unbalance, with frequency equals to the rotational speed. So in order to make levitation we choose the Q-parameterization controller free parameter Q such that the controller has poles on the unit circle at $z=1$. However, when rotor speed $p \neq 0$ there exist sinusoidal disturbance forces, with frequency equals to the rotational speed. So in order to achieve asymptotic rejection of these disturbance forces, the Q-parameterization controller free parameter Q is chosen such that the controller has poles on the unit circle at $z=\exp^{jpT_s}$ for a certain speed of rotation p (T_s is the sampling period). First, we introduce the experimental setup employed in this research. Second, we give a mathematical model for the magnetic bearing in difference equation form. Third, we explain the proposed discrete-time Q-parameterization controller design methodology. The controller free parameter Q is assumed to be a proper stable transfer function. Fourth, we show that the controller free parameter which satisfies the design objectives can be obtained by simply solving a set of linear equations rather than solving a complicated optimization problem. Finally, several simulation and experimental results are obtained to evaluate the proposed controller. The results obtained show the effectiveness of the proposed controller in eliminating the unbalance vibrations at the design speed of rotation.

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요 약

본 논문에서 자기 축수 시스템의 부상과 불평형 보상 제어기를 제안한다. 특정한 동작속도에서 불균형 진동과 부상을 제거하기위해 우리는 이산시간 Q 매개변수화 제어를 사용한다. 회전자의 속도가 $p=0$일 때, 회전속도와 같은 주파수로 회전자의 불균형은 없다. 그래서 부상을 만들기 위해 우리는 제어기가 $z=1$의 단위원에서 극점을 가지는 Q매개변수화 Q를 선택한다. 그러나 회전자의 속도가 $p\neq0$일 때, 회전속도와 같은 주파수로 정현파의 외란이 존재하게 된다. 그래서 이 외란의 점근적인 소멸을 하기위해 Q 매개화변수 제어기 자유 변수 Q가 제어기가 어떠한 회전속도 p에 대한 $z=\exp^{jpT_s}$에 있는 단위 원에서 극점을 가지도록 선택된다. 첫째로, 우리는 이 연구에서 적용된 실험적인 구성을 소개한다. 두 번째로, 우리는 차분 방정식의 형태로 자기축수 시스템의 수학적인 모델을 제안한다. 세 번째로 우리는 제안된 이산 시간 Q매개변수화 제어기 설계방법을 설명한다. 제어기의 자유 매개변수 Q는 안정한 전달함수가 된다고 가정한다. 네 번째로, 우리는 설계목적을 만족하는 자유 매개변수가 복잡한 최적문제를 풀기보다는 선형 방정식을 구함으로서 만족될 수 있다. 마지막으로 몇 개의 시뮬레이션과 실험적인 결과가 제언된 제어기를 평가하기 위해 구해진다. 획득된 결과는 회전설계 속도에서의 불균형 진동을 제거하는 제안된 제어기의 효과를 나타낸다.

1. Introduction

After making stable levitation, if we make rotor rotate, the shaft vibrations due to the sinusoidal disturbance forces generated by the unbalance are caused. This problem can be solved using active controlled magnetic bearing systems. There are several papers in the literature which deal with this problem (1)~(7) using feedback control to eliminate these vibrations.

The Q-parameterization theory (8)~(9) provide a good tool for the controller design of magnetic bearing systems in order to achieve levitation and elimination of rotor vibrations (5), (7). This is because the controller free Q-parameter can be chosen such that stable levitation states and asymptotic rejection of sinusoidal disturbances are achieved. The Q-parameterization control has several advantages such as: the closed loop poles can be located in a prescribed region in the open left half plane. This insures fast and well damped transient response. It requires easy choice of weighting functions which reflects the robust stability and performance goals compared with other robust control methods such as H_2, LQG, H_∞, H_∞/μ synthesis control (10). In (5) the Q-parameterization theory was utilized to design a controller for a magnetic bearing system to solve the problem of unbalance in two different ways. One way, is to compensate for the unbalance forces by generating electro-magnetic forces that cancel these forces, another way is to make the rotor rotates around its axis of inertia which is known as automatic balancing.

In this paper we extend the controller design methodology developed in (5) to achieve stable leviation and unbalance compensation for a certain speed of rotation using the controller designed in the discrete-time domain. When the plant speed $p=0$ we can design a controller which has poles on the unit circle at $z=1$, on the contrary, when plant speed $p\neq0$ since the frequency of the unbalance sinusoidal disturbance forces equals the rotational speed, we can achieve asymptotic rejection of the frequency disturbances by designing a controller which has poles on the unit circle at $z=\exp^{jpT_s}$ where T_s is the sampling period. This can be done by a suitable choice of the controller free parameter Q. We also show in this paper that the controller free parameter Q which satisfy our design objectives can be obtained by simply solving a set of linear equations rather than solving a complicated optimization problem as for example in the H_∞ synthesis control. A 20 states controller is obtained for the magnetic bearing for levitating and operating at a single design speed of rotation. Several simulation and experimental results were obtained. The results show that levitation states stable and vibrations are eliminated at the operating speed using the proposed controller.

2. Experimental Setup

The magnetic bearing system employed in this research is shown in Fig. 1. It is a 4-axis controlled horizontal shaft magnetic bearing with symmetric structure, the axial motion is not controlled actively. The diameter of the rotor is 96 mm and its span is equal 600 mm. A three-phase induction motor (1 kW, 4-poles) is located at the center of the rotor. Around the rotor there are four pairs of electromagnets arranged radially in both sides, and four pairs of eddy-current type gap sensors located outside the electromagnets. Further the system employs a tachometer to measure the rotor speed. The experimental machine is controlled by a digital

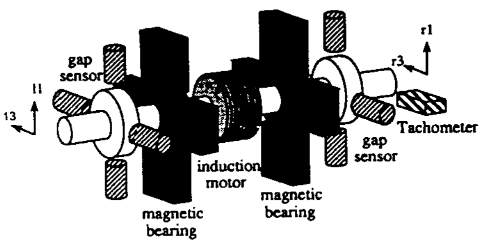

Fig. 1 Four axis magnetic bearing system

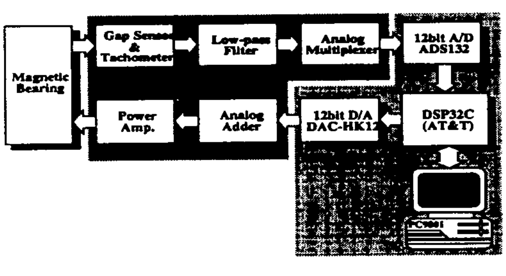

Fig. 2 Digital control system

controlled system that consists of a 32-bit floating point Digital Signal Processor (DSP) DSP32C(AT&T), 12-bit A/D and 12-bit D/A converters. Using these systems, the final discrete-time controller is computed on the DSP. The structure of the digital control system is shown in Fig. 2.

3. Mathematical Difference Equations

In order to make the transformation from continuous time domain to discrete time domain, we need to use some mehtods. There are many kinds of methods to do this transformation, however in this paper we use 'z.o.h'(zero order holder) method. The sampling time of z.o.h is 158 μsec. The mathematical model of discrete-time domain has been derived in reference (7), and the obtained results are such that:

$$\begin{aligned} x_g(k+1) &= A_d x_g(k) + B_d u(k) + E_d f_d(k) \\ y(k) &= C_d x_g(k) + n(k) \\ z_r(k) &= C_d x_g(k) \end{aligned} \tag{1}$$

where $z_r(k)$ is the variable that needs to be regulated, $n(k)$ represents sensor noise. And each matrices will be like this.

$$x_g(k) = \begin{pmatrix} x_z(k) \\ x_\theta(k) \\ x_y(k) \\ x_\varphi(k) \end{pmatrix}$$

$$x_z(k) = \begin{pmatrix} z_s(k) \\ z_s(k+1) \\ i_{l1}(k) + i_{r1}(k) \end{pmatrix} \quad x_\theta(k) = \begin{pmatrix} l_\varphi(k) \\ l_\theta(k) \\ i_{l1}(k) - i_{r1}(k) \end{pmatrix}$$

$$x_y(k) = \begin{pmatrix} y_s(k) \\ y_s(k+1) \\ i_{l3}(k) + i_{r3}(k) \end{pmatrix} \quad x_\varphi(k) = \begin{pmatrix} l_\varphi(k) \\ l_\varphi(k) \\ i_{l3}(k) - i_{r3}(k) \end{pmatrix}$$

$$u(k) = \begin{pmatrix} u_z(k) \\ u_\theta(k) \\ u_y(k) \\ u_\varphi(k) \end{pmatrix} = \begin{pmatrix} e_z(k) \\ e_\theta(k) \\ e_y(k) \\ e_\varphi(k) \end{pmatrix} = \begin{pmatrix} e_{l1}(k) + e_{r1}(k) \\ e_{l1}(k) - e_{r1}(k) \\ e_{l3}(k) + e_{r3}(k) \\ e_{l3}(k) - e_{r3}(k) \end{pmatrix}$$

$$y(k)=\begin{pmatrix} y_z(k) \\ y_\theta(k) \\ y_y(k) \\ y_\varphi(k) \end{pmatrix}\begin{pmatrix} z_s(k) \\ l_\theta(k) \\ y_s(k) \\ l_\varphi(k) \end{pmatrix}$$

$$f_d=\begin{pmatrix} f_{dz}(k) \\ f_{d\theta}(k) \\ f_{dy}(k) \\ f_{d\phi}(k) \end{pmatrix}=\begin{pmatrix} \varepsilon\sin(pT_s+k) \\ \tau\cos(pT_s+\lambda) \\ \varepsilon\cos(pT_s+k) \\ \tau\sin(pT_s+\lambda) \end{pmatrix}$$

$$z_s=\frac{g_{l1}(k)'+g_{r1}(k)'}{2} \quad l_\theta=\frac{-g_{l1}(k)'+g_{r1}(k)'}{2}$$

$$y_s=\frac{-(g_{l3}(k)'+g_{r3}(k)')}{2} \quad l_\varphi=\frac{-g_{l3}(k)'+g_{r3}(k)'}{2}$$

where

$g_j(k)'$: Deviations from the steady gap lengths between the electromagnets and the rotor

$i_j(k)'$: Deviations from the steady currents of the electromagnets

$e_j(k)'$: Deviations from the steady voltages of the electromagnets

$\varepsilon, \tau, k, \lambda$: Imbalance parameters

T_s : Sampling period. $(j=l_1, l_3, r_3)$

The subscripts 'l' and 'r' denote the left-hand side and the right-hand side of the magnetic bearing respectively, and the subscripts '1' and '3' denote the vertical directions and the horizontal directions of the rotor, respectively. The other matrices in equation (1) are defined as follows.

$$A(p)_d=\begin{pmatrix} A_{dz} & 0 & 0 & 0 \\ 0 & A_{d\theta} & 0 & 0 \\ 0 & 0 & A_{dy} & 0 \\ 0 & 0 & 0 & A_{d\varphi} \end{pmatrix}$$

$$B_d=\begin{pmatrix} b_{dz} & 0 & 0 & 0 \\ 0 & b_{d\theta} & 0 & 0 \\ 0 & 0 & b_{dy} & 0 \\ 0 & 0 & 0 & b_{d\phi} \end{pmatrix}$$

$$C_d=\begin{pmatrix} c_{dz} & 0 & 0 & 0 \\ 0 & c_{d\theta} & 0 & 0 \\ 0 & 0 & c_{dy} & 0 \\ 0 & 0 & 0 & c_{d\phi} \end{pmatrix}$$

$$A_{dz}=\begin{pmatrix} 0 & 1 & 0 \\ \frac{\alpha}{m}-\frac{2(d_{l1}+d_{r1})(L_l^2+l_r^2)}{m} & 0 & \frac{l_l^2(c_{l1}+c_{r1})}{J_y} \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}$$

$$A_{d\theta}=\begin{pmatrix} 0 & 1 & 0 \\ \frac{\alpha}{J_y}l_m^2-\frac{(d_{l1}+d_{r1})(l_l^2+l_r^2)}{J_y} & 0 & \frac{l_l^2(c_{l1}+c_{r1})}{J_y} \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}$$

$$A_{dy}=\begin{pmatrix} 0 & 1 & 0 \\ \frac{\alpha}{m}-\frac{4}{m}d_{l3} & 0 & \frac{2c_{l3}}{m} \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}$$

$$A_{d\varphi}=\begin{pmatrix} 0 & 1 & 0 \\ \frac{\alpha}{J_h}l_m^2-\frac{2d_{l3}(l_l^2+l_r^2)}{J_y} & 0 & \frac{l_l^2 2c_{l3}}{J_y} \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}$$

$$b_{dz} = b_{d\theta} = b_{dy} = b_{d\varphi} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix}$$

$$c_{dz} = c_{d\theta} = c_{dy} = c_{d\varphi} = (1\ 0\ 0)$$

$$E_{dg}=\begin{pmatrix} e_{ddz} & 0 & 0 & 0 \\ 0 & e_{dd\theta} & 0 & 0 \\ 0 & 0 & e_{ddy} & 0 \\ 0 & 0 & 0 & e_{dd\phi} \end{pmatrix}$$

$$e_{ddz} = e_{ddy} = \begin{pmatrix} 0 \\ \frac{1}{m} \\ 0 \end{pmatrix}$$

$$e_{dd\theta} = e_{dd\varphi} = \begin{pmatrix} 0 \\ \frac{l_l^2+l_l l_r}{2J_y} \\ 0 \end{pmatrix}$$

In the above equations, α denotes the coefficient of the force which occurs when the rotor eccentrically deviates, and hence we set $\alpha = 0$. And p denotes the rotor speed, c, d denote the linearization coefficients of electromagnet. The parameters of the magnetic bearing system

Table 1 Magnetic bearing parameters

Parameter	Symbol	Value	Unit
Mass of the Rotor	m	1.39×10^1	kg
Moment of Inertia about X	J_x	1.348×10^{-2}	kg · m^2
Moment of Inertia about Y	J_y	2.326×10^{-1}	kg · m^2
Distance between Center of Mass and Electromagnet	$l_{l,r}$	1.30×10^{-1}	m
Distance between Center of Mass and Motor	l_m	0	m
Steady Attractive Force	$F_{l1,r1}$	9.09×10	N
	$F_{l2\sim4,r2\sim4}$	2.20×10	N
Steady Current	$I_{l1,r1}$	6.3×10^{-1}	A
	$I_{l2\sim4,r2\sim4}$	3.1×10^{-1}	A
Steady Gap	W	5.5×10^{-4}	m
Resistance	R	1.07×10	Ω
Inductance	L	2.85×10^{-1}	H

used in this research are given in Table 1

4. Discrete-Time Q-Parametrization Control

The Q-parameterization theory (8) ~ (9) states that the set of all stabilizing controllers of a given plant $G(z)$ can be characterized by one free parameter control feedback.

Consider the one-parameter control feedback system shown in Fig. 3, for controlling any of the SISO subsystems described by Eq. (1), where $r \in \mathrm{R}$ is the reference (command) input signal, $n \in \mathrm{R}$ the sensor noise, $f_d \in \mathrm{R}$ is the disturbance force, $d \in \mathrm{R}$ is plant outout distrubance, $u \in \mathrm{R}$ is the controller output, $y \in \mathrm{R}$ is the plant output, and $K(z) \in \mathrm{R}$ is the stabilizing controller for G(z). Note that n, and f_d may also represent model uncertainties. In order to characterize the set of all stabilizing controllers K for G(z), first we need to construct a doubly coprime factorization (see (8) for details) $N, M, \overline{N}, \overline{M}$ $X, Y, \overline{X}, \overline{Y} \in RH_\infty$ for $G(z)$. First we choose real matrices F_1 and F_2 such that the matrices $A_0 := A_d - B_d F_1$ and $\overline{A_0} := A_d - F_2 C_d$ are stable (all the eigenvalues of A_0 and $\overline{A_0}$ lie inside the unit circle), then the coprime factorization $\overline{N}, \overline{M}, X(z), Y(z) \in RH_\infty$ for $G(z)$ is given as follows

$$\overline{N} = C_d(zI - \overline{A_0})^{-1} B_d$$

$$\overline{M} = I - C_d(zI - \overline{A}0)^{-1} F_2 \qquad (2)$$

$$X(z) = F_1(zI - \overline{A}_0)^{-1} F_2$$

$$Y(z) = I + F_1(zI - \overline{A}_0)^{-1} B_d$$

Then the set of all stabilizing controllers for $G(z)$ is given by

$$K(z) = \{ (Y(z) - Q(z)\overline{N}(z))^{-1} (X(z) + Q(z)(z)), \; |Y(z) - Q(z)\overline{N}(z)| \neq 0 \} \qquad (3)$$

where $Q(z) \in RH_\infty$

5. Controller Objectives

The following controller objectives are imposed.

(1) We need to achieve robust stability against speed and other parameters variation: and achieve fast and well damped transient response.

(2) We need to achieve rejection of low frequency disturbance(stable levitation).

(3) We need to achieve asymptotic rejection of the class of sinusoidal disturbance with frequency equal to the rotational speed p, in order to compensate for the unbalance.

6. Controller Synthesis

(1) In order to satisfy requirement No. 1, the closed loop poles must be located at a prescribed region in the open left half plane. This can be achieved by choosing $\overline{N}, \overline{M}, X, Y, Q \in D_s$ where D_s is a subset of RH_∞ defined as shown in Fig. 4.

(2) In order to satisfy requirement *No. 2*, the controller must have a pole at $z=1$. This can be achieved by choosing $Q(z)$ such that the following identity holds

$$K(z=1) = \infty \qquad (4)$$

From Eq. (3), $Q(z)$ must satisfy the following Equation.

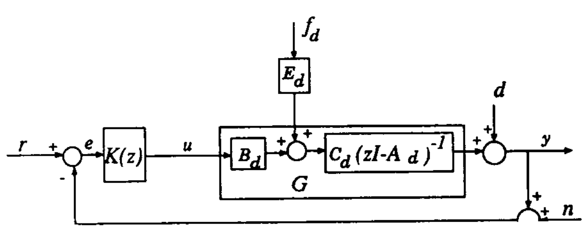

Fig. 3 One-parameter feedback control system

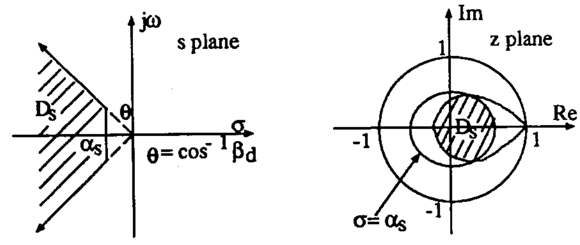

Fig. 4 Generalized region of stability

$$Y(z=1) - Q(z=1)\bar{N}(z=1) = 0 \tag{5}$$

(3) In order to satisfy requirement No. 3, the controller free parameter $Q(z)$ must be chosen such that the controller has poles at $z = exp^{jpTs}$. This can be achieved by choosing $Q(z)$ such that the following identity holds

$$K(z = \exp^{jpT_s}) = \infty \tag{6}$$

From Eq. (3), $Q(z)$ must satisfy the following Equation.

$$Y(z = zp) - Q(z = zp)\bar{N}(z = zp) = 0 \tag{7}$$

where $zp = exp^{jpTs}$.

Equation (7) is in fact 2 equations, 1 equation for the real part and 1 equation for the imaginary part. This suggests that $Q(z)$ can take the form for a single design speed of rotation

$$Q(z) = a_0 + \frac{a_1}{(z - z_{p1})} + \frac{a_2}{(z - z_{p2})} \tag{8}$$

where $a_0, a_1, a_2 \in \mathrm{R}$ are free design parameters and $z_{p1}, z_{p2} > \alpha_s \in R$ are fixed. Note that Q is a proper stable transfer function.

In this paper we design a controller for magnetic bearings rotate at design speed p, so $\{Q\}(z)$ is chosen as follows:

$$Q(z=1) = a_0 + \frac{a_1}{(1 - z_{p1})} + \frac{a_2}{(z - z_{p2})} \tag{9}$$

$$Q(z = zp) = a_0 + \frac{a_1}{(zp - z_{p1})} + \frac{a_2}{(zp - z_{p2})} \tag{10}$$

where $zp = exp^{jpTs}$.

Eqs. (9) and (10) are in fact three linear equation in the three unknown free design parameters a_0, a_1, a_2. In order to solve Eqs. (9) and (10) for a_0, a_1, a_2 we need first to solve Eqs. (5) and (7) for $Q(z=1)$ and $Q(z=exp^{jpTs}\})$. Eqs. (5) and (7) are also linear equations in $Q(z=1)$ and $Q(z=exp^{jpTs}\})$. From Eqs. (5) and (7) we have

$$\begin{aligned} Q(z=1) &= Y(z=1)\bar{N}^{-1}(z=1) \\ Q(z=zp) &= Y(z=zp)\bar{\mathrm{N}}\text{-}1(z=zp) \end{aligned} \tag{11}$$

where $zp = exp^{jpTs}$. Then the design parameters a_0, a_1, a_2 can easily be found by solving the following set of linear equations: Let $zp_k = 1/(zp - z_{pk})$, $k = 1,2$. Then we have

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 & 1/(1-z_{p1}) & 1/(1-zp_2) \\ 1 & \Re(zp_1) & \Re(zp_2) \\ 0 & \Im(zp_1) & \Im(zp_2) \end{pmatrix}^{-1} \times \begin{pmatrix} Q(z=1) \\ R(Q(z=zp)) \\ \Im(Q(z=zp)) \end{pmatrix} \tag{12}$$

where $R(\cdot)$ and $\Im(\cdot)$ denotes the real and imaginary parts of $(\cdot)$, and $zp = exp^{jpTs}$.

7. Simulation and Experimental Results

The method of controller design discussed in the previous section is applied to a magnetic bearing system whose parameters are given in Table 1. The μ synthesis Toolbox (14) with Simulink were used for the design and simulation. The system is discretized using a zero order hold at a sampling time $T_s = 158\,\mu sec$. The controller $K(z)$ is designed at a speed $p=0$[rad/sec] and must be able to keep the system stable for a speed range $(0\text{-}2\pi\,350)$[rad/sec] = (0-21000)[rpm]. The operating speed to suppress the imbalance is assumed to be $p = 2\pi\,16.67$[rad/sec] = 1000[rpm]. The numerical values of F_1, F_2 for each subsystems, Z, Y, Θ, Ψ, were obtained using the algebraic Riccati equation such that the eigenvalues of A_o, $\bar{A}_0$ lie in the domain D_s are:

$$F_{1z}^{-1} = \begin{pmatrix} -2.1885e+05 \\ -1.3119e+03 \\ 1.4106e+02 \end{pmatrix}, \quad F_{1z}^{-1} = \begin{pmatrix} 7.6856e-01 \\ 1.8915e+01 \\ -3.4372e-02 \end{pmatrix}$$

$$F_{1z}^{-1} = \begin{pmatrix} 3.2005e+05 \\ 1.3103e+03 \\ 1.41170e+02 \end{pmatrix}, \quad F_{1z}^{-1} = \begin{pmatrix} 7.6859e-01 \\ 1.9102e+01 \\ 3.4655e-02 \end{pmatrix}$$

$$F_{1z}^{-1} = \begin{pmatrix} 1.2663e+05 \\ 8.3460e+02 \\ 8.8330e+01 \end{pmatrix}, \quad F_{1z}^{-1} = \begin{pmatrix} 7.6678e-01 \\ 7.5848e+00 \\ 2.5181e-02 \end{pmatrix}$$

$$F_{1z}^{-1} = \begin{pmatrix} 1.2708e+05 \\ 8.3339e+02 \\ 8.8730e+01 \end{pmatrix}, \quad F_{1z}^{-1} = \begin{pmatrix} 7.6679e-01 \\ 7.6578e+00 \\ 2.5412e-02 \end{pmatrix}$$

And

$$Z_{zp1z} = 80, \quad Z_{zp2z} = 1400$$

$$Z_{zp1th} = 80, \quad Z_{zp3th} = 1400$$

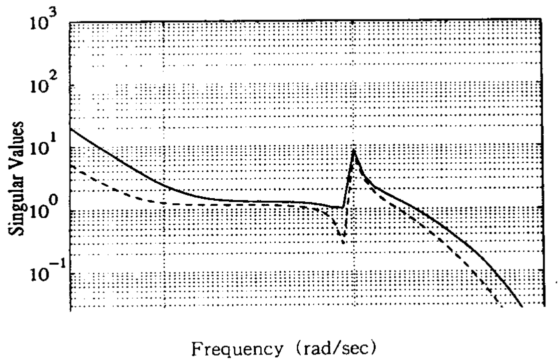

Fig. 5 Singular values of the loop gain GK, (-):Horizontal axis, (--):Vertical axis

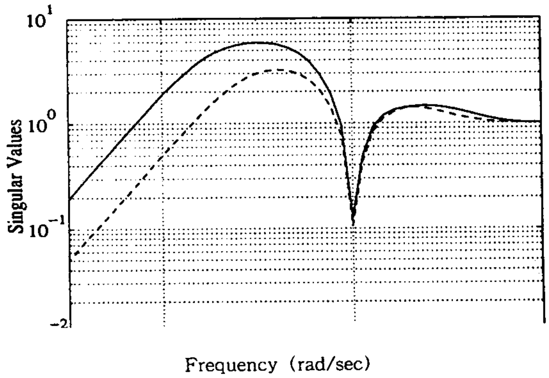

Fig. 6 Singular values of the sensitivity, (-):Horizontal axis, (- -):Vertical axis

$Z_{zp1y}=60,\ Z_{zp2y}=700$

$Z_{zp1\phi}=60,\ Z_{zp2\phi}=700$

The controller Q-paremeter that can satisfy (5), (7) for stable levitation and imbalance compensation was found to be

$$Q_z=-4.3910e+07-\frac{1.5085e+04}{z-80}+\frac{7.6103e+06}{z-1400}$$

$$Q_{th}=4.3686e+07+\frac{1.4958e+04}{z-80}-\frac{7.5649e+06}{z-1400}$$

$$Q_y=2.3515e+07+\frac{1.5755e+04}{z-60}-\frac{2.1984e+04}{z-700}$$

$$Q_\phi=2.3393e+07+\frac{1.5623e+04}{z-60}-\frac{2.1846e+06}{z-700}$$

Substituting the Q's in Eq. (3) we get (after model reduction) a 5 states controller for each of the four subsystems. The overall controller of the whole system has 20 states and is formulated as follows:

$$K_z=\begin{bmatrix} K_{dz} & 0 & 0 & 0 \\ 0 & K_{d\theta} & 0 & 0 \\ 0 & 0 & K_{dy} & 0 \\ 0 & 0 & 0 & K_{d\phi} \end{bmatrix} \tag{13}$$

7.1 Simulation Results

In Fig. 5 High loop gain at low frequency and low gain at high frequency are achieved. This means that the levitated state is very good and the steady state error is almost zero. In Fig. 6 we can see that the sensitivity is small at low frequencies which means good disturbance rejection for the class of step disturbances and approaches zero at the frequencies $\omega=2\pi$ 16.67[rad/sec]=1000[rpm] which means asymptotic rejection of the unbalance sinusoidal disturbance forces at this speed.

In this design, we ignored the interference terms, which express the gyroscopic effect, as $p=0$. We therefore verify the robust stability of the system against the changes in the rotor speed. Let the perturbed plant $p \neq 0$ be denoted by G_Δ and the additive perturbation Δ_p from G is as follows:

$$\Delta_p=G_\Delta-G \tag{14}$$

The robust stability is guaranteed if the following in equality holds:

$$\bar{\sigma}(\Delta_p)<\frac{1}{\bar{\sigma}(K(I+GK)^{-1})} \tag{15}$$

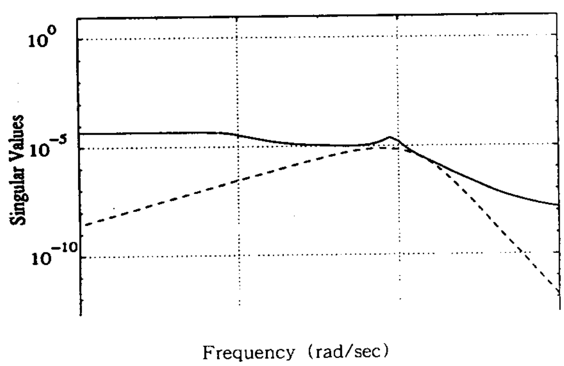

Fig. 7 $1/\bar{\sigma}(K(I+GK)^{-1}$ and $\bar{\sigma}(\Delta_p)(--)$ ($p=2\pi 350$[rad/sec](=21000[rpm])

In Fig. 7 we can see that the system is stable up to a speed $p = 2\pi 350$[rad/sec](=21000[rpm]).

7.2 Experimental Results

In Fig 8, 9, 10 we can see that the levitated state is very good. Fig. 10 shows that he gap displacements of vertical axis and horizontal axis. In this figure we can see that the levitated rotor shaft of magnetic bearing system is not in the center position. This is because of the sensor position. In order to make the imbalance we added a small weight (20[g]) to the left side of the rotor shaft.

Figs. 11, 12, and 13 show the experimental results of the gap displacements(left side) at different speeds for the imbalance compensation

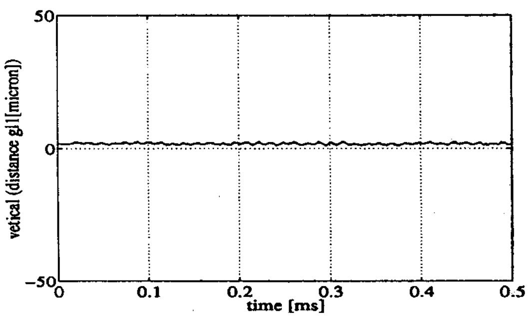

Fig. 8 Gap deviations of vertical axis(left side, p =0)

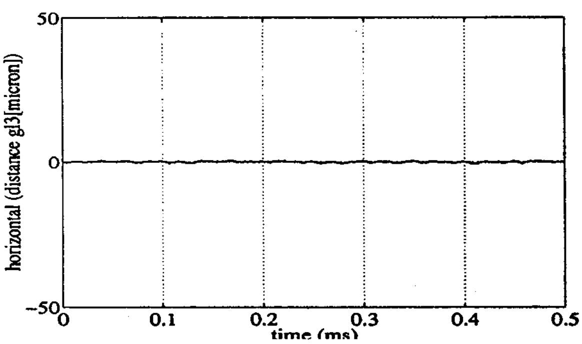

Fig. 9 Gap deviations of horizontal axis(left side, p =0)

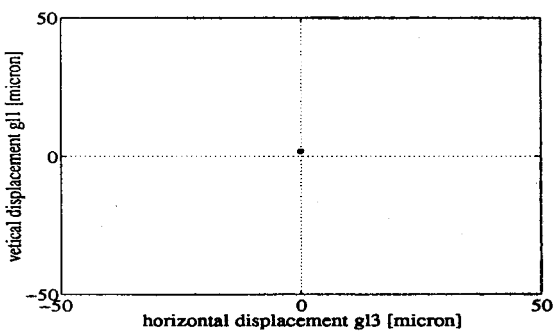

Fig. 10 Gap deviations of vertical and horizontal axis
(left side, p =0)

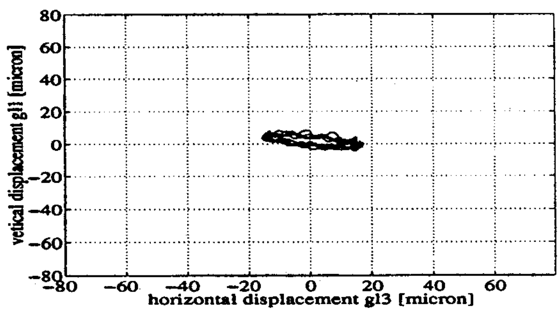

Fig. 11 Gap deviations of K1000 at speed 700[rpm] (Left side)

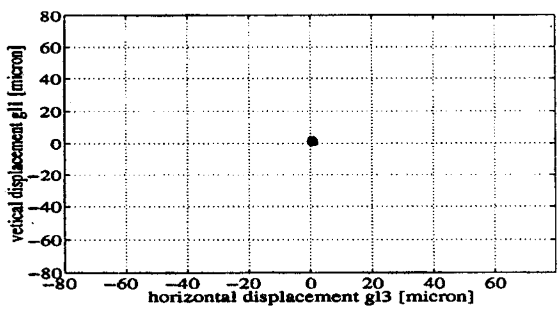

Fig. 12 Gap deviations of K1000 at speed 1000[rpm] (Left side)

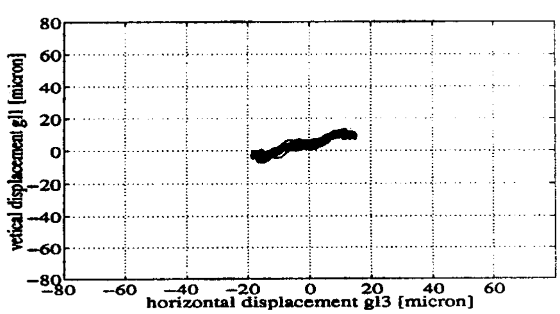

Fig. 13 Gap deviations of K1000 at speed 1300[rpm] (Left side)

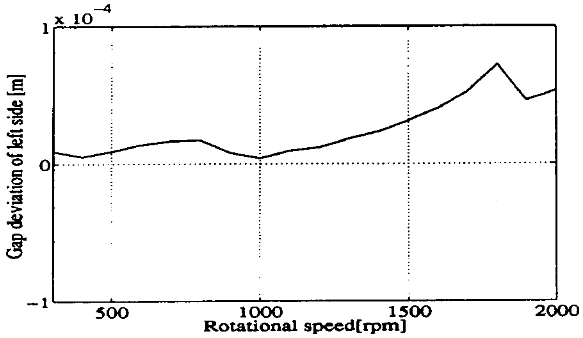

Fig. 14 Gap deviations of K1000 from 300[rpm] to 2000[rpm] (Left side)

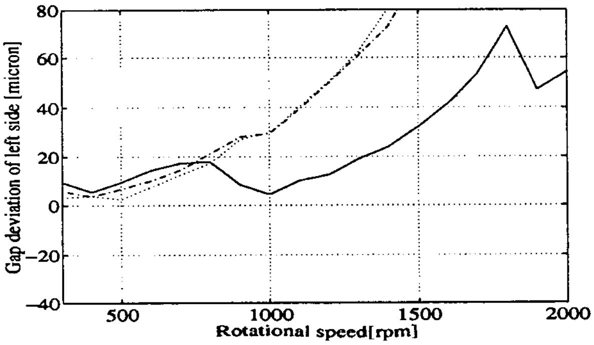

Fig. 15 Gap deviations of K_{1000}(-), K_{500}(· ·), K_{msp}(- · -) from 300[rpm] to 2000[rpm] (Left side)

design. Comparing three figures we can see the very good suppression of imbalance at design speed (1000rpm). Fig. 14 shows the gap deviations of left side from 300 rpm to 2000 rpm. In this figure we can also see the very good imbalance compensation at design speed (1000 rpm). From these figures we can see that the unbalance vibration due to the sinusodal disturbance forces generated by the imbalance is completely eliminated at the design speed $p=2\pi 16.67$[rad/sec](=1000[rpm]).

In order to investigate robust stability and robust performance of the proposed Q-parametrization control method, we have obtained experimental results for K_{1000}, K_{500}, K_{msp}. K_{1000} is the controller to suppress the imbalance at speed 1000[rpm], K_{500} is the controller to suppress the imbalance at speed 500[rpm], K_{msp} is the mixed sensitivity problem controller just for the levitation. K_{msp} is not the imbalance compensation controller. In this figure we can see the good suppression of imbalance at speed 500[rpm] and 1000[rpm].

8. Conclusion

In this paper we employed the discrete-time Q-parameterization control to design a controller which achieves stable levitation and elimination of unbalance vibrations at design speed (1000[rpm]). The free controller parameter is chosen such that the controller has poles on the unit circle at $z=\exp^{jpT_s}$ for the design speed of rotation p, $z=1$ for the stable levitation, and satisfy other control objectives. This insures asymptotic rejection of the unbalance disturbance forces generated by the unbalance and rejection of the low frequency disturbances. The controller free parameter Q is assumed to be a proper stable transfer function. We showed that the free controller parameter is obtained by solving a set of linear equations rather than solving a complicated optimization problem. The controller is designed at speed $p=0$ and the good simulation and experimental results that were obtained at design speed $p=2\pi 16.67$[rad/sec] showed the robustness of the proposed controller.

Elimination of unbalance vibrations using discrete domain Q-parameterization control can also be achieved by making the rotor rotates around its axis of inertia at the design speed (automatic balancing). In this case the rotor will be free from vibrations. This can be done using the same procedures explained in the previous section and (5).

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