◉ 研究論文

Numerical Analysis of a Diffuser Flow with Expansion and Streamline Curvature

Y. W. Lee

확대 및 유선곡률을 가진 디퓨저 흐름의 수치해석

이 연 원*

Key words : Conical Diffuser, Modified Combination Model, Pope' s Vortex Stretching Mechanism, Hanjalic-Launder' s Preferential Normal Strain

Abstract

A diffuser, an important equipment to change kinetic energy into pressure energy, has been studied for a long time. Though experimental and theoretical researches have been done, the understanding of energy transfer and detailed mechanism of energy dissipation is unclear.

As far as numerical prediction of diffuser flows are concerned, various numerical studies have also been done. On the contrary, many turbulence models have constraint to the applicability of diffuser-like complex flows, because of anisotropy of turbulence near the wall and of local nonequilibrium induced by an adverse pressure gradient.

The existing k-ε turbulence models have some problems in the case of being applied to complex turbulent flows. The purpose of this paper is to propose the new modified turbulence model applicable to diffuser-like flows with expansion and streamline curvature. In order to obtain the reliability of k-ε turbulence model, modified combination turbulence models composed of the anisotropic k-ε model with Hanjalic-Launder' s preferential normal strain and Pope' s vortex stretching mechanism are proposed. The results of the present proposed models prove the fact that the coefficient of pressure and the shear stress are well predicted at the diffuser flow.

* 부경대학교 기계자동차공학부, 열유체공학연구소(원고접수일 : 98년 6월)

Nomenclature

x, r : axial and radial coordinates
R_{in} : radius of inlet straight pipe
R_{loc} : maximum radius at each position
D : diameter of inlet straight pipe
U_m : mean axial velocity
δ_{ij} : Kronecker delta
ρ : density
μ, μ_t : viscosity, eddy viscosity
ν, ν_t : kinematic viscosity, kinematic eddy viscosity
C_m : constant of the nonlinear term in anisotropic expression
C_l : constant used in the inlet condition of
$C_{\varepsilon s}$: constant of vortex-stretching term

1. Introduction

Diffusers are the fluid-mechanical equipment converting kinetic energy into pressure energy. The importance of the diffuser has been widely known ,especially, to the design of turbomachineries. Though many experimental and theoretical studies[1-3] have been done, the turbulence of a conical diffuser flow is very complicated and the understanding of the detailed mechanism of energy transfer and energy dissipation is still unclear. Numerical studies of diffuser flows using k-ε turbulence models also show that they have some constraint to the applicability of diffuser-like complex flows, because of anisotropy of turbulence near the wall and of local nonequilibrium induced by the adverse pressure gradients.

By the way, extra strain rates considering of no importance are present in a complex shear flow, even small values of them can have a significant effect on the turbulence field, thus invalidating the applicability of many turbulence models[4-5]. Therefore, these kinds of diffuser flows influenced by the severe adverse pressure gradient and streamline curvature are one of the important research fields of numerical simulation of turbulence.

The purpose of this paper is to propose the new modified turbulence model applicable to diffuser-like flows with expansion and streamline curvature. In order to obtain the reliability of k-ε turbulence model, modified combination turbulence models composed of the anisotropic k-ε turbulence model expression with Hanjalic-Launder's prefertial normal strain[6] and Pope's vortex stretching mechanism[7] are proposed. The experimental data by Azad and Kassab[8] were used to compare with numerical computation results. The data are for a fully developed flow through the 8 degree total angle conical diffuser. This flow introduces a severe adverse axial pressure gradient and streamline curvature at the entrance of the diffuser.

2. Turbulence Models

2.1 Problems of the Standard k-ε Turbulence Model

The governing equations solved are the conservation of mass and momentum, expressed as

$$\frac{\partial \rho U_j}{\partial x_j} = 0 \qquad (1)$$

$$\rho \frac{D U_i}{Dt} = \frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\mu+\mu_t)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] \qquad (2)$$

where all the variables are ensemble averaged quantities and the repeated indices denote the summation over all directions. And the eddy viscosity μ_t is evaluated by turbulent kinetic energy(k) and its dissipation rate(ε) as

$\mu_t = C_\mu \rho \frac{k^2}{\varepsilon}$, and k and ε are governed by the following transport equations.

$$\rho \frac{Dk}{Dt} = \frac{\partial}{\partial x_j}\left[(\mu + \frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial x_j}\right] + P_k - \varepsilon \quad (3)$$

$$\rho \frac{D\varepsilon}{D_t} = \frac{\partial}{\partial x_j}\left[(\mu + \frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_j}\right] + \frac{\varepsilon^2}{k}\left[C_{\varepsilon 1}\frac{P_{ks}}{\varepsilon} + C_{\varepsilon 1}\frac{P_{kn}}{\varepsilon} - C_{\varepsilon 2}\right] \quad (4)$$

where the production term P_k and Reynolds stress are defined as

$$P_k = P_{ks} + P_{kn} = -\rho \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} \quad (5)$$

$$\rho \overline{u_i u_j} = \frac{2}{3}\rho k \delta_{ij} - \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_j}\right) \quad (6)$$

and the coefficients are chosen as follows[3] ;

$C_\mu = 0.09$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$.

In general, problems of the standard k-ε turbulence model are(1) assumption of eddy viscosity,(2) shortage of distribution mechanism of Reynolds stresses,(3) constant Cμ,(4)approximation of transport equation for the dissipation rate of turbulence energy(ε), (5) the law of the wall as a boundary condition[9]. The problem of ε-equation have been especially noted in diffuser-like flows having adverse pressure gradient effects and streamline curvature. It has been reported a few of suggestions for modifing the k-ε model, which aimed for the ε-equation.

Hanjalic and Launder[6] pointed out the special role that irrotational straining plays in the spectral transfort from the large, energy-containing to the small dissipating eddies. The generation term in the ε-equation involves in its general form both rotational and irrotational strain rates. Hanjalic and Launder multiplied irrotational term by a larger empir ical coefficient than the rotational one, in order to bring the irrotational part into promi nence. Rodi[10] scrutinized the k-ε turbulence model under adverse pressure gradient conditions. The modification gives rise to larger k − ε values, therefore reducing the length scale and also the shear stress.

Another modification suggested by Pope[7] is to introduce mean vortex stretching effects. Pope solved round-jet/plane-jet anomaly using the vortex-stretching invariant term ($X=(k/\varepsilon)^3 \Omega_{ij}\Omega_{jk}S_{ki}$), where Ω_{ij} and S_{ki} are the rate of mean rotation tensor and the rate of mean strain tensor respevtively. Recently Shon et al[11] showed that the vortex stretching invariant term brought the significant improvement to the prediction of symmetry boundary layers in the strong mean flow convergence and divergence.

However, previous modified models have used experimental results to compensate the shortage of distribution mechanism of normal stresses, and modified k-ε turbulence models are weak in generality. In this paper new modified models were proposed having distribution mechanism of normal stresses on the basis of an anisotropic expression.

2.2 Modified Combination Models

The modified combination model 1 is proposed, which is composed of the anisotropic k − ε model with Hanjalic-Launder' s preferential normal strain. With anisotropic Reynolds stresses expression, modified combination model 1 operates more effectively on the irrotational generation term than previous modified models having drawbacks of no distribution mechanism of Reynolds stresses. The k − ε equation of this model is as follows :

$$\rho \frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j}\left[(\mu + \frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_j}\right] + \frac{\varepsilon^2}{k}$$

$$\left[C_{\varepsilon 1}\frac{P_{ks}}{\varepsilon}+C_{\varepsilon 3}\frac{P_{kn}}{\varepsilon}-C_{\varepsilon 2}\right] \quad (7)$$

$$P_{ks}=-\overline{\rho u_i u_j}\frac{\partial U_i}{\partial x_j} \quad (i\neq j) \quad (8)$$

$$P_{kn}=-\overline{\rho u_i u_j}\frac{\partial U_i}{\partial x_j} \quad (i=j) \quad (9)$$

$$\overline{\rho u_i u_j}=\frac{2}{3}\rho k\delta_{ij}-\mu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)+\frac{k}{\varepsilon}\mu_t\sum_{m=1}^{3}C_m\left(S_{mij}-\frac{1}{3}S_{m\alpha\alpha}\delta_{ij}\right) \quad (10)$$

$$S_{1ij}=\frac{\partial U_i}{\partial x_m}\frac{\partial U_j}{\partial x_m} \quad (11)$$

$$S_{2ij}=\frac{1}{2}\left(\frac{\partial U_m}{\partial x_i}\frac{\partial U_j}{\partial x_m}+\frac{\partial U_m}{\partial x_j}\frac{\partial U_i}{\partial x_m}\right) \quad (12)$$

$$S_{3ij}=\frac{\partial U_m}{\partial x_i}\frac{\partial U_m}{\partial x_j} \quad (13)$$

Here C_m is the model constant defined in usual anisotropic k-ε turbulence model[12].

The modified combination model 2 has Pope's vortex stretching mechanism added to the modified combination model 1 as follows ;

$$\rho\frac{D\varepsilon}{Dt}=\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\mu_t}{\sigma_t}\right)\frac{\partial\varepsilon}{\partial x_j}\right)+\frac{\varepsilon^2}{k}\left[C_{\varepsilon 2}\frac{P_{ks}}{\varepsilon}+C_{\varepsilon 3}\frac{P_{kn}}{\varepsilon}-C_{\varepsilon 2}+C_{\varepsilon s}X\right] \quad (14)$$

$$X=\left(\frac{k}{\varepsilon}\right)^3\Omega_{ij}\Omega_{jk}S_{ki} \quad (15)$$

$$\Omega_{ij}=\frac{1}{2}\left[\frac{\partial U_i}{\partial x_j}-\frac{\partial U_j}{\partial x_i}\right] \quad (16)$$

$$S_{ij}=\frac{1}{2}\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right] \quad (17)$$

An anisotropic Reynolds stress expression has the second order nonlinear terms which play as source term, and this model does not become robust. So, calculations with the law of the wall are unstable and diverge sometimes. In the present paper Chen-Patel's two-layer model[13] more economical than low Reynolds number type was introduced to protect anisotropic informations near the wall against using the law of the wall and to enhance the stability of modified models.

3. Numerical Analysis and Boundary Condtion

The discretization of the governing equations are obtained by integrating the strong conservative form of differential equations over finite control volumes. The convection-diffusion formulation is based on a hybrid differencing scheme developed by Spalding[14]. A non-staggered variable arrangement is used here for all physical variables which are assumed to be located at the centroids of control volumes. In order to obtain the solution of Navier-Stokes equations, the linearized equations are converted to simple tridiagonal matrix systems and solved by a line-by-line relaxation mathod. The SIMPLE algorithm is used to update the new dependent variables. With a non-staggered grid system, a special treatment is required to obtain the cell face convection quantities to prevent the checker-board type oscillation. The cell face contravariant velocities are obtained using Peric's momentum interpolation method[14]. The coefficients are linearly interpolated, but neighboring cell node pressures are used rather than averaging the pressure gradients for the control volumes. This enforces strong velocity-pressure coupling. The grid system is generated by Eiseman's algebraic grid generation method[15] using 1-dimensional stretching function for the two-dimensional domain of a conical diffuser.

Four types of boundary conditions are needed at the axisymmetric conical diffuser flow ; these are inlet, outlet, solid wall and axis of symmetry. At the inlet, the axial velocity U and turbulent kinetic energy k are obtained from the measurements of Ref.8, whereas radial velocity V is taken to be zero and the dissipation rate of turbulent kinetic energy ε is approximated by $\varepsilon = k^{1.5}/L$ based on the equilibrium assumption[2] and the data of Laufer[16,17]. In general the $L=(C_l R_{in})$ is adopted. At the centerline, the normal gradient of all flow quantities, except radial velocity V which is set to zero at the boundary, is assumed to be zero. At the exit, Neumann condition for all variables is adopted. But a mass flow compens ation is also applied for the satisfaction of overall continuity at the exit. For the boundary conditions at the near-wall control volume, the wall function treatment is used for the standard k-ε turbulence model, whereas Chen and Patel' s two layer model[13] is used for the modified combination models. The first points along the wall are placed on the wall region $30 < y^+ < 200$ in the case of the standard k-ε turbulence model.

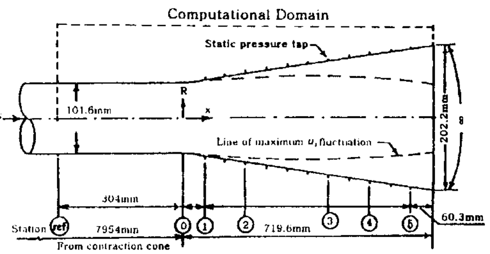


Fig. 1. Conical diffuser geometry and coordinate system

4. Results and Discussion

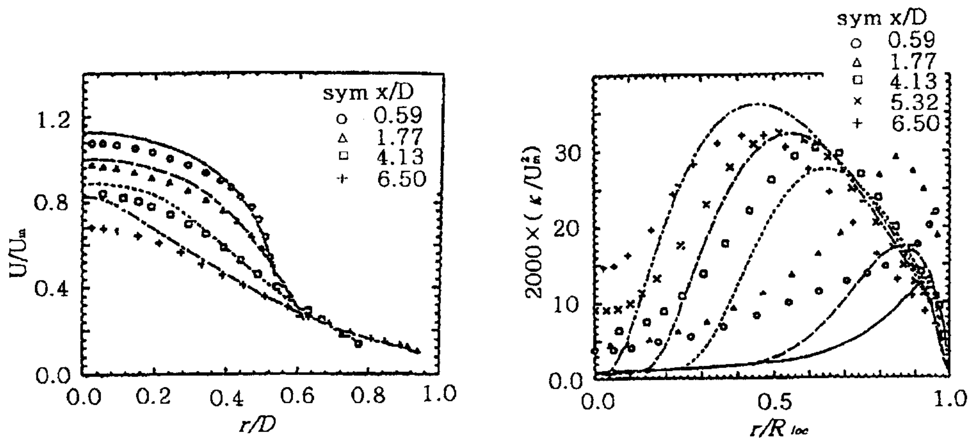
4.1 Results of Standard k-ε Turbulence Model

The diffuser geometry and the coordinate system used in the calculation are shown in fig.1. Reynolds number of the diffuser flow based on the pipe diameter and mean axial velocity is 115,000. The experimental values for all of inlet boundary conditions were adopted except ε. It was found that inlet ε distribution play an important role in the accurate prediction of the downstream mean velocity field and the centerline decay of k from the calculation of Lee and Kobayashi(3). Further information was that turbulent kinetic energy k is underpredicted rather than the measurement value, and shear stress is overpredicted rather than measured one through the interlinkage and feedback system. Therefore physically consistent profiles of ε are needed to obtain the optimum results.

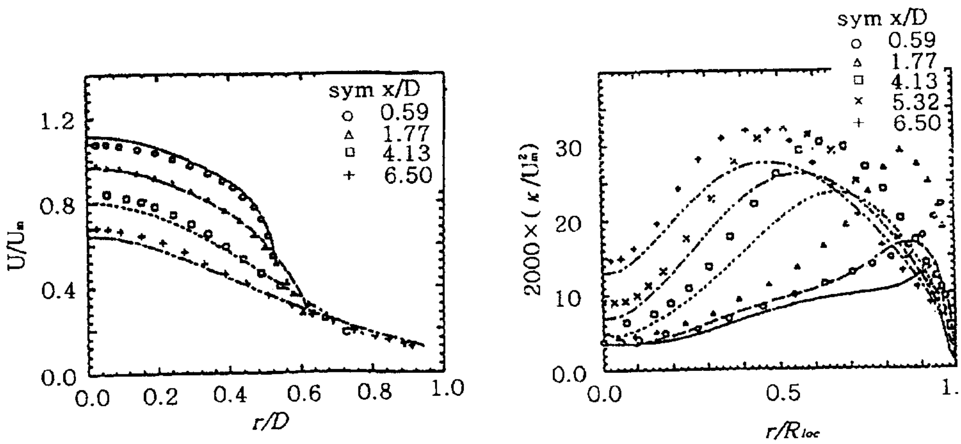
The effect of inlet ε was simulated for three cases of length scale, $L(=k^{1.5}/\varepsilon)$. The predicted radial velocity and turbulent kinetic energy at various positions are shown in fig.2(a,b,c) for the standard k-ε model. It is seen from the figures that the computational results are sensitive to the inlet ε values. Fig.2(a) shows that turbulent kinetic energy along the centerline of the diffuser develops very slowly compared with experiment and the decreasing rate of axial velocities is smaller than experiment. Fig.2(c) shows that, although the trend of the radial distribution of turbulent kinetic energy agrees with experiment, the peak values are smaller than fig.2(b) and the decreasing rate of axial velocities is larger than experiment. Fig.2(b) was adopted as the optimun inlet conditions.

Fig.3 shows the only shear stress $\overline{uv}$ in the axisymmetric diffuser flow. The predicted values are approximately overestimated to the maximum 25% at the downstream position. Rodi and Scheuerer[10] found that this fact exists at the plane boundary layer flow with adverse pressure gradient. The cause is due to the fact that the production of ε is relatively smaller than the production of k. The pro duction term in 2-dimensional axial flow can be written as follows ;

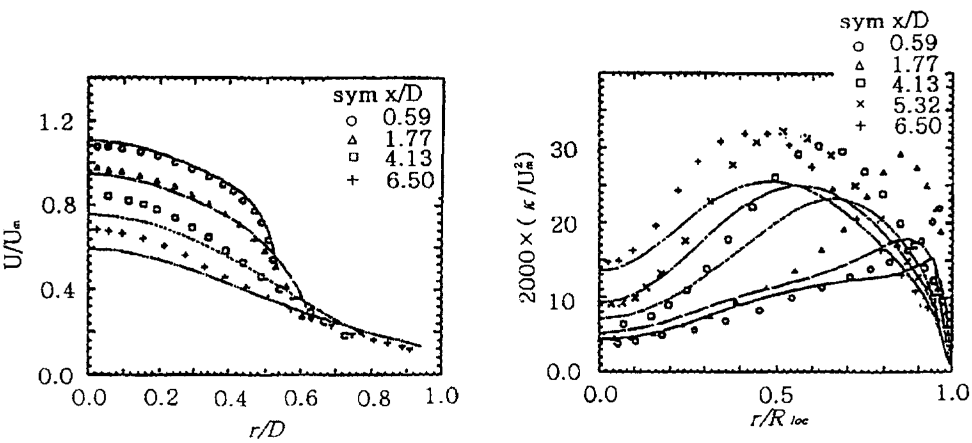
$$P_k = \underbrace{-\overline{uv}\left(\frac{\partial U}{\partial r} + \frac{\partial V}{\partial x}\right)}_{P_{ks}}$$



(a) $\varepsilon=\kappa^{1.5}/0.1R_{in}$



(b) $\varepsilon=\kappa^{1.5}/0.39R_{in}$



(b) $\varepsilon=\kappa^{1.5}/0.39R_{in}$

Fig. 2. Radial distribution of axial velocity and turbulent kinetic energy (line = computation, symbol = experiment)

$$\underbrace{-(\overline{uu}-\overline{vv})\frac{\partial U}{\partial x}-(\overline{ww}-\overline{vv})\frac{V}{r}}_{P_{kn}} \quad (18)$$

Here the most important term is usually the production term by shear stress. However, judging from fig.4 and Singh and Azad′s experiments[18], it should be noted that the production term by normal stress($\overline{uu}\,\frac{\partial U}{\partial x}$) is not negligible, especially important near the inlet. Fig.4 shows that the numerical results of normal strain rate $\frac{\partial U}{\partial x}$, which values are relatively large near the inlet having big pressure gradient and decrease downstream. The reason why plus values exist near the wall is due to the gradient along the axis, not the gradient along the streamline direction.

Oomachi[19] shows that $\overline{uu}\simeq\overline{vv}\simeq\overline{ww}$ in the isotropic k - ε turbulence model and the production by normal stress can seldom affect as compared with the production by shear stress. This is one of the weak points. The other is the underestimation of ε as noted by previous papers[2,3,6]. Rodi and Scheuerer used Hanjalic and Launder′ s idea for irrotational part to increase the production rate of the dissipation rate of turbulent kinetic energy. Hanjalic and Launder keep an eye on the irrotational contribution(the production by normal stress) to turbulent kinetic energy production P_k. They used $C_{\varepsilon 3}(=4.44)$ as the coefficient of the ε irrotational part of the production term in the equation, without using traditional $C_{\varepsilon 1}(=1.44)$;

- Standard k-ε turbulence model

$$P_\varepsilon=\frac{\varepsilon}{k}C_{\varepsilon 1}(P_{ks}+P_{kn}) \quad (19)$$

- Hanjalic and Launder model

$$P_\varepsilon=\frac{\varepsilon}{k}(C_{\varepsilon 1}P_{ks}+C_{\varepsilon 3}P_{kn})$$

$$=\frac{\varepsilon}{k}[C_{\varepsilon 1}P_k+(C_{\varepsilon 3}-C_{\varepsilon 1})P_{kn})] \quad (20)$$

They also adopted the experimental values, $\overline{uu}-\overline{vv}=0.33k$ to escape the isotropic defect concerning the distribution mechanism of normal stresses. By comparing the standard k-

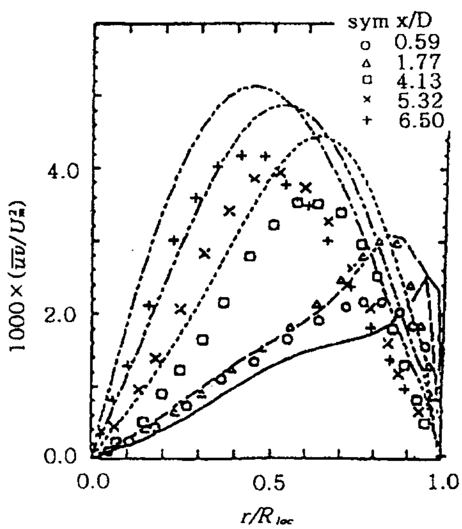


Fig. 3. Radial distribution of shear stress

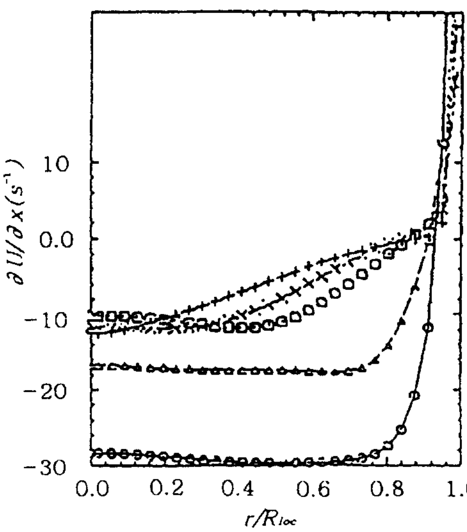


Fig. 4. Radial distribution of normal strain rate by numerical analysis

ε model and the Hanjalic and Launder model, one can see that their model is more sensitive to the decelerating flow field. However, it must be noted that the Hanjalic and Launder model has major problems with the violation of tensor invariance and realizability[20].

By the way, the nonlinear k-ε turbulence model which keeps tensor invariance shows the reliable predictability for the anisotropy of normal stresses. There are the possibility that the production term by the normal stress works well. This is the key point of the present model. Discrepancies in the shear stress between experiments and computations are mainly due to the discrepancies in the length scale resulting from the ε equation.

4.2 Results of Anisotropic(or nonlinear) k-ε Turbulence Model

To analyse the effects of nonlinear terms in axisymmetric diffuser flow one can simplify them by the order of magnitude using following assumption ; $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, $\frac{\partial U}{\partial x} \ll \frac{\partial U}{\partial y}$. Reynolds stresses are written as follows[17] ;

$$\overline{uu} = -\nu_t\left(2\frac{\partial U}{\partial x}\right)+\frac{2}{3}k+\frac{k}{\varepsilon}\nu_t \left[\left(\frac{\partial U}{\partial r}\right)^2\left(\frac{2}{3}C_1-\frac{1}{3}C_3\right)\right] \quad (21)$$

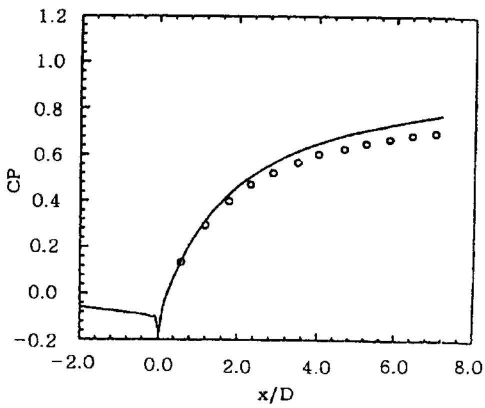
$$\overline{vv} = -\nu_t\left(2\frac{\partial V}{\partial r}\right)+\frac{2}{3}k+\frac{k}{\varepsilon}\nu_t \left[\left(\frac{\partial U}{\partial r}\right)^2\left(-\frac{1}{3}C_1+\frac{2}{3}C_3\right)\right] \quad (22)$$

$$\overline{ww} = -\nu_t\left(2\frac{V}{r}\right)+\frac{2}{3}k+\frac{k}{\varepsilon}\nu_t \left[\left(\frac{\partial U}{\partial r}\right)^2\left(-\frac{1}{3}C_1-\frac{1}{3}C_3\right)\right] \quad (23)$$

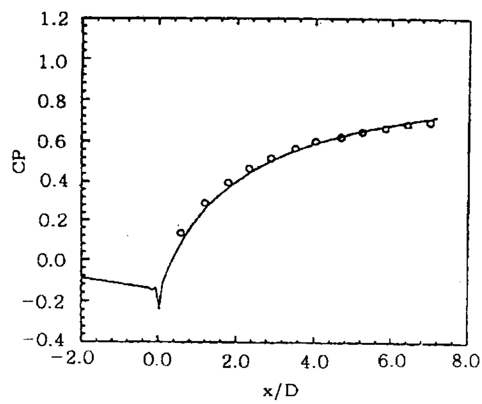
$$\overline{uv} = -\nu_t\left(\frac{\partial U}{\partial r}+\frac{\partial V}{\partial x}\right) \quad (24)$$

From above reduced equations, it is seen that though normal stresses have the second order nonlinear terms, shear stress is the same as the standard model.

It was confirmed from the previous papers[12,19,21] that the anisotropic model shows quite improvement for normal stresses due to the distribution mechanism by the second order nonlinear terms. This fact affects directly the pressure coefficient and turbulent kinetic energy. The coefficients of pressure along the pipe wall which are most important in the prediction of diffuser flows are seen in fig.5. The difference between computations and experiments increases downstream in the case of the standard k-ε turbulence model. But the anisotropic model reproduces well experimental

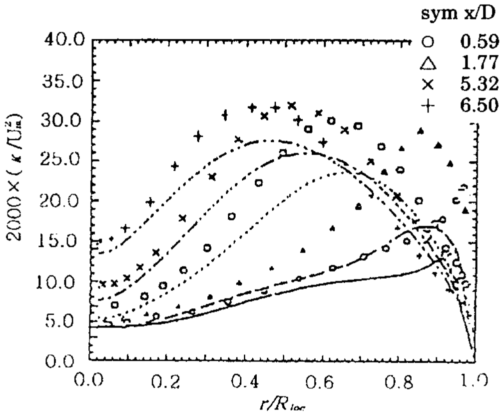


(a) isotropic model

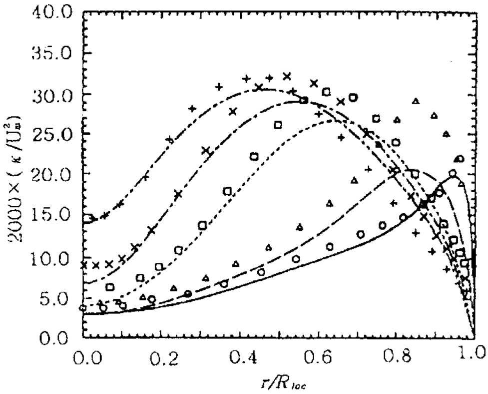


(b) anisotropic model

Fig. 5. Coefficient of pressure along the pipe wall

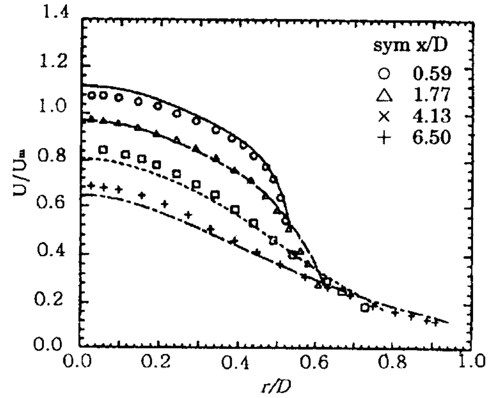


(a) isotropic model

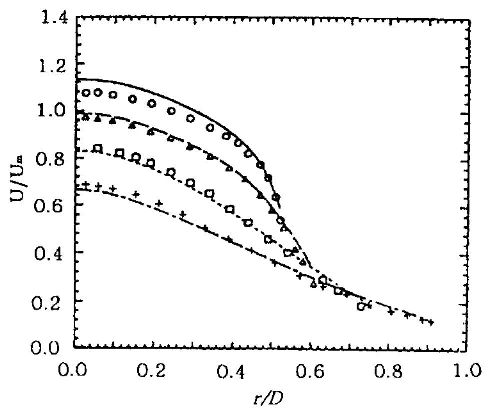


(b) anisotropic model

Fig. 6. Radial distribution of turbulent kinetic energy



(a) isotropic model



(b) anisotropic model

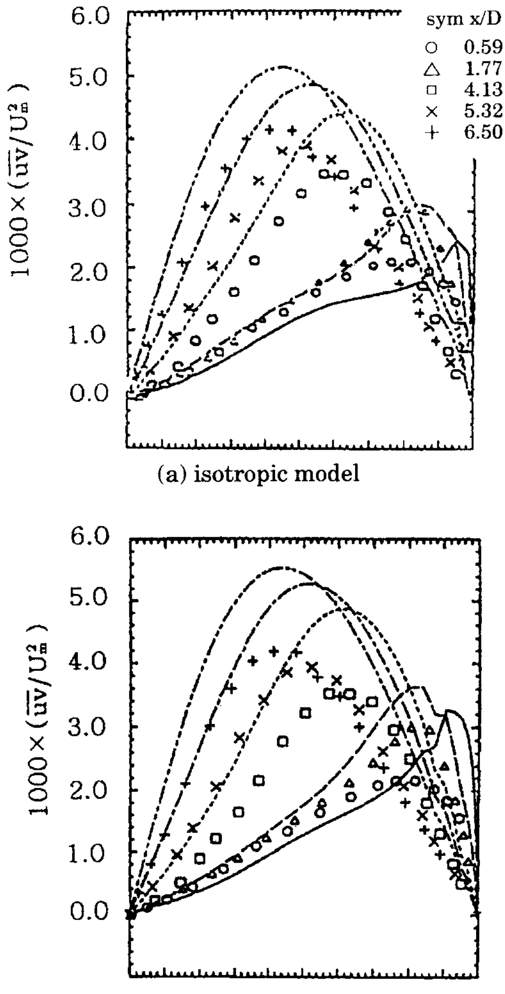
Fig. 7. Radial distribution of Axial velocity

trend because of correction of normal stresses. Fig.6(b) and 7(b) also show good prediction for turbulent kinetic energy and axial velocity respectively. However, it is seen from fig.8(b) that shear stress have little improvement as already noted in the equation (21).

4.3 Results of Modified k-ε Turbulence Models

It was recognized from the calculation by the standard model that shear stress was computated bigger than experiment ; nevertheless, turbulent kinetic energy is smaller than experiment. Therefore modified model 1 is adopted nonlinear terms as Reynolds stress expression for promoting turbulent kinetic energy. With correction on normal stresses, the turbulent kinetic energy production by normal stress terms will be improved so that turbulent kinetic energy can be computed more accurately. This could be recognized in fig.6(b). But the values of shear stress became still big. For solving such problem in model 1, Hanjalic and Launder' s preferen tial normal strain idea using approximately twice for the production of ε from irrotational part in the ε equation was adopted.

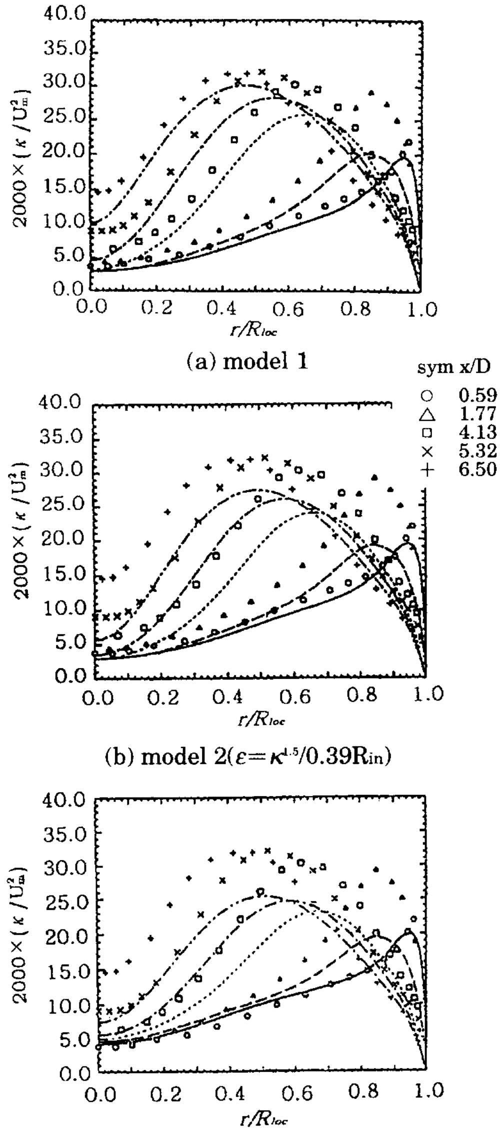
Fig.9(a) shows that turbulent kinetic energy was calcualted less than in anisotropic model due to the large production of the dissipation rate of turbulent kinetic energy(ε). Therefore



(a) isotropic model

(b) anisotropic model

Fig. 8. Radial distribution of shear stress



(a) model 1

(b) model 2($\varepsilon=\kappa^{1.5}/0.39R_{in}$)

(c) model 2($\varepsilon=\kappa^{1.5}/0.59R_{in}$)

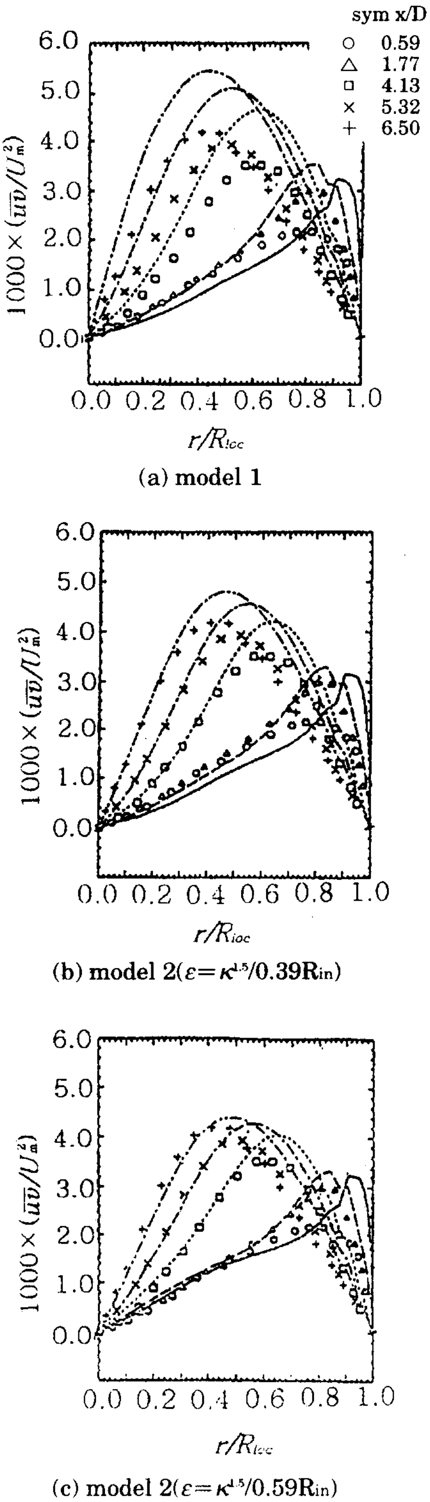
Fig. 9. Radial distribution of turbulent kinetic energy

shear stress became relatively small as compared with anisotropic model. This means that eddy viscosity was evaluated smaller than anisotropic model. By the small diffusion of mean velocity caused by relatively small eddy viscosity, it is seen from fig.11(a), 12(a), and 10(a) that velocity profiles and pressure coefficients were predicted more accurately but shear stresses have still problem.

For an axisymmetric conical diffuser flow, the nondimensional vortex stretching invariant term is written as follows.

$$X=\frac{1}{4}\left(\frac{k}{\varepsilon}\right)^3\left(\frac{\partial U}{\partial r}-\frac{\partial V}{\partial x}\right)^2\frac{V}{r} \tag{25}$$

Because of this vortex stretching, the generation of ε becomes large, and shear stress decreases. The fact causes turbulent kinetic energy to be decreased, therefore turbulent

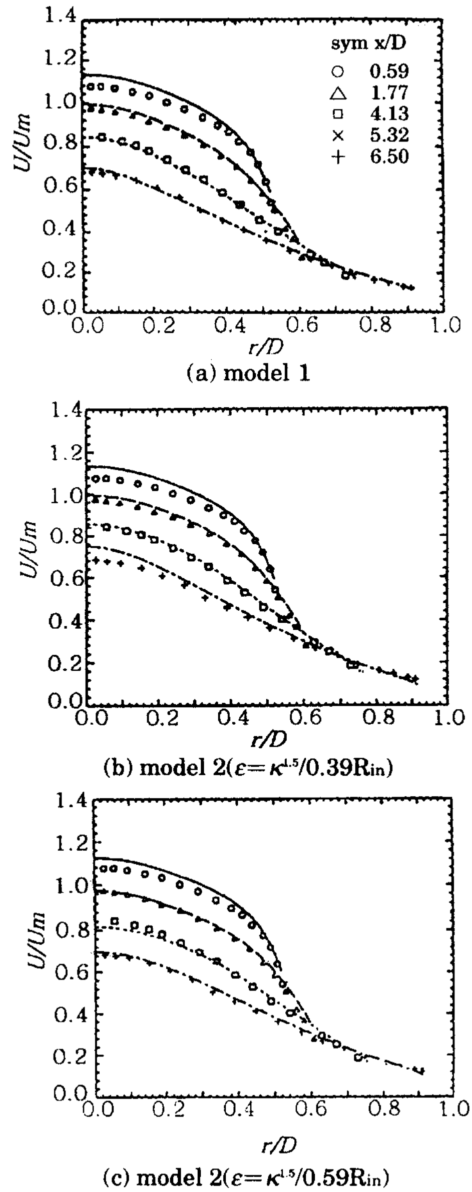


(a) model 1

(b) model 2($\varepsilon=\kappa^{1.5}/0.39R_{in}$)

(c) model 2($\varepsilon=\kappa^{1.5}/0.59R_{in}$)

Fig. 10. Radial distribution of shear stress



(a) model 1

(b) model 2($\varepsilon=\kappa^{1.5}/0.39R_{in}$)

(c) model 2($\varepsilon=\kappa^{1.5}/0.59R_{in}$)

Fig. 11. Radial distribution of axial velocity

diffusion becomes small. From fig.9(b) and 10(b) we can find out that shear stress is well predicted, however turbulent kinetic energy along the centerline is poorly predicted as going toward the exit because of one-sided increase of ε. It is also seen from fig.9(b) and 11(b) that turbulent diffusion caused by tur bulent viscosity is very small near the exit so that velocity profile does not agree with experi ments near the exit.

Fig.10(b) shows the radial distribution of shear stress by model 2. Although the model 1 does not show improvement, the model 2 shows dramatic correction. This means that the distribution mechanism of normal stresses based on anisotropic expression has still some problems. And the fine adjustment of the inlet ε reproduces very well against measurements without making an effect on the coefficient of pressure and velocity(fig.10(c), 11(c), and 12(c)). From the viewpoint of ASM(Algebraic Stress Model), we can obtain the following simplied shear stress[9].

$$-\overline{uv} = \left[\frac{1-C_2}{(P_k/\varepsilon + C_1 - 1)}\frac{\overline{v}^2}{k}\right]\left(\frac{k^2}{\varepsilon}\right)\frac{\partial U}{\partial r} \qquad (26)$$

where C_1 and C_2 are constants. We can conjecture from this equation (26) that it is very difficult to improve the shear stress by correcting only ε for various complex flow fields. The reliable function between produc tion and its dissipation rate is needed.

Fig.13 shows the ratios of normal Reynolds stresses($\overline{vv}/\overline{uu}$) of each model. If simplifying the nonlinear terms by the order of magnitude (in this case, we do not neglect $\frac{\partial U}{\partial x}$), normal Reynolds stresses are written as follows[17].

$$\overline{uu} = -\nu_t\left(2\frac{\partial U}{\partial x}\right) + \frac{2}{3}k + \frac{k}{\varepsilon}\nu_t \left[\left(\frac{\partial U}{\partial x}\right)^2\left(\frac{2}{3}C_1+\frac{2}{3}C_2+\frac{2}{3}C_3\right) + \left(\frac{\partial U}{\partial r}\right)^2\left(\frac{2}{3}C_1 - \frac{1}{3}C_3\right)\right] \qquad (27)$$

$$\overline{vv} = -\nu_t\left(2\frac{\partial V}{\partial r}\right) + \frac{2}{3}k + \frac{k}{\varepsilon}\nu_t \left[\left(\frac{\partial U}{\partial x}\right)\left(-\frac{1}{3}C_1+\frac{1}{3}C_2+\frac{1}{3}C_3\right) + \left(\frac{\partial U}{\partial r}\right)^2\left(-\frac{1}{3}C_1 + \frac{2}{3}C_3\right)\right] \qquad (28)$$

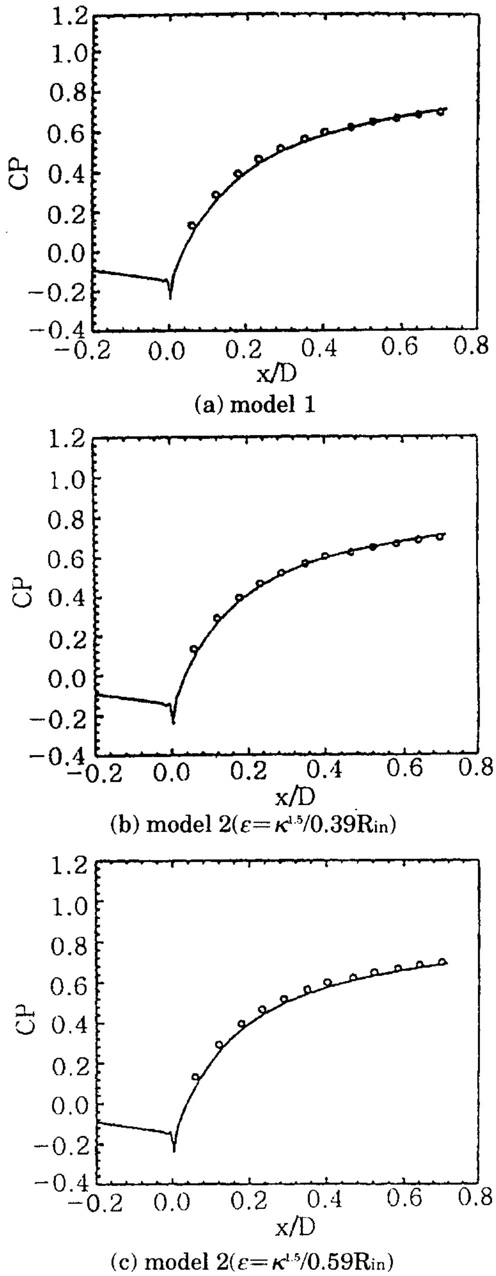


Fig. 12. Coefficient of pressure along the pipe wall

The standard k-ε turbulence model which does not have physically reliable distribution mechanism of normal stresses shows counter-trend in comparision to measurements. But

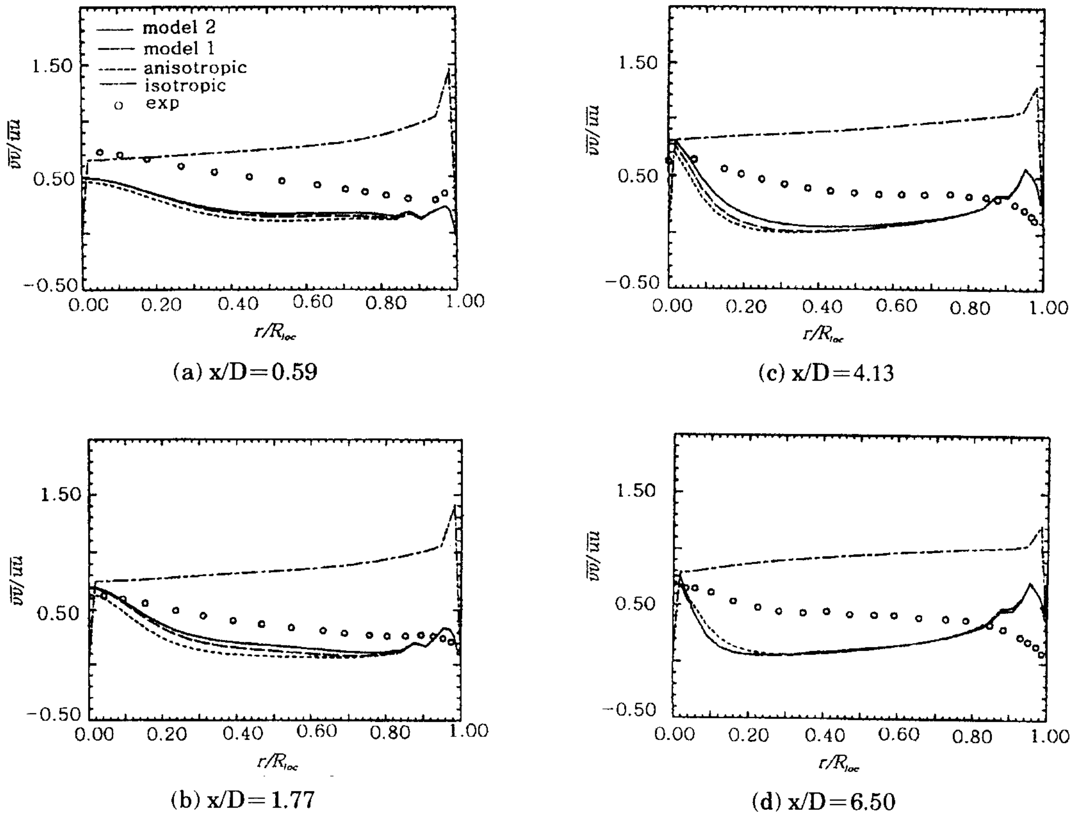


Fig. 13. Ratios of normal Reynolds stresses

modified models show correct-trend of experimental results because of second order nonlinear terms.

5. Conclusions

Evaluation of each model about axisymmetric diffuser flow which has an adverse pressure gradient and streamline curvature shows that modified k-ε turbulence model reproduces a better prediction than the standard k-ε turbulence model compared with the experimental results. Although modified k-ε turbulence models show a better prediction, they have some problems concerning the distribution mechanism of normal stresses as mentioned above. We can conjecture that it is very difficult to improve the shear stress by correcting only ε for various complex flow fields. The reliable function between production and its dissipation rate is needed.

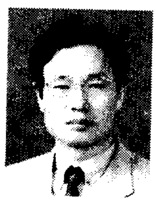
REFERENCES

1. Okwuobi, P. A. C. and Azad, R. S., Turbulence in a Conical Diffuser with Fully Developed Flow at Entry, J. Fluids Mechanics, Vol.57, pp.265-273, 1973.
2. Lai, Y. G., So, R. M. C. and Hwang, B. C., Calculation of Planar and Conical Diffuser Flows, AIAA J., Vol.27, pp.542-548, 1989.
3. Lee, Y. W. and Kobayashi, T. Numerical Simulation of a Conical Diffuser by Using the General Curvilinear Coordinate System, SEISAN-KENKYU, IIS, Univ. of Tokyo, Vol.44,

pp.62-65, 1992.

4. Lakshminarayana, B., Turbulence Modeling for Complex Shear Flows, AIAA J., Vol.24, pp.1900-1917, 1986.
5. Bradshaw, P., REVIEW-Complex Turbulent Flows, J. Fluids Engng, pp.146-154, 1975.
6. Hanjalic, K., Sensitizing the dissipation equation to irrotational strains, J. of Fluids Engng, Vol.102, pp.34-40, 1980.
7. Pope, S. B., An Explanation of the Turbulent Round-Jet/Plane-Jet Anomaly, AIAA J., Vol.23, pp.1308-1319, 1978.
8. Azad, R. S. and Kassab, S. Z. Turbulent Flow in a Conical Diffuser : Overview and Implications, Phys. Fluids A 1(3), pp.564-573, 1989.
9. Kawamura, Y., Introduction of Turbulence Model – Basic Idea, A Report of the Evaluation and Establishment of Turbulence Models, JSME, pp.73-81, 1993.
10. Rodi, W. and Scheuerer, G., Scrutinizing the k-ε Turbulence Model under Adverse Pressure Gradient Conditions, J. of Fluids Engng, Vol.108, pp.174-179, 1986.
11. Shon, C. H. et al., Comparison of Various Turbulence Models for the Calculation of Plane of Symmetry Flows, AIAA J., Vol.16, pp.591-597, 1991.
12. Myong, H. K., Fundamental Studies on Two Equation Turbulence Model for Numerical Predictions of Wall-Bounded Shear Flow and Heat Transfer, Ph.D.Dissertation, The University of Tokyo, 1988.
13. Chen, H. C. and Patel, V. C., Near-Wall Turbulence Models for Complex Flows Including Separation, AIAA Journal, Vol.26, pp.641-648.
14. Peric, M., A Finite Volume Method for the Prediction of Three-Dimensional Fluid Flow in Complex Duct s, Ph.D. Thesis, Imperial College, Univ. of London, 1985.
15. Eiseman, P. R., Alternative Direction Adaptive Grid Generation, AIAA Journal, Vol.22, pp.551-560,1985.
16. Laufer, J., The Structure of Turbulence of Fully Developed Pipe Flow, NACA Rept. 1174, 1954.
17. Lee, Y. W., Kobayashi, T. and Chung, B. S., Numerical Analysis of a Diffuser Flow – Comparison and Verification of k-ε Turbulence Models, The 71th JSME Fall Annual Meeting, Vol.3, pp.479-481, 1993.
18. Singh, D. and Azad, R. S., Turbulent Kinetic Energy Balance in a Conical Diffuser, Proc. of the 7th Biennial Symposium on Turbulence, pp.21-33, 1981.
19. Oomachi, M., Numerical Analysis of Backward Facing Step Flow Using the Anisotropic k-ε Turbulence Model, Master Thesis, The University of Tokyo, 1990.
20. Speziale, C. G., Modeling the Dissipation Rate in Rotating Turbulent Flows, Proceeding of Turbulent Shear Flow, pp.129-151, 1991.

저 자 소 개



이연원(李蓮源)

1958년 8월생. 1981년 경북대학교 공과대학 기계공학과 졸업. 1983년 경북대학교대학원 기계공학과 졸업(석사), 1993년 일본 동경대학 대학원 기계공학과 졸업(박사), 1984년~1986년 포스코개발 설계팀장, 1993년~현재 부경대학교 기계자동차공학부 조교수, 당학회 회원