論文 한국공작기계학회지 제 7 권 제 3 호 1998. 6.
Journal of the Korean Society of Machine Tool Engineers

A Configuration Design Sensitivity Analysis for Kinematically driven Mechanical Systems

D. W. Kim*, S. M. Yang*, H. W. Kim**, D. S. Bae**

Abstract

A continuum-based configuration design sensitivity analysis method is developed for kinematically driven mechanical systems. The configuration design variable for mechanical systems is defined. The 3-1-3 Euler angle is employed as the orientation design variable. Kinematic admissibility conditions of the velocity field are proposed to eliminate the reassembly of a mechanical system after a configuration design change. Direct differentiation method is used to derive the governing equations of the design sensitivity. Numerical examples are presented to demonstrate the validity and effectiveness of the proposed method.

Keyword : Configuration design sensitivity(배치 설계 민감도), direct differential method(직접 미분법), backward difference formula(후진 차분 공식), kinematic admissibility condition(기구학적 허락 조건), velocity field(속도역)

1. Introduction

Analytic sensitivity of a mechanical system due to a design change provides useful information in optimization and what-if analysis.

Configuration design sensitivity analysis methods are well developed in the area of the structural mechanics. Sensitivity analysis of the static response and eigenvalue were performed in Refs. 1 through 7. Twu and Choi[6] developed a continuum configuration design sensitivity analysis

* Chonbuk National University
** Hanyang University

method for static response and eigenvalue, using the material derivative idea developed for shape design sensitivity analysis. Two basic assumptions were used through the development of orientation design sensitivity analysis ; (1) the design component rotates without shape changes and (2) only a small design perturbation is considered. A line and surface design components are considered. This paper extends the Chois' previous works such that the design components can undergo shape change for a general design component. The extended theory is then applied to the sensitivity analysis of a kinematically driven system due to a configuration change.

Design propagation analysis due to a design change of mechanical system has been presented in Ref. 8. Design perturbation of mechanical systems was introduced and all generalized coordinates were reassembled such that all kinematic constraint equations are satisfied. However, the reassembly process may not yield a unique position and orientation. To avoid the nonunique initial position and orientation, this research proposes a configuration design change method. The velocity field of the configuration design change is defined such that all kinematic constraints are satisfied, which eliminates the reassembly process. The body reference frame is fixed during the design change. As a result, the generalized coordinates are not affected by the design change.

The kinematics and design variables of a body are presented in Section 2. Section 3 defines the configuration change of a body. The kinematic admissibility conditions of the velocity field is presented Section 4. Section 5 derives the governing equation of design sensitivity due to a configuration change. The numerical examples are presented in Section 6. Finally, conclusions are drawn in Section 7.

2. Kinematics and Design variables of a body

Orientation matrix of a body in Fig. 2.1 is given as

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = [\mathbf{f} \ \mathbf{g} \ \mathbf{h}] \tag{2.1}$$

where $\mathbf{f}$, $\mathbf{g}$, and $\mathbf{h}$ are the unit vectors on the axes x', y', and z', respectively. The $x'-y'-z'$ frame is the body reference frame and the $X-Y-Z$ frame is the inertial reference frame. The design reference frame on which a design is defined must be specified. The body reference frame $x'-y'-z'$ is chosen as the design reference frame in this paper for convenience. Note that the position and orientation of the $x'-y'-z'$ frame are not affected by a design change.

Since the body shown in Fig. 2.1 is a continuum, each point and orientation on the domain of the body can be theoretically an independent design variable. As an example, the components of vector $\mathbf{s}'_o$ and orientation matrix of the $x''-y''-z''$ frame with respect to the $x'-y'-z'$ frame are candidate design variables. The design orientation matrix can be parameterized by the 3-1-3 Euler angle(θ_1, θ_2, θ_3)as follows.

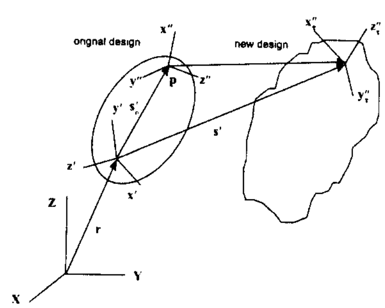

Fig. 2.1 A body and its new design

$$
\mathbf{C} = \begin{bmatrix} c\theta_1 c\theta_3 - s\theta_1 c\theta_2 c\theta_3 & -c\theta_1 s\theta_3 - s\theta_1 c\theta_2 c\theta_3 & s\theta_1 s\theta_2 \\ s\theta_1 c\theta_3 + c\theta_1 c\theta_2 s\theta_3 & -s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3 & -c\theta_1 s\theta_2 \\ s\theta_2 s\theta_3 & s\theta_2 c\theta_3 & c\theta_2 \end{bmatrix} \tag{2.2}
$$

where $s \equiv \sin$ and $c \equiv \cos$. If every points and orientations on the body are taken as independent design variables, these design variables are too many for a practical design consideration and there exists many design constraints among these variables due to the kinematic admissibility conditions. Therefore, configuration design change of a body is proposed in this research. Benefits of the configuration design variable is twofold. First, the number of the design variables can be significantly reduced. Secondly, the velocity field can be selected such that kinematic constraints are satisfied, which eliminates the reassembly process after a design change.

3. Configuration change of a body

Suppose that only one parameter τ defines the transformation $\mathbf{T}$, as shown in Fig. 3.1,

where $\boldsymbol{\Omega}_\tau$, $\boldsymbol{\Gamma}_\tau$, and the $x''_\tau - y''_\tau - z''_\tau$ frame denote the variations of $\boldsymbol{\Omega}$, $\boldsymbol{\Gamma}$, and the $x'' - y'' - z''$ frame by the mapping $\mathbf{T}$,respectively. The mapping, $\mathbf{T} : \begin{bmatrix} s'_o \\ \boldsymbol{\theta}_o \end{bmatrix} \rightarrow \begin{bmatrix} s' \\ \boldsymbol{\theta} \end{bmatrix}$ is given by

$$
\begin{bmatrix} s' \\ \boldsymbol{\theta}(s'_o) \end{bmatrix} = \mathrm{T}(s'_o, \boldsymbol{\theta}_o, \tau) = \begin{bmatrix} s'_o \\ \boldsymbol{\theta}_o \end{bmatrix} + \tau \begin{bmatrix} \mathrm{V}_\Omega(s'_o) \\ \mathrm{U}_\Theta(s'_o) \end{bmatrix} \tag{3.1}
$$

where the subscript "o" denotes the original configuration and $\boldsymbol{\theta}$ is the 3-1-3 Euler angle for the x″-y″-z″ frame with respect to the $x' - y' - z'$ frame. $\mathbf{V}_\Omega$ and $\mathbf{U}_\Theta$ are the shape and orientation design velocity fields, respectively, and are defined by

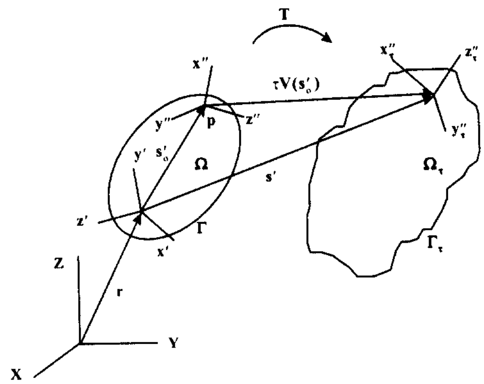

Fig. 3.1 Configuration change by Mapping T

$$
\mathbf{V}_\Omega(s'_o) \equiv \frac{ds'}{d\tau} \tag{3.2}
$$

$$
\mathbf{U}_\Theta(s'_o) \equiv \frac{d\boldsymbol{\theta}}{d\tau} \tag{3.3}
$$

Using the differentiation rule of an orthonormal matrix, $\delta\mathbf{C}$ is obtained as follows.

$$
\delta\mathbf{C} = \widetilde{\delta\xi'}\,\mathbf{C} \tag{3.4}
$$

where $\delta\xi'$ is the virtual rotation of the $x'' - y'' - z''$frame with respect to the $x' - y' - z'$ frame and the skew-symmetric operator tilde associated with of a vector $\mathbf{a} = [a_x\ a_y\ a_z]^T$ is defined as

$$
\tilde{\mathbf{a}} \equiv \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \tag{3.6}
$$

Relationship between $\delta\boldsymbol{\theta}$ and $\delta\xi'$ is given in Ref. 9 as follows.

$$
\delta\xi' = \mathbf{R}\,\delta\boldsymbol{\theta} \tag{3.7}
$$

where

$$
\mathbf{R} \equiv \begin{bmatrix} 0 & -s\theta_1 & s\theta_1 s\theta_2 \\ 0 & c\theta_1 & -c\theta_1 s\theta_2 \\ 1 & 0 & c\theta_2 \end{bmatrix} \tag{3.8}
$$

Using $\delta\boldsymbol{\theta} = \mathbf{U}\,\delta\tau$ from Eq. 3.3, Eq. 3.7 is written as

$$\delta\xi' = \mathbf{RU}\,\delta\tau \tag{3.9}$$

and the differentiation of ξ' with respect to τ is written as follows.

$$\frac{d\xi'}{d\tau} = \mathbf{RU} \tag{3.10}$$

4. Kinematic admissibility conditions of velocity field

When a mechanical system undergoes a configuration design change, the kinematic admissibility conditions at a joint interface must be preserved so that the kinematic constraints are satisfied after a configuration design change. To achieve this goal, the velocity fields must satisfy some geometric conditions at the joint interface. Joints were characterized by a set of elementary constraints[9]. The velocity field must be defined such that kinematic admissibility conditions are satisfied after a design change. They are derived in the following subsections.

4.1 Dot-1 constraint

The Dot-1 constraint represents an orthogonal condition between vectors $\mathbf{a}_i$ and $\mathbf{a}_j$; that is,

$$\Phi^{d1}(\mathbf{a}_i, \mathbf{a}_j) \equiv \mathbf{a}_i^T \mathbf{a}_j = 0 \tag{4.1}$$

Vector $\mathbf{a}_i, \mathbf{a}_j$ are changed by configuration change of bodies i and j using Eq. 3.1 as follow.

$$\mathbf{a}_i = \mathbf{A}_i \mathbf{C}_{ij}(\theta_{0ij} + \tau \mathbf{U}(\mathbf{s}'_{oij}))\mathbf{a}''_i \tag{4.2a}$$

$$\mathbf{a}_j = \mathbf{A}_j \mathbf{C}_{ji}(\theta_{0ji} + \tau \mathbf{U}(\mathbf{s}'_{oji}))\mathbf{a}''_j \tag{4.2b}$$

where $\mathbf{a}''_i$, $\mathbf{a}''_j$ is the vector with respect to $x''-y''-z''$ frame. After a configuration change, Eq. 4.1 must be also satisfied with Eqs. 4.2 and can be rewritten as

$$\Phi^{d1}(\mathbf{a}_i, \mathbf{a}_j) = (\mathbf{a}''_i)^T \mathbf{C}_{ij}^T(\theta_{0ij} + \tau \mathbf{U}(\mathbf{s}'_{oij}))\mathbf{A}_i^T \mathbf{A}_j \mathbf{C}_{ji}(\theta_{0ji} + \tau \mathbf{U}(\mathbf{s}'_{oji}))\mathbf{a}''_j \tag{4.3}$$

4.2 Dot-2 constraint

Orthogonality of the body-fixed vector $\mathbf{a}_i$ and the vector $\mathbf{d}_{ij}$ between body i and body j can be written as

$$\Phi^{d2}(\mathbf{a}_i, \mathbf{d}_{ij}) \equiv \mathbf{a}_i^T \mathbf{d}_{ij} = 0 \tag{4.4}$$

Vector $\mathbf{a}_i, \mathbf{s}_{ij}, \mathbf{s}_{ji}$ are changed by configuration change of bodies i, j using Eq. 3.1 as follow.

$$\mathbf{a}_i = \mathbf{A}_i \mathbf{C}_{ij}(\theta_{0ij} + \tau \mathbf{U}(\mathbf{s}'_{oij}))\mathbf{a}''_i \tag{4.5a}$$

$$\mathbf{s}'_{ij} = \mathbf{s}'_{oij} + \tau \mathbf{V}(\mathbf{s}'_{oij}) \tag{4.5b}$$

$$\mathbf{s}'_{ji} = \mathbf{s}'_{oji} + \tau \mathbf{V}(\mathbf{s}'_{oji}) \tag{4.5c}$$

After a configuration change, Eq. 4.4 must be also satisfied with Eqs. 4.5 and can be rewritten as

$$\Phi^{d2}(\mathbf{a}_i, \mathbf{d}_{ij}) = \mathbf{a}_i^T(\mathbf{r}_j + \mathbf{A}_j(\mathbf{s}'_{oji} + \tau \mathbf{V}(\mathbf{s}'_{oji})) - \mathbf{r}_i - \mathbf{A}_i(\mathbf{s}'_{oij} + \tau \mathbf{V}(\mathbf{s}'_{oij}))) \tag{4.6}$$

4.3 Spherical constraint

A spherical constraint requires a pair of points, O'_{ij} and O'_{ji} on two bodies to coincide. A necessary and sufficient condition is that $\mathbf{d}_{ij} = \mathbf{0}$; i.e.,

$$\Phi^{s}(O'_{ij}, O'_{ji}) \equiv \mathbf{r}_j + \mathbf{A}_j \mathbf{s}'_j - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}'_i = \mathbf{0} \tag{4.7}$$

Vector $\mathbf{s}_{ij}, \mathbf{s}_{ji}$ are changed by configuration change of bodies i, j using Eq. 3.1 as follow.

$$\mathbf{s}'_{ij} = \mathbf{s}'_{oij} + \tau \mathbf{V}(\mathbf{s}'_{oij}) \tag{4.8a}$$

$$\mathbf{s}'_{ji} = \mathbf{s}'_{oji} + \tau \mathbf{V}(\mathbf{s}'_{oji}) \tag{4.8b}$$

After a configuration change, Eq. 4.7 must be also satisfied with Eqs. 4.8 and can be rewritten as

$$\begin{aligned}\boldsymbol{\Phi}^{s}(O_{ij}', O_{ji}') &= \mathbf{r}_j + \mathbf{A}_j(\mathbf{s}_{oji}' + \tau \mathbf{V}(\mathbf{s}_{oji}')) \\ &\quad - \mathbf{r}_i - \mathbf{A}_i(\mathbf{s}_{oij}' + \tau \mathbf{V}(\mathbf{s}_{oij}')) \\ &= 0\end{aligned} \quad (4.9)$$

4.4 Distance constraint

A distance constraint requires a specified distance between a pair of points, O_{ij}' and O_{ji}' on two bodies. A necessary and sufficient condition is that $\mathbf{d}_{ij}^T \mathbf{d}_{ij} = C$; i.e.,

$$\boldsymbol{\Phi}^{dis}(O_{ij}', O_{ji}', C) = \mathbf{d}_{ij}^T \mathbf{d}_{ij} - C = 0 \quad (4.10)$$

Vector $\mathbf{s}_{ij}$, $\mathbf{s}_{ji}$ are changed by configuration change of bodies i, j using Eq. 3.1 as follow.

$$\mathbf{s}_{ij}' = \mathbf{s}_{oij}' + \tau \mathbf{V}(\mathbf{s}_{oij}') \quad (4.11a)$$

$$\mathbf{s}_{ji}' = \mathbf{s}_{oji}' + \tau \mathbf{V}(\mathbf{s}_{oji}') \quad (4.11b)$$

After a configuration change, Eq. 4.10 must be also satisfied with Eqs. 4.11.

5. Governing equation of design sensitivity due to a configuration change

When a mechanical system is kinematically driven, the governing sensitivity equations are obtained by differentiating the constraint equations. Geometry of a system is represented by the kinematic constraints and the driving constraints that uniquely determines the system configuration. The kinematic and driving constraints are expressed in general as follows.

$$\boldsymbol{\Phi}(\mathbf{r}_i, \mathbf{A}_i, \mathbf{s}_i, \mathbf{C}_{ij}, \mathbf{r}_j, \mathbf{A}_j, \mathbf{s}_j, \mathbf{C}_{ji}, t) = \mathbf{0} \quad (5.1)$$

Taking differentiation of Eq. 5.1 with respect to τ yields

$$\boldsymbol{\Phi}_Z \mathbf{Y} + \left(\boldsymbol{\Phi}_D \frac{d\mathbf{D}}{d\tau} \right) = \mathbf{0} \quad (5.2)$$

where $\boldsymbol{\Phi}_z$, $\boldsymbol{\Phi}_D$, $\mathbf{Y}$, and $\frac{d\mathbf{D}}{d\tau}$ are defined as

$$\boldsymbol{\Phi}_Z = [\ \boldsymbol{\Phi}_{r_i}\ \ \boldsymbol{\Phi}_{\pi_i}\ \ \boldsymbol{\Phi}_{r_j}\ \ \boldsymbol{\Phi}_{\pi_j}\ ] \quad (5.3a)$$

$$\boldsymbol{\Phi}_D = [\ \boldsymbol{\Phi}_{s_i'}\ \ \boldsymbol{\Phi}_{\xi_i'}\ \ \boldsymbol{\Phi}_{s_j'}\ \ \boldsymbol{\Phi}_{\xi_j'}\ ] \quad (5.3b)$$

$$\mathbf{Y} \equiv \frac{d\mathbf{Z}}{d\tau} = \left[\left(\frac{d\mathbf{r}_i}{d\tau}\right)^T \left(\frac{d\boldsymbol{\pi}_i}{d\tau}\right)^T \left(\frac{d\mathbf{r}_j}{d\tau}\right)^T \left(\frac{d\boldsymbol{\pi}_j}{d\tau}\right)^T \right]^T \quad (5.3c)$$

$$\frac{d\mathbf{D}}{d\tau} = \left[\left(\frac{d\mathbf{s}_i'}{d\tau}\right)^T \left(\frac{d\boldsymbol{\xi}_i'}{d\tau}\right)^T \left(\frac{d\mathbf{s}_j'}{d\tau}\right)^T \left(\frac{d\boldsymbol{\xi}_j'}{d\tau}\right)^T \right]^T \quad (5.3d)$$

Equation 5.2 can be rewritten as

$$\boldsymbol{\Phi}_Z \mathbf{Y} = -\left(\boldsymbol{\Phi}_D \frac{d\mathbf{D}}{d\tau} \right) \quad (5.4)$$

Using Eq. 3.2 and Eq. 3.10, $\frac{d\mathbf{D}}{d\tau}$ in Eq. 5.3.d can be calculated as

$$\frac{d\mathbf{D}}{d\tau} = \begin{bmatrix} \mathbf{V}(\mathbf{s}_{io}') \\ \mathbf{R}\,\mathbf{U}(\mathbf{s}_{io}') \\ \mathbf{V}(\mathbf{s}_{jo}') \\ \mathbf{R}\,\mathbf{U}(\mathbf{s}_{jo}') \end{bmatrix} \quad (5.5)$$

Considering all bodies of a system, Eq. 5.4 can be written as

$$\boldsymbol{\Phi}_Z \mathbf{Y} = -\left(\boldsymbol{\Phi}_D \frac{d\mathbf{D}}{d\tau} \right) \quad (5.6)$$

where $\mathbf{Y}$ and $\frac{d\mathbf{D}}{d\tau}$ are defined as

$$\mathbf{Y} \equiv [\ \mathbf{Y}_0^T\ \ \mathbf{Y}_1^T\ \ \mathbf{Y}_2^T\ \cdots\ \mathbf{Y}_n^T\ ]^T \quad (5.7a)$$

$$\frac{dD}{d\tau} \equiv \left[\left(\frac{dD_0}{d\tau}\right)^T \left(\frac{dD_1}{d\tau}\right)^T \cdots \left(\frac{dD_n}{d\tau}\right)^T \right]^T \tag{5.7b}$$

6. Numerical Examples

6.1 Slider crank

The slider crank mechanism, shown in Fig. 6.1, is modeled by using three bodies, three revolute joints, one translational joint, and one relative driving constraint. The system is kinematically driven. Joint 1 is driven by sin $2\pi t$. Analysis was carried out for 2.0 sec. X -acceleration of body 1 is given in Fig. 6.2.

The position of Joint 1 is moved from point P to P′, as shown in Fig 6.1. The proposed sensitivity analysis is carried out. The analytic sensitivity and FDM sensitivity of X-acceleration of the $x_1' - y_1'$ frame are shown to be identical in Fig. 6.3, which validates the purposed method.

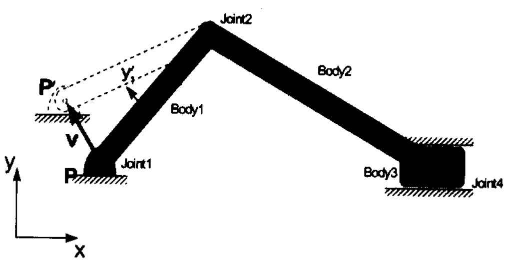

Fig. 6.1 Slider crank mechanism

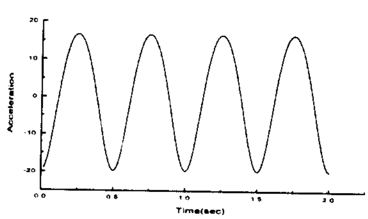

Fig. 6.2 x Acceleration of Body 1

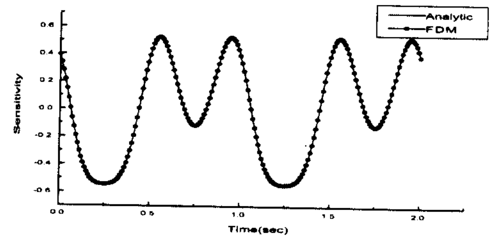

Fig. 6.3 Sensitivity of x-acceleration of Body 1

6.2 MacPherson strut suspension

A macpherson strut suspension system consists of a wheel knuckle, a lower control arm, and a strut, as shown in Fig. 6.4. The chassis and lower control arm are connected by a revolute joint. The steering mechanism is modeled by a rack and pinion, which are connected to the wheel knuckle by a steering rod. The strut is mounted on a chassis with a spherical joint. Since an anti-roll bar would have little influence on suspension kinematics, this is not included in the model. For kinematic analysis, the constraint equations that are generated from two cut-joints, namely spherical constraint for point E and distance constraint from point C to point D and two driving constraint, namely rack and strut drivers are required. Analyses were carried out for 10.0 sec. Z-acceleration of knuckle is given in Fig. 6.6. The position of joint(between lower control arm and base body) is defined as design variable, as shown in Fig 6.5. The proposed sensitivity analysis is carried out and sensitivity of acceleration of knuckle with respect to the position of joint(between lower control arm and base body) change is obtained, as shown in Fig. 6.7. The analytic sensitivity and FDM sensitivity are shown to be identical in Fig. 6.7, which validates the purposed method.

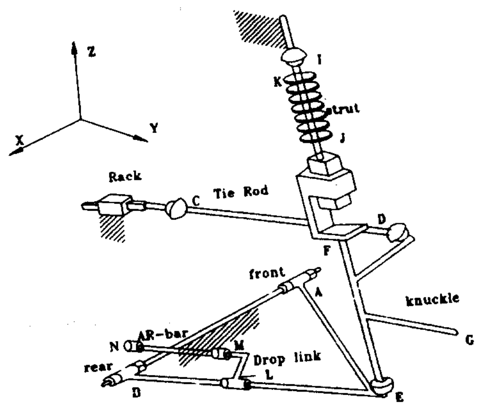

Fig. 6.4 MacPherson strut suspension

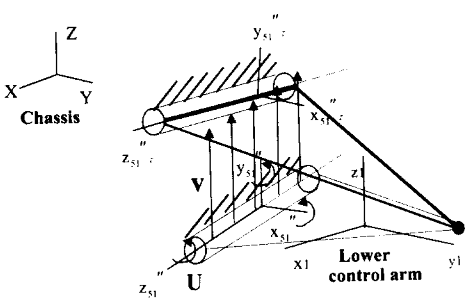

Fig. 6.5 Velocity field applied to the model

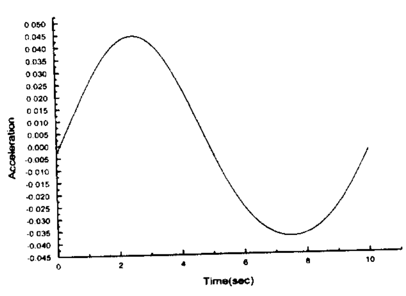

Fig. 6.6 Z acceleration of knuckle

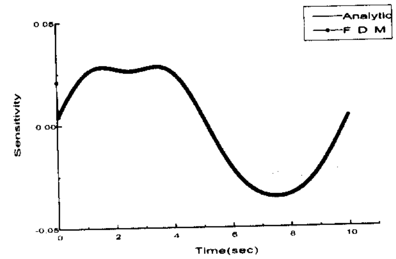

Fig. 6.7 Sensitivity of z acceleration of knuckle

7. Conclusions

A continuum-based configuration design sensitivity analysis method is proposed in this paper. The configuration design variable for mechanical system is defined. The sensitivity equations of motion are formulated, using the direct differentiation method. The design sensitivity analysis of a slider crank and MacPherson strut suspension due to a configuration design change is successfully performed.

References

1. Edward J. Haug, Kyung K. Choi, Vadim Komkov, "Design Sensitivity Analysis of Structural Systems", Mathematics in Science and Engineering Vol. 177, 1986
2. Choi, K. K. and Seong, H. G., "A Domain Method for Shape Design Sensitivity Analysis of Built-up Structures", Computer Methods in Applied Mechanics and Engineering, Vol. 57, pp. 1-15, 1986
3. Choi, K. K. and Seong, J. L. T., "Design Sensitivity Analysis of Nonlinear Structural Systems. Part I : Theory, "International Journal of Numerical Methods in Engineering, Vol. 24, pp. 2039-2055, 1987
4. Park, J. S. and Choi, K. K., "Design Sensitivity Analysis and Optimization of Nonlinear Structural Systems with Critical Loads",

ASME journal of Mechanical Design, to appear, 1991; also presented at the 16th ASME Design Automation Conference, September 1990
5. Choi, K. K. and Lee, J. H., "Sizing Design Sensitivity Analysis of Dynamic Frequency Response of Vibrating Structures", ASME journal of Mechanical Design, to appear, 1990; also presented at the 15th ASME Design Automation Conference, September 1989.
6. Twu, S-L. and Choi, K. K., "Configuration Design Sensitivity Analysis Optimization of Built-up Structures", Technical Report R-91, Center for Simulation and Design Optimization of Mechanical Systems, The University of Iowa, 1990
7. Choi, K. K. and Chang, K-H., "A Prototype of Shape Design Sensitivity Analysis and What-if Workstation", Proc. of 32nd SDM Conference, Paper No. 91-1206, pp.578-587, 1991
8. Hong-Liu Zou, Karim Abdel-Malek, Jia-Yi Wang, "Analytical Criteria for Analysis of Design Propagations in Mechanisms", ASME Design Engineering Technical Conferences and Computers in Engineering Conference, 96-DETC/MECH-1172, pp. 1-10, 1996
9. Parviz E. Nikravesh, "Computer-Aided Analysis of Mechanical Systems", Prentice-Hall International, Inc, 1988
10. Edward J. Haug, "Computer-Aided Kinematics and Dynamics of Mechanical Systems, Volume I: Basic Methods, Allyn and Bacon, Needham Heights, Massachusetts.