研究論文

샤시 프레임에 용접한 스트러트 접합부의 설계 민감도 해석

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Design Sensitivity Analysis of Welded Strut Joints on Vehicle Chassis Frame

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Abstract

Design sensitivity analysis of a vehicle system is an essential tool for design optimization and trade-off studies. Most optimization algorithms require the derivatives of cost and constraint function with respect to design in order to calculate the next improved design. This paper presents an efficient algorithm application for the design sensitivity analysis, using the direct differentiation method. A mounting area of suspension that welded on chassis frame is analyzed to show the validity and the efficiency of the proposed method.

1. Introduction

The theory of sensitivity functions and its applicability to the parameter identification or estimation problem has been established and used successfully more than a decade. Recently, increasing concern has shown to the sensitivity analysis for the design and modification of mechanical system.

The performance of mechanical system design is strongly affected by the accuracy of the model and the influence of the design parameters. Hence, whenever a physical response is calculated from a

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mathematical model there also arises an interest in the sensitivity of that response with respect to parameters of the problems.

This sensitivity information may be used to assess the effect of uncertainties in the mathematical model, to predict the change in response due to a change in parameters, and to optimize a system with the aid of mathematical optimization techniques.

Analytic synthesis methods in designing mechanical systems were proposed by Erdman and his colleges in Ref. 1. Function generation, trajectory generation, and rigid body guidance problems were considered. Though the analytic synthesis methods are powerful in designing a specific mechanism, the numerical methods have been preferred in coping with design of general mechanical systems[2-4].

Numerical optimization has become a routine procedure in designing a structural system. The design optimization method for the structural systems are developed in size, shape, configuration, and topology optimization[5]. In contrast to the structural design area, there exist few general purpose codes that have design optimization capability of mechanical systems. One of the major difficulties is an efficient and reliable analysis of the design sensitivity of a dynamic response due to a design change. As a result, objective of this paper is to develop an efficient and reliable analysis method of the design sensitivity for on welded joints on chassis frame.

Even though the formulations proposed in the previous studies[6]. were general, their applications were relatively simple due to complexity of the formulations. The first fully three dimensional applications are demonstrated by Mani in Ref. 7. The velocity transformation method was used to derive the governing equations of design sensitivity. The formulation complexity problem was resolved by using a symbolic language.

Contribution of this research is as follows. The algorithm for design sensitivity calculation is applied for a practical vehicle system that is consisted of many different types of welded joints to demonstrate its validity and efficiency.

2. Macpherson strut Suspension

The Macpherson strut suspension is shown in Fig. 1. Earle S.Macpherson developed a suspension with geometry similar to the unequal-arm front suspensions using a strut configuration. The strut is a telescopic member incorporating damping with the wheel rigidly attached at its lower end, such that the strut maintains the wheel in the camber direction. The upper end is fixed to the chassis, and the lower end is located by linkages which pick up the lateral and longitudinal forces.

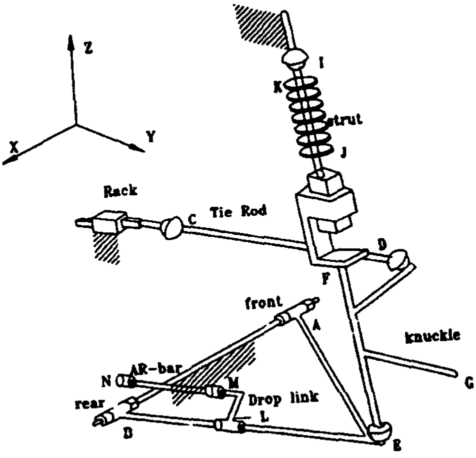

Fig. 1 The MacPherson strut suspension

3. Governing Equations of Design Sensitivity

3.1 Implicit Numerical Integration of Equations of Motion

The variational form of the equations of motion for a constrained mechanical system is as follows

$$\delta q^T (M \dot{v} - Q + \Phi_q^T \lambda) = 0$$

where $\delta\mathbf{q}$ is the virtual displacement vector in Euclidean space $\mathbf{R}^n$, $\dot{\mathbf{v}}$ is the acceleration vector, and λ is the Lagrange multiplier vector for joints in $\mathbf{R}^m$. $\boldsymbol{\Phi}$ represents the position level constraint vector in $\mathbf{R}^m$, and the Jacobian is expressed by $\boldsymbol{\Phi}_q \in \mathbf{R}^{m\times n}$ that is assumed to have full row-rank. The mass matrix $\mathbf{M}$ and the force vector $\mathbf{Q}$ are defined as follow

$$\mathbf{M} = \mathrm{diag}(\mathbf{M}_1, \mathbf{M}_2, \cdot\ \cdot, \mathbf{M}_{nbd})$$
$$\mathbf{Q} = (\mathbf{Q}_1^T, \mathbf{Q}_2^T, \cdot\ \cdot, \mathbf{Q}_{nbd}^T)$$

where nbd denotes the number of bodies. Since $\delta\mathbf{q}$ is arbitrary, the equations of motion are obtained as follow.

$$\mathbf{F}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}, \lambda) = \mathbf{M}\dot{\mathbf{v}} - \mathbf{Q} + \boldsymbol{\Phi}_q^T\lambda = 0$$

The equations of motion for a constrained mechanical system can be implicitly described as

$$\mathbf{v} - \dot{\mathbf{q}} = 0 \tag{3.1.a}$$
$$\mathbf{F}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}, \lambda) = 0 \tag{3.1.b}$$
$$\boldsymbol{\Phi}(\mathbf{q}) = 0 \tag{3.1.c}$$

Successive differentiations of Eq. 3.1.c yield

$$\dot{\boldsymbol{\Phi}}(\mathbf{q}, \mathbf{v}) = \boldsymbol{\Phi}_q\mathbf{v} - \mathbf{v} = 0 \tag{3.2.a}$$
$$\ddot{\boldsymbol{\Phi}}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}) = \boldsymbol{\Phi}_q\dot{\mathbf{v}} - \mathbf{v} = 0 \tag{3.2.b}$$

Equations 3.1 and 3.2 comprise a system of overdetermined differential algebraic equations (ODAEs). An algorithm for the backward differentiation formula (BDF) to solve the ODAEs is given in Ref. 1 as follows.

$$\mathbf{H}(\mathbf{x}) = \begin{bmatrix} \mathbf{F}(\mathbf{x}) \\ \ddot{\boldsymbol{\Phi}} \\ \dot{\boldsymbol{\Phi}} \\ \boldsymbol{\Phi} \\ \mathbf{U}_0^T(\frac{h}{b_0}\mathbf{R}_1) \\ \mathbf{U}_0^T(\frac{h}{b_0}\mathbf{R}_2) \end{bmatrix} = \begin{bmatrix} \mathbf{F}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}, \lambda) \\ \boldsymbol{\Phi}_q\dot{\mathbf{v}} - \mathbf{v} \\ \boldsymbol{\Phi}_q\mathbf{v} - \mathbf{v} \\ \boldsymbol{\Phi}(\mathbf{q}) \\ \mathbf{U}_0^T(\frac{h}{b_0}\dot{\mathbf{v}} - \mathbf{v} - \zeta_1) \\ \mathbf{U}_0^T(\frac{h}{b_0}\mathbf{v} - \mathbf{q} - \zeta_2) \end{bmatrix} = 0 \tag{3.3}$$

where $\zeta_1 \equiv \frac{1}{b_0}\sum_{i=1}^{k} b_i \mathbf{v}_{n-i}$ and $\zeta_2 \equiv \frac{1}{b_0}\sum_{i=1}^{k} b_i \mathbf{q}_{n-i}$ in which k is the order of integration and b_i's are the BDF coefficients. $\mathbf{x} = [\lambda^T, \dot{\mathbf{v}}^T, \mathbf{v}^T, \mathbf{q}^T]^T$ and the columns of $\mathbf{U}_o \in \mathbf{R}^{nr\times(n-m)}$ constitute bases for the parameter space of the position level constraints. $\mathbf{U}_o$ is chosen as $\begin{bmatrix} \boldsymbol{\Phi}_q \\ \mathbf{U}_o^T \end{bmatrix}$, the inverse of which exists. Therefore, the parameter space spanned by the columns of $\mathbf{U}_o$ and the subspace spanned by the columns of $\boldsymbol{\Phi}_q^T$ constitute the entire $\mathbf{R}^n$ space.

The number of equations and the number of unknowns in Eqs. 3.3 are same, so Eqs. 3.3 can be solved. The Newton's numerical method can be applied to obtain the solution $\mathbf{x}$.

$$\mathbf{H}_x\Delta\mathbf{x} = -\mathbf{H} \tag{3.4.a}$$
$$\mathbf{x}^{i+1} = \mathbf{x}^i + \Delta\mathbf{x} \tag{3.4.b}$$

3.2 Implicit Numerical Integration of Equation of Design Sensitivity

A mechanical system consists of bodies, joints, and force elements. Physical properties of these elements are described by various parameters. The parameters are defined as design variables in this research and are denoted by

$$\mathbf{b} = [\, b_1, b_2, \cdot \cdot \cdot, b_k \,]^T \qquad (3.5)$$

Taking the derivative of Eq. 3.3 with respect to the design parameter vector **b** and appending the BDF integration formula yield the following governing equations of design sensitivity$^{(8)}$

$$G(x, x_b) = \begin{bmatrix} \dfrac{dF}{db} \\ \dfrac{d\Phi}{db} \\ \dfrac{d\dot{\Phi}}{db} \\ \dfrac{d\ddot{\Phi}}{db} \\ h' U_0^T (R_3)_b \\ h' U_0^T (R_4)_b \end{bmatrix} = \begin{bmatrix} F_q q_b + F_v v_b + F_{\dot{v}} \dot{v}_b + F_\lambda \lambda_b + F_b \\ \Phi_q q_b + \Phi_b \\ \dot{\Phi}_q q_b + \dot{\Phi}_v v_b + \dot{\Phi}_b \\ \ddot{\Phi}_q q_b + \ddot{\Phi}_v v_b + \ddot{\Phi}_{\dot{v}} \dot{v}_b + \ddot{\Phi}_b \\ U_0^T (h' \dot{v}_b - v_b - \zeta_1) \\ U_0^T (h' v_b - q_b - \zeta_2) \end{bmatrix} = 0 \qquad (3.6)$$

where $h' \equiv \dfrac{h}{b_0}$ and $x_b = [\, \lambda_b^T, \dot{v}_b^T, v_b^T, q_b^T \,]^T$.

Equations 3.6 comprises the same number of equations as the unknowns. The Newton's numerical method can be applied to Eqs. 3.6 for the solution x_b as

$$G_{x_b} \Delta x_b = -G \qquad (3.7.a)$$

$$x_b^{i+1} = x_b^i + \Delta x_b \qquad (3.7.b)$$

The below subset equations of Eqs. 3.7.a will be considered first,

$$F_q \Delta q_b + F_v \Delta v_b + F_{\dot{v}} \Delta \dot{v}_b + F_\lambda \Delta \lambda + F_b = -\frac{dF}{db} \qquad (3.8.a)$$

$$\Phi_q \Delta q_b + \Phi_b = -\frac{d\Phi}{db} \qquad (3.8.b)$$

$$h' \Delta \dot{v}_b - \Delta v_b + h' R_3(x) = 0 \qquad (3.8.c)$$

$$h' \Delta v_b - \Delta q_b + h' R_4(x) = 0 \qquad (3.8.d)$$

It can be easily shown that any solution Δx_b satisfying Eqs. 3.8 is also the solution of Eq. 3.7.a. The Δv_b and $\Delta \dot{v}_b$ in Eqs. 3.8.c and 3.8.d are obtained in terms of Δq_b as follows.

$$\Delta v_b = \frac{1}{h'} \Delta q_b - R_4(x) \qquad (3.9.a)$$

$$\Delta \dot{v}_b = \frac{1}{h'^2} \Delta q_b - \frac{1}{h'} R_4(x) - R_3(x) \qquad (3.9.b)$$

Substituting Eqs. 3.9 into Eq. 3.8.a and multiplying both sides of Eq. 3.8.a by h'^2 yields

$$K^* \Delta q_b + h'^2 \, \Phi_q{}^T \Delta \lambda_b = R_5 \qquad (3.10)$$

where $F_\lambda = \Phi_q$ is used and K^* and R_5 are defined by

$$K^* \equiv h'^2 F_q + h' F_v + F_{\dot{v}}$$

$$R_5 \equiv -h'^2 \frac{dF}{db} + h'(h' F_v + F_{\dot{v}}) R_4 + h'^2 F_{\dot{v}} R_3$$

Equations 3.10 and 3.8.b can be rewritten in a matrix form as

$$\begin{bmatrix} K^* & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \Delta q_b \\ \Delta \lambda_b \end{bmatrix} = \begin{bmatrix} R_5 \\ -\dfrac{d\Phi}{db} \end{bmatrix} \qquad (3.11)$$

Equation 3.12 can be solved for Δq_b, $\Delta \lambda_b$.

To solve Δv_b, Eq. 3.7.a is rewritten as

$$U_0^T (h' \Delta v_b) = U_0^T (\Delta q_b - h' R_4) \qquad (3.12)$$

Without loss of generality U_0^T can be chosen as $N^T K^*$ and Eq. 3.12 can be rewritten as follows.

$$N^T K^*(h'\Delta v_b) = N^T K^*(\Delta q_b - h' R_4) \qquad (3.13)$$

where the N is chosen such that $\Phi_q N = 0$. Since the N is a null space of the Φ_q, Eq. 3.13 can be rewritten in a different form as

$$K^*(\Delta v_b) + \Phi_q^T \tau_1 = K^*(cj\,\Delta q_b - R_4) \qquad (3.14)$$

Equations 3.14 and 3.7.a can be solved for Δv_b by

$$\begin{bmatrix} K^* & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \Delta v_b \\ \tau_1 \end{bmatrix} = \begin{bmatrix} K^*(cj\,\Delta q_b - R_4) \\ -\dfrac{d\dot{\Phi}}{db} - \dot{\Phi}_q \Delta q_b - \dot{\Phi}_b \end{bmatrix} \qquad (3.15)$$

The Δv_b can be obtained by taking a similar process to the Δv_b as

$$\begin{bmatrix} K^* & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \Delta \dot{v}_b \\ \tau_2 \end{bmatrix} = \begin{bmatrix} K^*(cj\,\Delta v_b - R_3) \\ \dfrac{d\ddot{\Phi}}{db} - \ddot{\Phi}_v \Delta v_b - \ddot{\Phi}_q \Delta q_b - \ddot{\Phi}_b \end{bmatrix} \qquad (3.16)$$

The formulation presented above are implemented as in Fig. 2

4. Numerical Example

To show the validity of the proposed formulation, dynamic analysis of a passenger vehicle is performed. The Macpherson strut and multi-link suspensions are employed as its front and rear suspensions. The list of vehicle parameters and their nominal values assumed in analysis is given in Table 1.

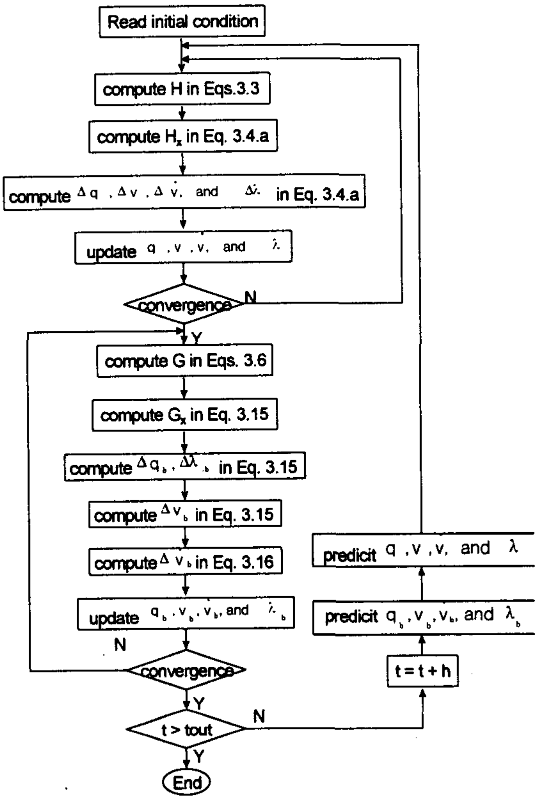

Fig. 2 Solution algorithm of design sensitivity analysis

J-turn simulation of the vehicle is carried out with the initial velocity of 80km/h and the step steering input shown in Fig. 3.

Generally the mounting area is welded and exposed to a fracture due to the load transfer from the suspension system. Fig. 4 shows reaction force acting on the mounting point of the strut. When the applied fluctuating force has varying amplitude, it will cause enlargement of the welded defect. The damping coefficients of the suspension system are chosen as the design variable to observe the effect of the damping coefficient on the welded strut joints. The proposed sensitivity analysis is carried out and sensitivities of the welded strut joints with respect to the damping coefficient change are obtained.

Table 1. Vehicle data for sensitivity analysis

	Body	Mass(kg)	Moment of inertia($kg \cdot m^2$)
Front	Chassis	1460.0	484, 2344 , 2245
	Rack	1.0	1.0 , 1.0 , 2.0
	Lower control arm	3.0	2.0 , 4.0 , 2.0
	Tie rod	5.0	4.0 , 4.0 , 4.0
	Knuckle	4.0	3.0 , 6.0 , 3.0
	strut	2.0	1.0 , 1.0 , 2.0
	Spring constant	18639 N/m	
	Damping coefficient	1386 Ns/m	
Rear	Body	Mass(kg)	Momemt of inertia($kg \cdot m^2$)
	Strut	2.0	2.0 , 3.0 , 2.0
	Knuckle	3.0	3.0 , 4.0 , 3.0
	Camber control arm	2.0	2.0 , 3.0 , 2.0
	Toe control arm	2.0	1.0 , 1.0 , 2.0
	Trail link	2.0	2.0 , 2.0 , 2.0
	Spring constant	21582 N/m	
	Damping coefficient	1021 Ns/m	

The sensitivity results are validated against these obtained from the finite difference method(FDM). Since the sensitivities are very small, the error tolerance of the integration must be maintained to be very small. Otherwise, accurate FDM results can not be obtained. The error tolerance of 10^{-5} was used for this example. The analytic sensitivity and FDM sensitivity are shown to be identical in Fig. 5, which validates the purposed method. The sensitivity analysis was performed on a IBM compatible computer(266 Mhz) and took 10 min. This indicates that the sensitivity analysis of a fairly complicated system can be done quickly.

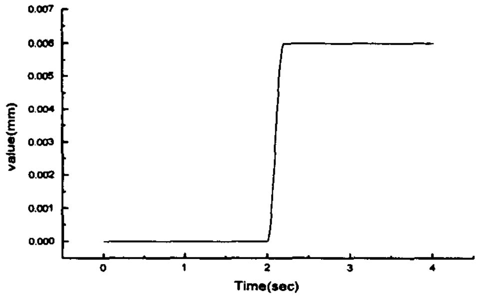

Fig. 3 Step function

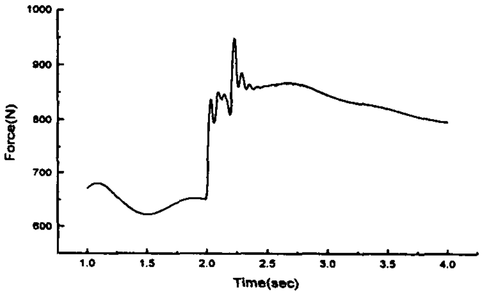

Fig. 4 Reaction force acting on the mounting point of the strut

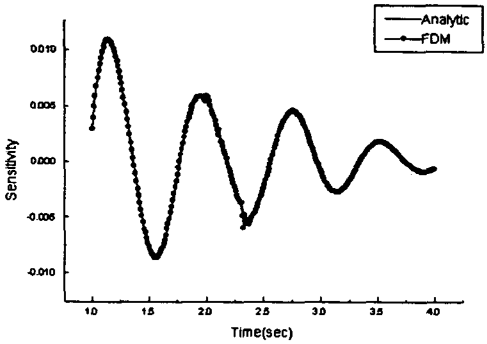

Fig. 5 Sensitivity of the reaction force on the welded area of the strut

5. Conclusions

A design sensitivity analysis method is proposed in this paper. Algorithms needed for sensitivity analysis is developed, and this makes possible to predict the parametric sensitivity. Sensitivities of the reaction force on welded chassis mounting area due to a damping coefficient change are obtained. The computing time indicated that sensitivity based design iteration of large scale mechanical systems is possible on the PC level computers with the proposed method.

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