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ON FUZZY FC-COMPACTNESS

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ABSTRACT. The purpose of this paper is to introduce and study the concept of fuzzy FC-compactness for fuzzy topological spaces.

1. Introduction and Preliminaries

The concept of fuzzy compactness was introduced by Chang [2] and soon after various covering properties of fuzzy topological spaces were discussed by many authors. In general topology, feebly open sets and relevant topological theory were introduced in [10] and [15]. In this paper, we introduce and study the concepts of fuzzy feebly open sets, fuzzy feebly closed sets and by using them, we explain the notions of fuzzy FC-compactness and several fuzzy mappings(fuzzy feebly irresolute and fuzzy feebly continuous, etc) for fuzzy topological spaces and show their basic properties. Throughout this paper $(X, \mathcal{T})$ (or simply X), we shall mean a fuzzy topological spaces(fts, for short) in Chang's [2] sense. A fuzzy point [11] with support $x \in X$ and the value r $(0 < r \leq 1)$ at $x \in X$ will be denoted by x_r, and for fuzzy set A, $x_r \in A$ iff $r \leq A(x)$. For two fuzzy sets A and B, we shall write AqB to mean that A is quasi-coincident(q-coincident, for short) with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$ [11], and B is said to be a q-neighborhood (q-nbd,for short) of A if there is a fuzzy open set U with $AqU \leq B$. If A is not q-coincident with B, then we write $A \not q B$. For a fuzzy set A in an fts X, $\mathrm{Cl}A, \mathrm{Int}A, A'$(or $1 - A$) denote the closure, interior, complement of A, respectively. By 0_X and 1_X we mean the constant fuzzy sets taking on the values 0 and 1 on X, respectively.

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DEFINITION 1.1. ([1]). Let A be a fuzzy set of an fts $(X, \mathcal{T})$. Then A is said to be

(a) *fuzzy semiopen* if there is an $U \in \mathcal{T}$ such that $U \leq A \leq \mathrm{Cl}U$,
(b) *fuzzy semiclosed* if there is an $F' \in \mathcal{T}$ such that $\mathrm{Int}F \leq A \leq F$.

LEMMA 1.2. ([1]). *The following are equivalent:*

(a) *A is a fuzzy semiclosed set,*
(b) *A' is a fuzzy semiopen set,*
(c) $\mathrm{IntCl}A \leq A$,
(d) $A' \leq \mathrm{ClInt}A'$.

DEFINITION 1.3. ([17]). Let A be a fuzzy set of an fts X. Then:

(a) the *fuzzy semi-closure of* A, denoted by $\mathrm{Scl}A$, is the intersection of all fuzzy semiclosed sets containing A,
(b) the *fuzzy semi-interior of* A, denoted by $\mathrm{Sint}A$, is the union of all fuzzy semiopen sets contained in A.

LEMMA 1.4. ([6]). *For a fuzzy set A in an fts X, $\mathrm{Scl}A$ is the union of all fuzzy points x_r such that every fuzzy semiopen U with $x_r q U$ is q-coincident with A.*

LEMMA 1.5. *Let A be a fuzzy set of an fts X. Then:*

(a) $\mathrm{Scl}A = A$ *iff A is fuzzy semiclosed,*
(b) $\mathrm{Sint}A = A$ *iff A is fuzzy semiopen,*
(c) $\mathrm{Int}A \leq \mathrm{Sint}A \leq A \leq \mathrm{Scl}A \leq \mathrm{Cl}A$ ([18]).

LEMMA 1.6. ([9]). *Let A and B be fuzzy sets in an fts X. Then:*

(a) *if $A \cap B = 0_X$, then $A \not{q} B$,*
(b) *$A \leq B$ iff $x_r q B$ for each $x_r q A$,*
(c) *$A \not{q} B$ iff $A \leq B'$,*
(d) *$x_r q(\cup_{\alpha\in\Lambda} A_\alpha)$ iff there is $\alpha_0 \in \Lambda$ such that $x_r q A_{\alpha_0}$.*

DEFINITION 1.7. A mapping $f : X \longrightarrow Y$ is said to be

(a) *fuzzy continuous* ([2]) if $f^{-1}(U)$ is fuzzy open in X for each fuzzy open set U in Y,
(b) *fuzzy irresolute* ([13]) if $f^{-1}(U)$ is fuzzy semiopen in X for each fuzzy semiopen set U in Y,
(c) *fuzzy semi-continuous* ([1]) if $f^{-1}(U)$ is fuzzy semiopen in X for each fuzzy open set U in Y.

LEMMA 1.8. ([1]). *Let* $\{A_\alpha \mid \alpha \in \Lambda\}$ *be a family of fuzzy sets of an fts* X. *Then* $\cup \text{Cl} A_\alpha \leq \text{Cl} \cup A_\alpha$ *and* $\cup \text{Int} A_\alpha \leq \text{Int} \cup A_\alpha$. *In case* Λ *is a finite set,* $\cup \text{Cl} A_\alpha = \text{Cl} \cup A_\alpha$.

2. Fuzzy feebly open sets and fuzzy feebly closed sets

DEFINITION 2.1. Let A be a fuzzy set of an fts X. Then A is said to be

(a) *fuzzy feebly open* if there exists fuzzy open set U in X such that $U \leq A \leq \text{Scl} U$,
(b) *fuzzy feebly closed* if its complement is fuzzy feebly open.

REMARK 2.2. It is obvious that every fuzzy open set is fuzzy feebly open and every fuzzy feebly open set is fuzzy semiopen, but the converses may not be true.

EXAMPLE 2.3. Let A, B, C be fuzzy sets of $X = \{a, b, c\}$ defined as follows:

$$A(a) = 0.5 \quad A(b) = 0.4 \quad A(c) = 0.4,$$
$$B(a) = 0.6 \quad B(b) = 0.7 \quad B(c) = 0.8,$$
$$C(a) = 0.5 \quad C(b) = 0.4 \quad C(c) = 0.6.$$

(1) Consider the fuzzy topology $\mathcal{T}_1 = \{0_X, 1_X, C\}$ on X. Then $C \leq B \leq \text{Scl} C = 1_X$. Thus B is fuzzy feebly open, but B is not fuzzy open.
(2) Consider the fuzzy topology $\mathcal{T}_2 = \{0_X, 1_X, A\}$ on X. Then $A \leq C \leq \text{Cl} A$. Thus C is fuzzy semiopen. Clearly $\text{Scl} A = A$, then $A \leq C \not\leq \text{Scl} A$. Hence C is not fuzzy feebly open.

DEFINITION 2.4. Let A be a fuzzy set of an fts X. Then:

(a) the *fuzzy feeble closure of* A, denoted by $\text{Fcl} A$, is the intersection of all fuzzy feebly closed sets containing A,
(b) the *fuzzy feeble interior of* A, denoted by $\text{Fint} A$, is the union of all fuzzy feebly open sets contained in A.

It is evident that $\text{Fcl} A = A$ iff A is fuzzy feebly closed set and $\text{Fint} A = A$ iff A is fuzzy feebly open.

DEFINITION 2.5. A fuzzy set A in an fts X is called a *fuzzy feeble q-nbd of a fuzzy point* x_r if there exists a fuzzy feebly open U in X such that $x_r q U \leq A$.

LEMMA 2.6. (a) *Let x_r and A be a fuzzy point, a fuzzy set, resp., in a fts X. Then $x_r \in A$ iff x_r is not q-coincident with A'* ([11]).

(b) *Let A, B be fuzzy open sets in an fts X with $A \not{q} B$. Then $A \not{q} \mathrm{Cl}B$ and $\mathrm{Cl}A \not{q} B$* ([14]).

THEOREM 2.7. *Let A be a fuzzy set of an fts X. Then $\mathrm{Fcl}A$ is the set of all fuzzy point x_r such that every fuzzy feebly open feeble q-nbd of x_r is q-coincident with A.*

PROOF. Let F be the intersection of all fuzzy feebly closed sets containing A and let $x_r \in F$.

Suppose there is a fuzzy feebly open feeble q-nbd V of x_r such that $V \not{q} A$. Then $A \leq 1 - V$. Since $1 - V$ is fuzzy feebly closed, $F \leq 1 - V$. But $x_r \notin 1 - V$. Thus $x_r \notin F$, which contradicts that $x_r \in F$.

Conversely, suppose $x_r \notin F$. Then there is a fuzzy feebly closed set $G \geq A$ with $x_r \notin G$, so G' is fuzzy feebly open with $x_r q G'$ and $A \not{q} G'$. Hence $x_r \notin \mathrm{Fcl}A$. □

LEMMA 2.8. *Let A be a fuzzy set in an fts X. Then:*

(a) $\mathrm{IntClIntCl}A = \mathrm{IntCl}A$ *and* $\mathrm{ClIntClInt}A = \mathrm{ClInt}A$,

(b) $(\mathrm{IntCl}A)' = \mathrm{ClInt}A'$ *and* $(\mathrm{ClInt}A)' = \mathrm{IntCl}A'$.

PROOF. (a) Let's show that $\mathrm{IntCl}A = \mathrm{IntClIntCl}A$. Since $\mathrm{IntCl}A$ is fuzzy open and $\mathrm{IntCl}A \leq \mathrm{ClIntCl}A$, $\mathrm{IntCl}A = \mathrm{Int}(\mathrm{IntCl}A) \leq \mathrm{Int}(\mathrm{ClIntCl}A)$.

Conversely, since $\mathrm{IntCl}A \leq \mathrm{Cl}A$ and $\mathrm{Cl}A$ is fuzzy closed, $\mathrm{Cl}(\mathrm{IntCl}A) \leq \mathrm{Cl}(\mathrm{Cl}A) = \mathrm{Cl}A$, so $\mathrm{Int}(\mathrm{ClIntCl}A) \leq \mathrm{IntCl}A$. Thus $\mathrm{IntCl}A = \mathrm{IntClIntCl}A$. The other proof is similar.

(b) It is clear and hence is omitted. □

LEMMA 2.9. *Let A be a fuzzy set in an fts X. Then* $\mathrm{IntCl}A \leq \mathrm{Scl}A$.

PROOF. Let $x_r \in \mathrm{IntCl}A$. Then $r \leq \mathrm{IntCl}A(x)$, i.e., $r \leq \mathrm{Cl}A(x)$, so $x_r \in \mathrm{Cl}A$. Thus for each fuzzy open q-nbd V of x_r, VqA.

Furthermore, since V is fuzzy open , V is fuzzy semiopen. Therefore $x_r \in \mathrm{Scl}A$. □

THEOREM 2.10. *Let A be a fuzzy open set in an fts X. Then* $\mathrm{IntCl}A = \mathrm{Scl}A$.

PROOF. By Lemma 2.9, it suffices to show that $\mathrm{Scl}A \leq \mathrm{IntCl}A$. Let $x_r \notin \mathrm{IntCl}U$. Then $x_r q(\mathrm{IntCl}A)'$ i.e., $x_r q \mathrm{ClInt}A'$.

Furthermore, by Lemma 2.8, $\mathrm{ClInt}A' = \mathrm{ClIntClInt}A'$ and hence $\mathrm{ClInt}A' \leq \mathrm{ClInt}(\mathrm{ClInt}A')$, so $\mathrm{ClInt}A'$ is fuzzy semiopen. But $A \not{q} \mathrm{ClInt}A'$; hence $x_r \notin S\mathrm{Cl}A$. This shows that $\mathrm{Scl}U \leq \mathrm{IntCl}U$. Thus $\mathrm{IntCl}A = \mathrm{Scl}A$. □

THEOREM 2.11. *Let A be a fuzzy set in an fts X. Then A is fuzzy feebly open iff $A \leq \mathrm{IntClInt}A$.*

PROOF. ($\Rightarrow$) Let A be a fuzzy feebly open set in an fts X. Then there is a fuzzy open set U such that $U \leq A \leq \mathrm{Scl}U$. By Theorem 2.10, $U \leq A \leq \mathrm{IntCl}U$. Since U is fuzzy open, $U = \mathrm{Int}U \leq \mathrm{Int}A$. Thus $A \leq \mathrm{IntCl}U \leq \mathrm{IntClInt}A$.

($\Leftarrow$) Let $A \leq \mathrm{IntClInt}A$. By Lemma 2.9, $\mathrm{Int}A \leq A \leq \mathrm{Scl}(\mathrm{Int}A)$. Thus A is fuzzy feebly open in X. □

THEOREM 2.12. *Let A be a fuzzy set in an fts X. Then:*

(a) $\mathrm{Int}A \leq \mathrm{Fint}A \leq \mathrm{Sint}A \leq A \leq \mathrm{Scl}A \leq \mathrm{Fcl}A \leq \mathrm{Cl}A$,
(b) $\mathrm{Fcl}(1-A) = 1 - \mathrm{Fint}A$,
(c) $\mathrm{Fint}(1-A) = 1 - \mathrm{Fcl}A$.

PROOF. (a) Note that $\mathrm{Int}A \leq \mathrm{Sint}A \leq A \leq \mathrm{Scl}A \leq \mathrm{Cl}A$ ([18]). Then we show that only $\mathrm{Scl}A \leq \mathrm{Fcl}A$.

Let $x_r \notin \mathrm{Fcl}A$. Then there is a fuzzy feeble q-nbd V of x_r such that $V \not{q} A$, so there is a fuzzy feebly open set U in X such that $x_r q U \leq V$ and $U \not{q} A$. Since U is fuzzy feebly open, U is fuzzy semiopen. Thus $x_r \notin \mathrm{Scl}A$. This shows that $\mathrm{Scl}A \leq \mathrm{Fcl}A$.

(b) By definitions of fuzzy feeble closure and interior, we can let

$$\mathrm{Fcl}A = \cap\{F \mid F \text{ is fuzzy feebly closed set and } A \leq F\}$$

and

$$\mathrm{Fint}A = \cup\{U \mid U \text{ is fuzzy feebly open set and } U \leq A\}.$$

Then

$$\begin{aligned} 1 - \mathrm{Fint}A &= 1 - \cup\{U \mid U \text{ is fuzzy feebly open set and } U \leq A\} \\ &= \cap\{1 - U \mid U \text{ is fuzzy feebly open set and } U \leq A\} \\ &= \cap\{V \mid V \text{ is fuzzy feebly closed set and } V \geq 1 - A\} \\ &= \mathrm{Fcl}(1-A). \end{aligned}$$

(c) The proof is similar to (b). □

THEOREM 2.13. *Let A be a fuzzy set in an fts X. Then A is fuzzy feebly closed iff* $\mathrm{ClIntCl}A \leq A$.

PROOF. It follows from Theorem 2.11 and Theorem 2.12. □

LEMMA 2.14. *Let A be a fuzzy set in an fts X. Then*
(a) $\mathrm{Cl}(\mathrm{Fcl}A) = \mathrm{Cl}A$,
(b) $\mathrm{ClIntCl}A \leq \mathrm{Fcl}A$.

PROOF. (a) Since $A \leq \mathrm{Fcl}A \leq \mathrm{Cl}A$, $\mathrm{Cl}A \leq \mathrm{Cl}(\mathrm{Fcl}A) \leq \mathrm{Cl}A$. Thus $\mathrm{Cl}A = \mathrm{Cl}(\mathrm{Fcl}A)$.

(b) Since $\mathrm{Fcl}A$ is fuzzy feebly closed, $\mathrm{ClIntClFcl}A \leq \mathrm{Fcl}A$ by Theorem 2.13. Thus $\mathrm{ClIntCl}A \leq \mathrm{Fcl}A$ by (a). □

THEOREM 2.15. *Let A be a fuzzy open set in an fts X. Then* $\mathrm{Fcl}A = \mathrm{ClIntCl}A$.

PROOF. By Lemma 2.14, $\mathrm{ClIntCl}A \leq \mathrm{Fcl}A$.

Conversely, we show that $\mathrm{Fcl}A \leq \mathrm{ClIntCl}A$. Let $x_r \notin \mathrm{ClIntCl}A$. Then $x_r q \mathrm{IntClInt}A'$. Since A is a fuzzy open, $A \leq \mathrm{ClIntCl}A$, so $A' = 1 - A \geq 1 - \mathrm{ClIntCl}A = \mathrm{IntClInt}A'$. But $\mathrm{Int}A' \leq \mathrm{IntClInt}A' \leq \mathrm{Scl}(\mathrm{Int}A')$. It follows that $\mathrm{IntClInt}A'$ is a feebly open with $x_r q \mathrm{IntClInt}A'$ and $A \not{q} \mathrm{IntClInt}A'$. Therefore $x_r \notin \mathrm{Fcl}A$. This means that $\mathrm{Fcl}A \leq \mathrm{ClIntCl}A$. □

THEOREM 2.16. *Let A and B be fuzzy sets in an fts X. Then the following are true:*
(a) $\mathrm{Fcl}(0) = 0$,
(b) *if* $A \leq B$, *then* $\mathrm{Fcl}A \leq \mathrm{Fcl}B$,
(c) *if U is a fuzzy feebly open set, then UqA iff* $Uq\mathrm{Fcl}A$,
(d) $\mathrm{Fcl}A = \mathrm{FclFcl}A$,
(e) $\mathrm{Fcl}(A \cup B) = \mathrm{Fcl}A \cup \mathrm{Fcl}B$.

PROOF. (a) and (b) are obvious.

(c) Let $U \not{q} A$. Then $A \leq U'$ and hence $\mathrm{Fcl}A \leq \mathrm{Fcl}U' = U'$. Thus $U \not{q} \mathrm{Fcl}A$.

Conversely, let $U \not{q} \mathrm{Fcl}A$. Then $\mathrm{Fcl}A \leq U'$, so $A \leq U'$. Thus $U \not{q} A$.

(d) Since $\mathrm{Fcl}A \leq \mathrm{FclFcl}A$, it suffices to show that $\mathrm{FclFcl}A \leq \mathrm{Fcl}A$. Let $x_r \notin \mathrm{Fcl}A$. Then there is a fuzzy feebly open feeble q-nbd U of x_r such that $U \not{q} A$. By (c), there is fuzzy feebly open feeble q-nbd U of x_r such that $U \not{q} \mathrm{Fcl}A$. Thus we have $x_r \notin \mathrm{FclFcl}A$ and hence $\mathrm{FclFcl}A \leq \mathrm{Fcl}A$.

(e) Since $A \leq (A \cup B)$ and $B \leq (A \cup B)$, $\mathrm{Fcl}A \leq \mathrm{Fcl}(A \cup B)$ and $\mathrm{Fcl}B \leq \mathrm{Fcl}(A \cup B)$. Then $\mathrm{Fcl}A \cup \mathrm{Fcl}B \leq \mathrm{Fcl}(A \cup B)$.

Conversely, let $x_r \in \mathrm{Fcl}(A \cup B)$. Then for each fuzzy feebly open feeble q-nbd U of x_r, $Uq(A \cup B)$. Hence UqA or UqB, so $x_r \in \mathrm{Fcl}A$ or $x_r \in \mathrm{Fcl}B$. Thus $\mathrm{Fcl}(A \cup B) \leq \mathrm{Fcl}A \cup \mathrm{Fcl}B$. □

The fuzzy feeble interior of a fuzzy set can be characterized as follows:

THEOREM 2.17. *Let A and B be fuzzy sets in an fts X. Then the following hold:*

(a) $\mathrm{Fint}(0) = 0$,
(b) *if* $A \leq B$, *then* $\mathrm{Fint}A \leq \mathrm{Fint}B$,
(c) $\mathrm{Fint}\mathrm{Fint}A = \mathrm{Fint}A$,
(d) $\mathrm{Fint}(A \cap B) = \mathrm{Fint}A \cap \mathrm{Fint}B$.

PROOF. It is obvious. □

3. Fuzzy FC-compactness

DEFINITION 3.1. An fts X is said to be *fuzzy FC − compact* if for every fuzzy feebly open cover $\{V_\alpha \mid \alpha \in \Lambda\}$ of X, there is a finite subset Λ_0 of Λ such that $\cup\{\mathrm{Fcl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$.

Now we give a characterization of a fuzzy FC-compactness

THEOREM 3.2. *An fts X is fuzzy FC-compact iff for every family $\{V_\alpha \mid \alpha \in \Lambda\}$ of fuzzy feebly closed sets with $\cap\{V_\alpha \mid \alpha \in \Lambda\} = 0_X$, there is a finite subset Λ_0 of Λ such that $\cap\{\mathrm{Fint}V_\alpha \mid \alpha \in \Lambda_0\} = 0_X$.*

PROOF. ($\Rightarrow$) Suppose X is fuzzy FC-compact. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a family fuzzy feebly closed sets such that $\cap\{V_\alpha \mid \alpha \in \Lambda\} = 0_X$. Then $\{1 - V_\alpha \mid \alpha \in \Lambda\}$ is a family of fuzzy feebly open sets such that $\cup\{1 - V_\alpha \mid \alpha \in \Lambda\} = 1_X$. Since X is fuzzy FC-compact, there is a finite subset Λ_0 of Λ such that $\cup\{\mathrm{Fcl}(1 - V_\alpha) \mid \alpha \in \Lambda_0\} = 1_X$. Thus $\cap\{\mathrm{Fint}V_\alpha \mid \alpha \in \Lambda_0\} = 1 - \cup\{\mathrm{Fcl}(1 - V_\alpha) \mid \alpha \in \Lambda_0\} = 0_X$.

($\Leftarrow$) Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy feebly open cover of X. Then $\{1 - V_\alpha \mid \alpha \in \Lambda\}$ is a family of fuzzy feebly closed sets such that $\cap\{1 - V_\alpha \mid \alpha \in \Lambda\} = 0_X$. By hypothesis, there is a finite subset Λ_0 of Λ such that $\cap\{\mathrm{Fint}(1 - V_\alpha) \mid \alpha \in \Lambda_0\} = 0_X$, i.e., $\cup\{\mathrm{Fcl}V_\alpha) \mid \alpha \in \Lambda_0\} = 1_X$. Thus X is fuzzy FC-compact. □

DEFINITION 3.3 ([5]). A collection $\mathcal{B}$ of fuzzy sets in an fts X is said to form a *fuzzy filterbase* in X if for every finite subcollection $mathcalB_0$ of $mathcalB$, $\cap\mathcal{B}_0 \neq 0_X$.

DEFINITION 3.4. A fuzzy point x_r in an fts X is said to be *fuzzy* $fc-accumulation\ point$ of a fuzzy filterbase $\mathcal{B}$ if for each fuzzy feebly open feeble q-nbd U of x_r and for each $B \in \mathcal{B}$, $Bq\mathrm{Fcl}U$.

The concept of fuzzy FC-compact can be characterized as follows:

THEOREM 3.5. *An fts X is fuzzy FC-compact iff each fuzzy filterbase in X has a fuzzy fc-accumulation point in X.*

PROOF. ($\Rightarrow$) Let X be fuzzy FC-compact. Suppose that a fuzzy filterbase $\mathcal{B} = \{B_\alpha \mid \alpha \in \Lambda\}$ has no fuzzy fc-accumulation point in X. Then for each $x \in X$ and for each $r(0 < r \leq 1)$, x_r is not a fuzzy fc-accumulation point of $\mathcal{B}$, so there are a fuzzy feebly open feeble q-nbd U_x of x_r and a $B_{\alpha(x)} \in \mathcal{B}$ such that $B_{\alpha(x)} \not{q} \mathrm{Fcl}U_x$. Now $U_x(x) > 1 - r$ for each $(0 < r \leq 1)$, and hence $\{U_x \mid x \in X\}$ is fuzzy feebly open cover of X. Since X is a fuzzy FC-compact, there is a finite subfamily $\{U_{x_1}, U_{x_2}, \cdots, U_{x_n}\}$ such that $\cup_{i=1}^n \mathrm{Fcl}U_{x_i} = 1_X$. Then there is a finite subfamily $\{B_{\alpha(x_1)}, B_{\alpha(x_2)}, \cdots, B_{\alpha(x_n)}\}$ of $\mathcal{B}$ such that $(\cap_{i=1}^n B_{\alpha(x_i)}) \not{q} 1_X$. This implies that $(\cap_{i=1}^n B_{\alpha(x_i)}) = 0_X$, contradicting the fact that $\mathcal{B}$ is a fuzzy filterbase.

($\Leftarrow$) Using Theorem 3.2, we shall prove that the statement holds. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a family of fuzzy feebly closed sets such that $\cap\{V_\alpha \mid \alpha \in \Lambda\} = 0_X$.

Suppose that for each finite subset Λ_0 of Λ, $\cap\{\mathrm{Fint}V_\alpha \mid \alpha \in \Lambda_0\} \neq 0_X$.

Then $\mathcal{B} = \{\mathrm{Fint}V_\alpha \mid \alpha \in \Lambda_0\}$ forms a fuzzy filterbase in X, and hence by hypothesis, $\mathcal{B}$ has a fuzzy fc-accumulation point x_r in X. Since $x_r \notin \cap\{V_\alpha \mid \alpha \in \Lambda\}$, there is an $\alpha_0 \in \Lambda$ with $x_r \notin V_{\alpha_0}$, so $x_r q(1 - V_{\alpha_0})$. Thus $1 - V_{\alpha_0}$ is a fuzzy feebly open feeble q-nbd of x_r such that $\mathrm{Fint}V_{\alpha_0} / q\,(1 - \mathrm{Fint}V_{\alpha_0}) = \mathrm{Fcl}(1 - V_{\alpha_0})$.

It follows that x_r is not a fuzzy fc-accumulation point of $\mathcal{B}$ and hence we have a contradiction. □

DEFINITION 3.6. An fts X is said to be

(a) *fuzzy compact* ([2]) if for every fuzzy open cover $\{U_\alpha \mid \alpha \in \Lambda\}$ of X, there is a finite subset Λ_0 of Λ such that $\cup\{V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$,

(b) *fuzzy almost compact* ([3]) if for every fuzzy open cover $\{V_\alpha \mid \alpha \in \Lambda\}$ of X, there is a finite subset Λ_0 of Λ such that $\cup\{\mathrm{Cl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$,

(c) *fuzzy nearly compact* ([8]) if for every fuzzy open cover $\{V_\alpha \mid \alpha \in \Lambda\}$ of X, there is a finite subset Λ_0 of Λ such that $\cup\{\mathrm{IntCl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$.

THEOREM 3.7. *Every fuzzy compact space X is fuzzy FC-compact.*

PROOF. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy feebly open cover of an fts X. Then for each $\alpha \in \Lambda$, $A \leq \mathrm{IntClInt}V_\alpha$ and hence $\{\mathrm{IntClInt}V_\alpha \mid \alpha \in \Lambda\}$ is fuzzy open cover of X.

Since X is fuzzy compact, there is a finite subset Λ_0 of Λ such that

$$\cup\{\mathrm{IntClInt}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X, \text{ i.e.,}$$
$$\cup\{\mathrm{ClIntClInt}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X, \text{ i.e.,}$$
$$\cup\{\mathrm{FclInt}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X, \text{ (Lemma 2.14), i.e.,}$$
$$\cup\{\mathrm{Fcl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X.$$

Thus X is fuzzy FC-compact. □

REMARK 3.8. The converse of Theorem 3.7 need not be true in general as shown by the following example:

EXAMPLE 3.9. Consider an fts $(X, \mathcal{T})$, where $X = \{a, b\}$ is a set and $\mathcal{T}$ consists of all constant fuzzy sets $C_\alpha(\frac{1}{2} < \alpha \leq 1)$, i.e., $C_\alpha(x) = \alpha$ for each $x \in X$. For each $C_\alpha \in \mathcal{T}$, $\mathrm{ClIntCl}C_\alpha = 1_X$ for each $\alpha \in (\frac{1}{2}, 1]$. By Theorem 2.15, $\mathrm{Fcl}C_\alpha = 1_X$. Thus for any feebly open set A, $\mathrm{Fcl}A = 1_X$ and hence X is fuzzy FC-compact. We note that the fuzzy open cover $\{C_\alpha \mid \frac{1}{2} < \alpha < 1\}$ of X has no finite subcover. Thus X is not fuzzy compact.

DEFINITION 3.10. An fts X is said to be *$f-regular$* if each fuzzy feebly open set A in X is a union of fuzzy feebly open U_α's in X such that $\mathrm{Fcl}U_\alpha \leq A$ for each α.

THEOREM 3.11. *Let X be a fuzzy f-regular and fuzzy FC-compact space. Then X is fuzzy compact.*

PROOF. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy open cover of X, i.e., $\cup_{\alpha\in\Lambda} V_\alpha = 1_X$. Then $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy feebly open cover of X. Since X is f-regular, $V_\alpha = \cup_{\beta\in\Lambda} U_\beta^\alpha$, where U_β^α is a fuzzy feebly open set such that $\mathrm{Fcl}U_\beta^\alpha \leq V_\alpha$ for each β.

It follows that $1_X = \cup_{\alpha\in\Lambda} V_\alpha = \cup_{\beta\in\Lambda} U_\beta^\alpha$.

By the fuzzy FC-compactness of X, there is a finite subset Λ_0 of Λ such that $\cup_{\beta\in\Lambda_0}\mathrm{Fcl}U_\beta^\alpha = 1_X$. But $\cup_{\beta\in\Lambda_0}\mathrm{Fcl}U_\beta^\alpha \leq \cup_{\alpha\in\Lambda_0} V_\alpha$. Thus $\cup_{\alpha\in\Lambda_0} V_\alpha = 1_X$, and hence X is fuzzy compact. □

THEOREM 3.12. *Every fuzzy nearly compact space is fuzzy FC-compact.*

PROOF. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy feebly open cover of X. Then for each $\alpha \in \Lambda$, $V_\alpha \leq \mathrm{IntClInt}V_\alpha$, so $\{\mathrm{IntClInt}V_\alpha \mid \alpha \in \Lambda\}$ is a fuzzy open cover of X. Since X is fuzzy FC-compact, there is a finite subset Λ_0 of Λ,

$$\cup\{\mathrm{IntCl}(\mathrm{IntClInt}V_\alpha) \mid \alpha \in \Lambda_0\} = 1_X, \text{ i.e.,}$$

$$\cup\{\mathrm{IntClInt}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X, \text{(Lemma 2.8), i.e.,}$$

$$\cup\{\mathrm{ClIntCl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X, \text{ i.e.,}$$

$$\cup\{\mathrm{Fcl}V_\alpha) \mid \alpha \in \Lambda_0\} = 1_X.$$

Thus X is fuzzy FC-compact. □

THEOREM 3.13. *Every fuzzy FC-compact space is fuzzy almost compact.*

PROOF. Since fuzzy open set is fuzzy feebly open, it is obvious. □

DEFINITION 3.14 ([7]). An fts X is said to be *fuzzy extremally disconnected* if the closure of every fuzzy open set in X is fuzzy open in X.

LEMMA 3.15. *Let A be a fuzzy set in a fuzzy extremally disconnected space. Then A is fuzzy feebly open iff A is fuzzy semiopen.*

PROOF. Since a fuzzy feebly open is fuzzy semiopen, it suffices to show that a fuzzy semiopen is fuzzy feebly open. Let A be a fuzzy semiopen. Then $A \leq \mathrm{ClInt}A$. Since X is a fuzzy extremally disconnected, $A \leq \mathrm{ClInt}A = \mathrm{Int}(\mathrm{ClInt}A)$.Thus A is a fuzzy feebly open. □

THEOREM 3.16. *Let A be a fuzzy feebly open set in a fuzzy extremally disconnected space. Then* $\mathrm{Fcl}A = \mathrm{Cl}A$.

PROOF. By Theorem 2.12 (a), $\mathrm{Fcl}A \leq \mathrm{Cl}A$.

Conversely, let A be a fuzzy feebly open set in X and let $x_r \notin \mathrm{Fcl}A$. Then there is a fuzzy feebly open feeble q-nbd U of x_r such that $U \not{q} A$, so Int$U \not{q}$ Int$A$. Since X is fuzzy extremally disconnected, by Lemma 2.6, we get ClInt$U \not{q}$ ClInt$A$. Hence $x_r \notin$ ClInt$A$. But $A \leq$ IntClInt$A$ = ClInt$A$, so Cl$A \leq$ ClInt$A$. Thus $x_r \notin$ ClA. This shows that Cl$A \leq$ Fcl$A$. Therefore Fcl$A$ = ClA. □

THEOREM 3.17. *Every fuzzy extremally disconnected, almost compact space is fuzzy FC-compact.*

PROOF. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy feebly open cover of X. Then for each $\alpha \in \Lambda$, $V_\alpha \leq \text{IntClInt}V_\alpha$, so $\{\text{IntClInt}V_\alpha \mid \alpha \in \Lambda\}$ is a fuzzy open cover of X. Since X is fuzzy almost compact, there is a finite subset Λ_0 of Λ such that $\cup\{\text{Cl}(\text{IntClInt}V_\alpha) \mid \alpha \in \Lambda_0\} = 1_X$.

It follows that $\cup\{\text{Cl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$. By Theorem 3.16, $\text{Fcl}V_\alpha = \text{Cl}V_\alpha$ for each $\alpha \in \Lambda_0$. Thus $\cup\{\text{Fcl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$ which proves the fuzzy FC-compactness of X. □

THEOREM 3.18. *Every fuzzy extremally disconnected, FC-compact space is fuzzy nearly compact.*

PROOF. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be a fuzzy open cover of X. Then $\{V_\alpha \mid \alpha \in \Lambda\}$ is a fuzzy feebly open cover of X. Since X is fuzzy FC-compact, there is a finite subset Λ_0 of Λ such that $\cup\{\text{Fcl}V_\alpha \mid \alpha \in \Lambda_0\} = 1_X$.

Furthermore, since X is fuzzy extremally disconnected, then $\text{Fcl}V_\alpha = \text{Cl}V_\alpha$ and $\text{Cl}V_\alpha = \text{IntCl}V_\alpha$ for each $\alpha \in \Lambda_0$. Hence $1_X = \cup\{\text{Fcl}V_\alpha \mid \alpha \in \Lambda_0\} = \cup\{\text{Cl}V_\alpha \mid \alpha \in \Lambda_0\} = \cup\{\text{IntCl}V_\alpha \mid \alpha \in \Lambda_0\}$. Thus X is fuzzy nearly compact. □

DEFINITION 3.19. A mapping $f : X \longrightarrow Y$ is said to be

(a) *fuzzy irresolute* ([13]) if $f^{-1}(U)$ is fuzzy semiopen in X for each fuzzy semiopen set U in Y,
(b) *fuzzy semi – continuous* ([1]) if $f^{-1}(U)$ is fuzzy semiopen in X for each fuzzy open set U in Y.

It is known ([13]) that every fuzzy irresolute mapping is fuzzy semi-continuous.

DEFINITION 3.20. A mapping $f : X \longrightarrow Y$ is said to be

(a) *fuzzy feebly irresolute* if $f^{-1}(U)$ is fuzzy feebly open in X for each fuzzy feebly open set U in Y,
(b) *fuzzy feebly continuous* if $f^{-1}(U)$ is fuzzy feebly open in X for each fuzzy open set U in Y.

REMARK 3.21. (a) Every fuzzy feebly irresolute mapping is fuzzy feebly continuous.

(b) Every fuzzy continuous mapping is fuzzy feebly continuous and every fuzzy feebly continuous mapping is fuzzy semi-continuous.

THEOREM 3.22. *Let $f : X \longrightarrow Y$ be a mapping. Then the following are equivalent :*

(a) *f is fuzzy feebly irresolute,*
(b) *$f^{-1}(G)$ is fuzzy feebly closed in X for each fuzzy feebly closed set G in Y,*
(c) *for each fuzzy set A in X, $f(\mathrm{Fcl}A) \leq \mathrm{Fcl}(f(A))$,*
(d) *for each fuzzy set B in Y, $\mathrm{Fcl}(f^{-1}(B)) \leq f^{-1}(\mathrm{Fcl}(B))$.*

PROOF. $(a) \Longleftrightarrow (b)$ Let G be a fuzzy feebly closed in Y. Then $1 - G$ is fuzzy feebly open in Y, so $f^{-1}(1 - G)$ is fuzzy feebly open in X, i.e., $1 - f^{-1}(G)$ is fuzzy feebly open in X. Thus $f^{-1}(G)$ is fuzzy feebly closed in X. The proof of the converse is obvious.

$(b) \Longrightarrow (c)$ Let A be a fuzzy set in X. Then $\mathrm{Fcl}(f(A))$ is fuzzy feebly closed in Y. By (b), $f^{-1}(\mathrm{Fcl}f(A))$ is fuzzy feebly closed in X. Since $A \leq f^{-1}f(A) \leq f^{-1}(\mathrm{Fcl}f(A))$, $\mathrm{Fcl}A \leq \mathrm{Fcl}(f^{-1}(\mathrm{Fcl}f(A))) = f^{-1}(\mathrm{Fcl}f(A))$. Thus $f(\mathrm{Fcl}A) \leq f(f^{-1}(\mathrm{Fcl}f(A))) \leq \mathrm{Fcl}f(A)$.

$(c) \Longrightarrow (d)$ Let B be a fuzzy set in Y. By (c), $f(\mathrm{Fcl}(f^{-1}(B))) \leq \mathrm{Fcl}(f(f^{-1}(B))) \leq \mathrm{Fcl}B$. Thus $f^{-1}(f(\mathrm{Fcl}(f^{-1}(B)))) \leq f^{-1}(\mathrm{Fcl}B)$, i.e., $\mathrm{Fcl}f^{-1}(B) \leq f^{-1}(\mathrm{Fcl}B)$.

$(d) \Longrightarrow (b)$ Let G be a fuzzy feebly closed in Y. Then $\mathrm{Fcl}f^{-1}(G) \leq f^{-1}(\mathrm{Fcl}G) = f^{-1}(G)$, so $\mathrm{Fcl}f^{-1}(G) = f^{-1}(G)$ and hence $f^{-1}(G)$ is fuzzy feebly closed in X. □

THEOREM 3.23. *Let $f : X \longrightarrow Y$ be a mapping. Then the following are equivalent:*

(a) *f is fuzzy feebly continuous,*
(b) *$f^{-1}(G)$ is fuzzy feebly closed set in X for each fuzzy closed set G in Y,*
(c) *$f(\mathrm{Fcl}A) \leq \mathrm{Cl}(f(A))$ for each fuzzy set A in X,*
(d) *$\mathrm{Fcl}(f^{-1}(B)) \leq f^{-1}(\mathrm{Cl}(B))$ for each fuzzy set B in Y.*

PROOF. It is analogous to proof of Theorem 3.22. □

THEOREM 3.24. *Let $f : X \longrightarrow Y$ be a fuzzy feebly irresolute surjection mapping from a fuzzy FC-compact space X to an fts Y. Then Y is also fuzzy FC-compact.*

PROOF. Let $f : X \longrightarrow Y$ be a fuzzy feebly irresolute mapping and let $\{V_\alpha\}$ be a fuzzy feebly open cover of Y. Then $\{f^{-1}(V_\alpha)\}$ is a fuzzy feebly open cover of X. Since X is fuzzy FC-compact, there is a finite subfamily $\{f^{-1}(V_{\alpha_i}) \mid i = 1, 2, \cdots, n\}$ such that $\cup_{i=1}^n \mathrm{Fcl}(f^{-1}(V_{\alpha_i})) = 1_X$. Thus $1_Y = f(1_X) = f(\cup_{i=1}^n \mathrm{Fcl}(f^{-1}(V_{\alpha_i})) = \cup_{i=1}^n (f(\mathrm{Fcl}(f^{-1}(V_{\alpha_i})))) \leq \cup_{i=1}^n \mathrm{Fcl}(f(f^{-1}(V_{\alpha_i}))) \leq \cup_{i=1}^n \mathrm{Fcl} V_{\alpha_i}$. This shows that Y is fuzzy FC-compact. □

THEOREM 3.25. *Let $f : X \longrightarrow Y$ be a fuzzy feebly continuous surjection mapping from a fuzzy FC-compact space X to an fts Y. Then Y is fuzzy almost compact.*

PROOF. Let $f : X \longrightarrow Y$ be a fuzzy feebly continuous mapping and let $\{V_\alpha\}$ be a fuzzy open cover of Y. Then $\{f^{-1}(V_\alpha)\}$ is a fuzzy feebly open cover of X. Since X is fuzzy FC-compact, there is a finite subfamily $\{f^{-1}(V_{\alpha_i}) \mid i = 1, 2, \cdots, n\}$ such that $1_X = \cup_{i=1}^n (\mathrm{Fcl}(f^{-1}(V_{\alpha_i}))) = \mathrm{Fcl}(\cup_{i=1}^n f^{-1}(V_{\alpha_i}))$. Thus $1_Y = f(1_X) = f(\mathrm{Fcl}(\cup_{i=1}^n f^{-1}(V_{\alpha_i}))) \leq \mathrm{Cl} f(\cup_{i=1}^n (f^{-1}(V_{\alpha_i}))) = \mathrm{Cl}(\cup_{i=1}^n f(f^{-1}(V_{\alpha_i}))) \leq \mathrm{Cl}(\cup_{i=1}^n V_{\alpha_i}) = \cup_{i=1}^n \mathrm{Cl} V_{\alpha_i}$. Therefore Y is fuzzy almost compact. □

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