■ 論 文 ■

Signal Control Policies in Saturated Network

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목 차

요 약

교통신호제어는 교차로상에서 유입 차량들을 관제할 뿐만 아니라 교통사고를 예방하는 역할을 하고 있다. 또한 최근에는 네트워크의 최적화 도구로도 활용되는 추세에 있다. 1950년대 후반 신호제어에 대한 Webster의 제어전략이 발표된 이후 관련 학자들에 의해 몇 개의 신호제어 전략들이 발표되었지만, 이들 전략들은 대부분 비포화시를 반영하는 전략들이었다. 그러나 최근의 도심 가로망들은 근포화나 과포화 등 극심한 교통혼잡을 겪고 있기 때문에 이에 부합하는 새로운 신호제어전략이 요구되고 있다. 본 연구는 네트워크 차원의 신호제어전략을 수립하기 위한 첫 번째 연구로서 혼잡시를 고려한 2가지의 새로운 교통신호제어전략을 제시하였다. 첫 번째 신호전략은 과포화시 차량들의 대기행렬을 수용할 수 있는 접근로의 수용용량을 고려한 전략이며, 두 번째 전략은 overflow에 의한 차량들의 지체시간을 최소화시키는 전략이다. 본 연구에서는 기존의 2개 신호전략과 새로운 전략들을 예제 가로망을 대상으로 서로 비교, 평가하였으며, 신호제어전략이 통행배정모형(traffic assignment)과 결합하는 경우 유일해(unique solution)가 존재하는 지(monotonicity condition)도 검토하였다.

I. INTRODUCTION

Mutual influence between traffic signal control and route choice behaviour is well known. Traffic signal settings are readjusted according to actual traffic observations regularly, in turn this signal settings also influence on the drivers' behaviour. The effect of intersection on travel cost is of primary importance in urban networks because most of the travel time be spent at the intersection, but conventional equilibrium route choice is not fully considered this fact. Since Allsop(1974) suggested that the effects of signal settings on the traffic flow pattern should be taken into account by combining traffic control and route choice, many researchers have developed these kinds of combined models.

Traffic signal control policy is a method for selecting the signal timings such as cycle time, green split according to the traffic flow on that link. Two types of signal policy, fixed time policy and responsive policy are known widespread. In fixed time policy the timings will switch between different signal timing plans at pre-defined times of the day. In responsive policy, however, the timings are adjusted on and during each day, based at least partially on prevailing road conditions, estimated from detectors in the road.

One of the signal control policies is Webster signal policy, proposed by Webster(1958) which calculate the green splits in order to equalize the degree of saturation flow rates for each approaches. In conjunction with signal control policy, there exist several traffic control systems such as OPAC(USA), SCOOT(UK), UTOPIA(Italy), PRODYN(France), SCATS(Australia) and TRACS(Korea). These control systems are a kind of UTC(Urban Traffic Control) system and they have adaptive signal control subsystems respectively to reduce the delay and to avoid a spill-back. Most of the control systems, however, follow Webster signal policy or its modified version. More detailed reviews on traffic control systems are found in Shepherd(1994) and Lee(1995).

Since Webster, a few signal control policies are suggested, but these policies do not fully consider the oversaturated conditions. In this paper, oversaturated condition is defined as a situation that traffic demand exceeds link capacity, or the degree of saturation rate has more than the value of 1.0.

In the paper, two new signal control policies are suggested in terms of oversaturated conditions and assessed the properties of them. One of them is based on the length of approach enable to accommodate queues in oversaturated condition and the other based on the minimization of the queue delay on the intersection. Four signal policies, including Webster and Po suggested by Smith(1979), are compared each other with an example network.

The paper is structured as follows. In section 2, intersection delay performance functions are explained briefly. In section 3, four signal control policies are described including 2 new ones proposed by author in this paper and section 4 will compare the signal control policies with an example network. In the case of that traffic assignment combine the signal control policy, the monotonicity is important because it is guaranteed a unique solution. The monotonicity of each signal policy is evaluated in section 5. Finally some conclusions are presented in section 6.

II. SIGNALIZED INTERSECTION DELAY FORMULATION

1. Notation

Let the variables and notation be defined in the paper as follows.

d_i average signal delay function for link i $(=d_{ui}+d_{oi})$

d_{ui} uniform signal delay function for link i

d_{oi} overflow signal delay function for link i

$\overline{d_i}$ expanded signal delay function for link i $(\overline{d_{ui}}+\overline{d_{oi}})$

$\overline{d_{ui}}$ expanded uniform signal delay function for link i

$\overline{d_{oi}}$ expanded overflow signal delay function for link i

f_i approach volume for link i

λ_i green time split for link i

x_i degree of saturation flow for link i $(=\frac{f_i}{C_i}=\frac{f_i}{\lambda_i s_i})$

c_i capacity of approach for link i $(=\lambda_i s_i)$

s_i saturation flow for link i

C cycle time in seconds

g_i green time in seconds for link i

r_i effective red time in second for link i $[=C-g_i=C-\lambda_i(C-L)]$

L lost time in seconds

l_i length of approach i, or link length

q_{oi} average overflow queue for link i

N_m maximum queues at intersection

Note that in some cases, the subscript i is omitted for simplicity.

2. Signal delay function

The principal parameter used to describe the performance of signalized intersections is delay. There have been developed some signalized delay function since Webster(1958). The Webster formulation has emerged as the most quoted, and has a two-term equation as follows.

$$d = \frac{9}{10}\left[\frac{C(1-\lambda)^2}{2(1-\lambda x)} + \frac{x}{2\lambda s(1-x)}\right] \tag{1}$$

where the subscript i is omitted for simplicity, d is a average overall delay in seconds per vehicle and C is signal cycle time in seconds. x is the degree of saturation $(=\frac{f}{\lambda s})$, s is a saturation flow and λ is green time split. Webster delay function is, however, only valid for the case in which the demand is less than the capacity, or the degree of saturation(x) is less than the value of 1.0. For the cases in whichx >1.0, there exist several signal delay functions. Akcelik(1988) reviewed the signal delay functions proposed so far and unified them in a simple equation with different values of parameters. One of the most useful formulations is HCM(Highway Capacity Manual) delay function(1985), which has consisted of 2-terms as follows.

$$d = \frac{0.38\,C(1-\lambda)^2}{1-\lambda x} + 173x^2\left[(x-1) + \sqrt{(x-1)^2 + \frac{16x}{c}}\right] \tag{2}$$

Another delay function is Koti(The Korea Transport Institute) delay function, proposed by Kim, et.al.(1991), which can depict Korean traffic conditions. The Koti delay function is as follows.

$$d = \frac{0.5\,C(1-\lambda)^2}{1-\lambda x} + 225\left[(x-1) + \sqrt{(x-1)^2 + \frac{12x}{c}}\right] \tag{3}$$

where the subscript i is also omitted and c is the capacity for that link, or $c=\lambda s$. The two terms of the delay formula can be referred to as the uniform delay(d_u) and the overflow(d_o) terms.

$$d = d_u + d_o \tag{4}$$

The first term of the equation accounts for uniform delay, i.e., the delay that occurs if arrival demand is uniformly distributed over time. The second term of the equation accounts for incremental delay of random arrivals over uniform arrivals, and for the additional delay due to cycle failures. The HCM and Koti delay functions are, in this paper, used for the comparison among traffic signal policies in section 4.

The usefulness of these formulas are that an overflow queue formulation can be also used as a common base for the formulas to predict delay, number of stops and queue length. Akcelik(1988) suggested the relationship between the average overflow queue(q_o) in vehicles and the overflow delay(d_o) in seconds. This relationship come from the deterministic queueing theory in oversaturated condition, which overflow queues are dependent on only capacity of that link.

$$d_o = 3600\frac{q_o}{c} \tag{5}$$

where c is the capacity in vehicles/hr. From equation(5), the average overflow queue(q_o) can be obtained as

$$q_o = \frac{1}{3600}\,d_o\,c \tag{6}$$

where q_o is allowing for randomness. Akcelik also suggested the maximum back of the queue, N_m in vehicles as

$$N_{mi} = \left[\frac{f_i r_i}{3600\left(1 - \frac{f_i}{s_i}\right)}\right] + q_{oi} \tag{7}$$

where r_i is an effective red time in seconds and q_o is the average overflow queue given by equation(6).

III. SIGNAL CONTROL POLICY

1. Existing signal control policies

Since 1950s, various signal control policies have been proposed. Some of them are Webster' s equisaturation policy(1958), Allsop' s delay minimization policy(1971), Smith' s capacity maximization policy(P_o ; 1979) and so on. Most popular signal policy in traffic engineering is Webster' policy which is setting the green split to each approach in proportion to the traffic volumes. The policy is based on the premise that overall delays at a junction are approximately minimal when the f/c ratio of each approach is equal, which is known as equisaturation policy. Webster signal policy is as follows.

Webster signal policy :

$$Eq_i.\ \frac{f_i}{\lambda_i s_i} \quad i=1,\cdots,n \tag{8}$$

where Eq_i. stands here for " equate for all approach i".

Smith(1981) showed that his P_o signal policy possesses capacity-maximizing properties, so that it maximizes the total demand flow in the OD matrix that can be catered for by the network and also proved P_o should guarantee a feasible solution to the combined assignment/signal control problem. The capacity-maximizing property in P_o is the property of attracting traffic to higher capacity roads by allocating proportionally more green time to these. P_o signal policies is described as

P_o signal policy :

$$Eq_i.\ s_i d_i \quad i=1,\cdots,n \tag{9}$$

From the equation(9), it is easy to find that P_o signal policy determine the green splits on the bases of equalizing delays for each links with different weights. Weights are here the saturation flow(s_i) for each link respectively.

These two signal policies, however, were not developed for oversaturated traffic conditions, so such policies were inefficient for some oversaturated cases. It is well known at an oversaturated state that to minimize a queue-length or to avoid blocking upstream is more important than to minimize average delay for each vehicles. To copy with such limits, this paper propose two new signal control policies which consider the characteristics of oversaturated conditions.

2. Two new signal control policies

One of the new signal policies is Queue-length signal policy(P_{ql}) which is for reducing the queue-length in oversaturated cases. The signal policy determines the green time split according to the link-lengths, which can accommodate traffic queues, of the relating approaches. The Queue-length policy is similar to the P_o policy suggested by Smith(1979) mentioned above, but the signal policy include the length of each approach in the policy so as to accommodate the queues in oversaturated cases as follows.

Queue-length signal policy(P_{ql}) :

$$Eq_i.\ \overline{d_{oi}}\ s_i l_i \quad i=1,\cdots,n \tag{10}$$

where λ_i, s_i are the green time split and saturation flow rate on link i respectively. $\overline{d_{oi}}$ is the expanded second term, or random term, of the Webster' s delay function and l_i is the link length on approach i. In the long run, the signal policy is expected to reroute the traffics from the approach with short length into longer one like P_o signal policy.

The second signal control policy is developed to minimize queue delay in oversaturated traffic condition. This problem can be formulated as a minimization program.

$$\begin{aligned} \min\ & \sum_i q_{oi}\overline{d_{oi}} \\ s.t,\ & \sum_i \lambda_i = 1 \end{aligned} \tag{11}$$

where q_{oi} is a average overflow queues on link i. The Lagrangian of the minimization problem with the equality constraints can be formulated as

$$L(\lambda_i, \mu_i) = \sum_i q_{oi}\overline{d_{oi}}(\lambda_i) + \mu_i(1 - \sum_i \lambda_i) \tag{12}$$

where μ_i denotes the dual variable associated with the constraint. The optimality condition for this program is given by

$$\frac{\partial L(\lambda_i, \mu_i)}{\partial \lambda_i} = q_{oi}\frac{\partial \overline{d_{oi}}}{\partial \lambda_i} - \mu_i = 0 \tag{13}$$

and

$$\frac{\partial L(\lambda_i, \mu_i)}{\partial \mu_i} = 1 - \sum_i \lambda_i = 0 \quad (14)$$

equation(12) and (13) can be rewritten as

$$q_{oi}\frac{\partial \overline{d_{oi}}}{\partial \lambda_i} = \mu_i \quad (15)$$
$$\sum_i \lambda_i = 1$$

so that we have

$$Eq_i.\ q_{oi}\frac{\partial \overline{d_{oi}}}{\partial \lambda_i} \quad (16)$$

Equation(16) is another new signal control policy(P_{qd}) proposed in the paper so as to minimize queue delay in oversaturated conditions.

IV. COMPARISON OF THE POLICIES

This section compare the signal control policies in terms of the green time split, intersection delay and average/maximum queue with an example.

1. Mathematical equations and its solution algorithm

In order to calculate the green time split on each approach, four signal control policies; Webster, P_o and new two policies should be converted into nonlinear simultaneous equations. In the paper Webster delay function is used to determine the green time split because it can be differential with ease. In the case of n-phases, the signal control policies can be formulated respectively as follows.

Webster signal policy:

$F(\Lambda) = 0$:

$$\frac{f_1}{\lambda_1 s_1} - \frac{f_2}{\lambda_2 s_2} = 0$$
$$\frac{f_2}{\lambda_2 s_2} - \frac{f_3}{\lambda_3 s_3} = 0 \quad (17)$$
$$\cdots\cdots$$
$$\lambda_1 + \lambda_2 + \cdots + \lambda_n - 1 = 0$$

P_o signal control policy :

$F(\Lambda) = 0$:

$$s_1 d_1 - s_2 d_2 = 0$$
$$s_2 d_2 - s_3 d_3 = 0 \quad (18)$$
$$\cdots\cdots$$
$$\lambda_1 + \lambda_2 + \cdots + \lambda_n - 1 = 0$$

where, d_i is the Webster delay function in equation (1).

Queue-length signal control policy(P_{ql}) :

$F(\Lambda) = 0$:

$$\overline{d_{o1}} S_1 l_1 - \overline{d_{o2}} S_2 l_2 = 0$$
$$\overline{d_{o2}} S_2 l_2 - \overline{d_{o3}} S_3 l_3 = 0 \quad (19)$$
$$\cdots\cdots$$
$$\lambda_1 + \lambda_2 + \cdots + \lambda_n - 1 = 0$$

where, $\overline{d_{oi}}$ is the expanded overflow delay of Webster delay in equation(1). Webster delay is a steady-state intersection delay function, which can not describe the range of oversaturated condition. Webster delay function should be expanded by Taylor series for the purpose of calculating the green time splits in the oversaturated traffic conditions, that is the state of $x \geqq 1.0$. The expanded Webster delay function,bar $\overline{d}$ is the sum of uniform and overflow delays which are expanded at a certain point of the degree of saturation flow(x^*), set x^* be 0.98 in this paper.

$$\overline{d} = \overline{d_u} + \overline{d_o}$$
$$= \frac{9}{10}\left[\frac{C(1-\lambda)^2}{2(1-\lambda x^*)} + \frac{C\lambda(1-\lambda)^2}{2(1-\lambda x^*)^2}\cdot(x-x^*) + \frac{x^*}{2\lambda S(1-x^*)} + \frac{1}{2\lambda S(1-x^*)^2}\cdot(x-x^*)\right] \quad (20)$$

where the subscript i is also omitted for simplicity. In this paper to calculate the green time split for policy P_{ql}, the expanded overflow delay($\overline{d_o}$) in equation(20) is used. Therefore $\overline{d_{oi}}$ is as follows.

$$\overline{d_{oi}} = \frac{9}{10}\left[\frac{x^*}{2\lambda S(1-x^*)} + \frac{1}{2\lambda S(1-x^*)^2}\cdot(x-x^*)\right] \quad (21)$$

Queue-delay minimization signal control policy(P_{qd}) :

F(Λ) = 0 :

$$q_{o1}\frac{\partial \overline{d_{o1}}}{\partial \lambda_1} - q_{o2}\frac{\partial \overline{d_{o2}}}{\partial \lambda_2} = 0$$

$$q_{o2}\frac{\partial \overline{d_{o2}}}{\partial \lambda_2} - q_{o3}\frac{\partial \overline{d_{o3}}}{\partial \lambda_3} = 0 \qquad (22)$$

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n - 1 = 0$$

where, q_{oi} is the average overflow queue on linki, or $q_{oi} = \overline{d_{oi}}\,c = \overline{d_{oi}}\,\lambda s$, and $\frac{\partial \overline{d_{oi}}}{\partial \lambda_i}$ is the derivative of expanded overflow delay(d_{oi}) with respect to λ_i as follows.

$$\frac{\partial \overline{d_{oi}}}{\partial \lambda_i} = \frac{2\lambda_i S_i x^{*2} - \lambda_i S_i x^{*2} - 2f_i}{2\lambda_i^3 S_i^2 (1-x^*)^2} \qquad (23)$$

To solve the nonlinear simultaneous equations, Newton-Raphson method can be used. The solution procedure is as follows.

(step0) initialization
iteration k=1
set initial values of Λ^K

(step1) calculate δ^K
$\nabla F(\Lambda^K) \cdot \delta^K = -F(\Lambda^K)$
where, $\nabla F(\Lambda^K) = \frac{\partial F(\Lambda^K)}{\partial \Lambda^K}$

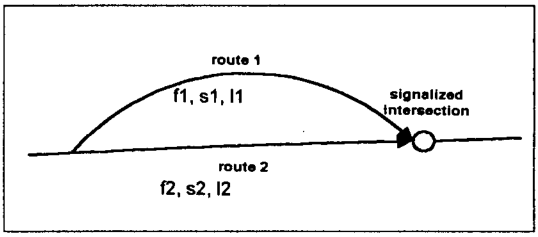
(step2) update Λ^K
$\Lambda^{K+1} = \Lambda^K + \delta^K$

(step3) convergence test
if $| F(\Lambda^{K+1}) | < \varepsilon$, stop : the solution is Λ^{K+1}
otherwise, return (step1).
where, ε is a predetermined small value.

More detailed solution procedures are found elsewhere(W.H.,Press, et.al. 1989). In the case of 2-phases, one dimensional line search method such as golden section method is also used for solving the problem.

2. Test network

To assess the four signal control signal policies, a numerical example is presented as in ⟨Figure 1⟩ The test network consists of only two routes. They have the same value of saturation flow of 2000vehicles/hr, but different link lengths : route 1 is longer than that of route 2. Both routes meet at the end of the each routes at signalized intersection with two phases. The input data such as route saturation flow, length and signal settings are shown in ⟨Table 1⟩ To compare the green split and total delay at intersection for each signal policy, approach flow of link 1, or f_1, will increase from 600vehicles/hr to 1800vehicles/hr gradually under the fixed value of f_2 (f_2 =1000vehicles/hr). In this section HCM and Koti delay functions in equation(2) and equation(3) are used to compare the signal policies. The results with Koti delay function are shown in appendix 1.



⟨Figure 1⟩ Test Network

⟨Table 1⟩ Input data

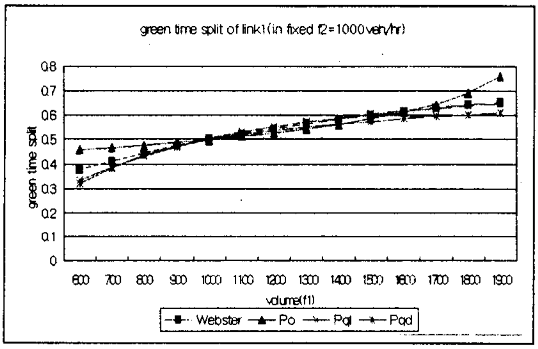
● network data		
route number	1	2
saturation flow (vehicle/hr)	2000	2000
link length(m)	500	300
● signal data		
cycle length	60 sec	
lost length	8 sec	
eff. green	52 sec	
min. green	7 sec	
2-phases, one for each route		
● traffic volume		
f_1 = 600vehs/hr ~ 1800vehs/hr		
f_2 = 1000vehs/hr(fixed)		

3. Comparison of the policies

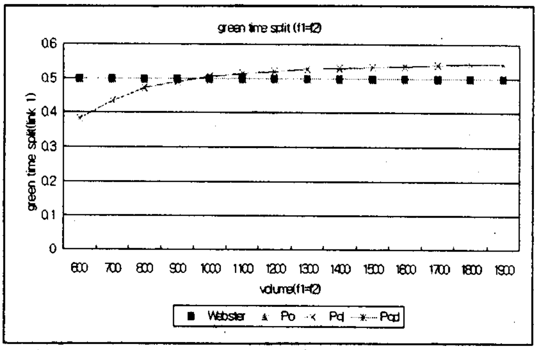
(1) green time split(λ_1)

⟨Figure 2⟩ and ⟨Table 2⟩ depict the relationship among the green time split of each signal policies with varying traffic volumes from 600vehs/hr to 1800vehs/hr on route 1 in the case of fixed traffic volume on route 2, or f_2 =1000vehs/hr. 〈Table 2〉 also show the values of the degree of saturation(x_1) on route 1. Each signal policy allocate somewhat different values of green time split λ_1 as f_1 increase, which is come from the objective of signal control policy : Webster signal policy calculate the green time split in order to equalize the degree of saturation rate for each approaches, that are routes in this example and P_o policy calculate the green time split to maximize through capacity. New policy P_{ql} is to maximize accommodation of queues and P_{qd} is to minimize queue delay. Note that compared with other signal policies, in the oversaturated region P_{ql} policy determine the longer green time split to long route (l_1) which can accommodate more queues.

This phenomenon occurs more explicitly in 〈Figure 3〉 as f_1 and f_2 increase with the same volumes simultaneously. As shown in 〈Figure 3〉, P_{ql} policy also determine longer green time split to longer route as traffic volume increase, but the other policies assign the same green time split, the value of 0.5.



〈Figure 2〉 Green time split of link 1 (λ_1)



〈Figure 3〉 Green time split(λ_1) in the case of $f_1=f_2$

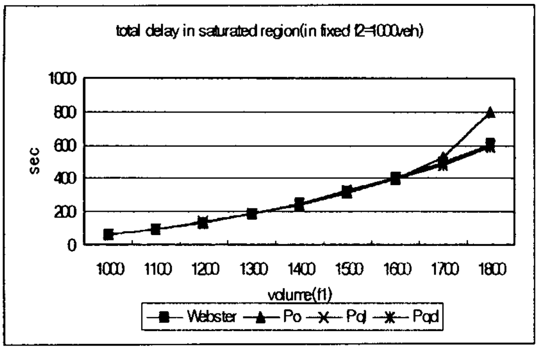
〈Table 2〉 Green time split(λ_1) and degree of sat.(x_1) in the case of fixed f_2 =1000

volume (f_1)	Webster		P_o		P_{ql}		P_{qd}	
	λ_1	degree of sat.	λ_1	degree of sat.	λ_1	degree of sat.	λ_1	degree of sat.
600	0.375	0.8	0.458	0.66	0.334	0.9	0.322	0.93
700	0.412	0.85	0.467	0.75	0.387	0.91	0.382	0.92
800	0.444	0.9	0.477	0.84	0.433	0.92	0.435	0.92
900	0.474	0.95	0.488	0.92	0.472	0.95	0.473	0.95
1000	0.5	1.0	0.498	1.0	0.505	0.99	0.5	1.0
1100	0.524	1.05	0.513	1.07	0.532	1.03	0.521	1.06
1200	0.545	1.1	0.528	1.14	0.555	1.08	0.538	1.12
1300	0.565	1.15	0.544	1.19	0.574	1.13	0.552	1.18
1400	0.583	1.2	0.564	1.24	0.59	1.19	0.565	1.24
1500	0.6	1.25	0.586	1.28	0.605	1.24	0.576	1.3
1600	0.615	1.3	0.613	1.31	0.617	1.3	0.586	1.37
1700	0.63	1.35	0.646	1.32	0.629	1.35	0.595	1.43
1800	0.643	1.4	0.691	1.3	0.639	1.41	0.603	1.49

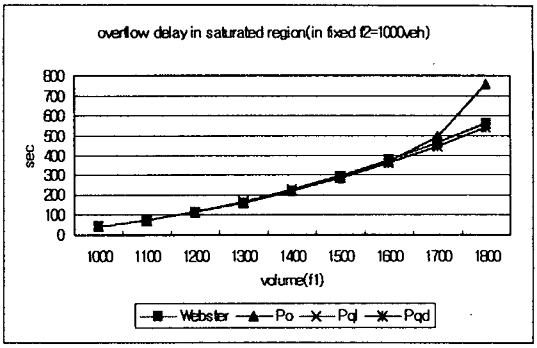
(2) total delay and overflow delay(d_{oi}) of intersection

Total delay is the summation of each approach delays with HCM delay function. 〈Figure 4〉 and 〈Table 3〉 display the total delay for signal policies. Compared with other three signal policies, P_{qd} policy has the lowest value of total delay as a whole in the oversaturated area. However, P_o policy show the highest value of total delay and the rests are not so different. These kinds of different values of total delays are stemmed out of the different green time split of each signal policies. In the respect of overflow delay(d_o) of the policies, P_{qd} also has the lowest as shown in

⟨Figure 5⟩ These results are accord with the objective of P_{qd} policy, which minimize queue delay in saturated state. Another new signal policy P_{ql} has a similar value of total delay but in some cases it has lower values than those of Webster or P_o signal policy



<Figure 4> Total delay in saturated region



<Figure 5> Overflow delay in saturated region

<Table 3> Total delays of the signal policies

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	23.5	27	29.2	33.3
700	27.4	30.6	30.6	31.7
800	33.8	36.1	35.0	34.7
900	44.7	45.6	44.8	44.8
1000	63.5	63.5	63.7	63.5
1100	92.9	93.5	94.3	92.8
1200	133.5	134.7	136.4	133
1300	184.8	184.8	188.9	183.2
1400	246.7	243.1	250.9	242.9
1500	318.0	311.5	322.2	312.1
1600	400.6	397.8	402.8	391.5
1700	495.0	527.2	493.6	482.2
1800	603.9	797.1	597.1	587.4

<Table 4> Overflow delays of the signal policies

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	6.9	10.4	12.3	16.3
700	10.2	13.3	13.2	14.3
800	15.8	18.1	17.0	16.7
900	25.9	26.8	26.0	26
1000	43.7	43.8	44.0	43.7
1100	72.1	72.7	73.5	72.0
1200	111.5	112.7	114.4	111.0
1300	161.5	161.4	165.6	159.8
1400	221.3	218.0	226.1	217.8
1500	291.1	284.4	295.4	284.8
1600	371.0	368.2	373.3	361.0
1700	461.3	494.3	459.8	446.5
1800	562.4	759.4	555.1	541.9

With the Koti delay function in equation(3), the results of total delay and overflow delay are shown in appendix 1. By using different delay function, total delay and overflow delay have different values from those of ⟨Table 3⟩ and ⟨Table 4⟩ The results for signal policies, however, are similar to those abovementioned.

(3) average overflow queue(q_o) and maximum queue(N_m)

⟨Table 5⟩ and ⟨Table 6⟩, with HCM delay function, show the average overflow queues and maximum queues at an intersection as the volume of f_i increases. The average overflow queues(q_o) can be calculated by equation (6) in section 2, which is allowing for randomness and oversaturation effects. As expected, P_{ql} policy has the small average queues in saturated conditions, because P_{ql} policy calculate the green time split so as to minimize queue length. ⟨Table 5⟩ also shows that the average queue of Webster policy is similar to that of P_{ql} policy. On the other hand, P_{qd} policy shows larger average queues than those of P_{ql} or Webster policy. The reason come from that the objective of P_{qd} is to minimize queue delay, not to minimize queue length.

Another important performance parameter in signal intersection is the maximum queues, which is useful in intersection design. The maximum back of queues is also given by equation(7) in section 2. As shown in ⟨Table 6⟩, maximum queues of the four policies increase steeply at near-saturated or over-saturated traffic condition. All signal policies, except P_{qd} represent the similar values as a whole. Note here that from the point of P_{ql}' rerouting property as mentioned earlier, P_{ql} policy shows promising results of minimizing queue lengths under the condition of not combing traffic assignment.

<Table 5> Average overflow queues[q_o]

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	1.8	3.0	2.6	3.2
700	2.7	3.8	3.2	3.4
800	4.3	5.1	4.5	4.4
900	7.2	7.5	7.2	7.2
1000	12.1	12.1	12.2	12.1
1100	20.0	20.3	20.1	20.0
1200	30.9	31.8	31.1	31.1
1300	44.7	46.0	44.9	45.2
1400	61.3	62.6	61.4	62.4
1500	80.6	81.4	80.7	82.7
1600	102.7	102.8	102.7	106.3
1700	127.7	129.0	127.7	133.2
1800	155.7	169.5	155.8	163.8

<Table 6> Maximum queues[N_m]

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	20.4	23.0	20.5	20.9
700	23.8	25.6	23.9	24.0
800	27.9	29.0	27.9	27.9
900	33.3	33.7	33.3	33.3
1000	41.0	41.0	41.1	41.0
1100	51.9	52.3	52.0	52.0
1200	66.4	67.5	66.4	66.6
1300	84.4	86.2	84.4	85.2
1400	106.2	108.3	106.1	108.1
1500	132.6	134.2	132.4	136.1
1600	165	165.2	164.8	171.1
1700	206.6	205.6	206.7	216.7
1800	267.2	269.8	268.2	284.4

The results of average overflow queues and maximum queues with Koti delay function are also shown in appendix 1. Most of the signal policies have the same values of average overflow queue as a whole and P_{ql} policy does not produce promising results than expected.

V. MONOTONICITY OF SIGNAL CONTROL POLICY

Traffic assignment combined with signal control policy is not sure if it has a unique solution because traffic assignment and traffic signal control influence each other. If the Jacobian of the combined delay function in traffic assignment is symmetric, the resulting problem reduces to one of convex minimization. Monotonicity of the function can be checked via the Jacobian J(the matrix of its first order derivatives), which must be positive semi-definite, or in other words, the determinant of the sum of the Jacobian and its transposed matrix should be greater than or equal to 0 as

$$\frac{1}{2}\ |J+J^T| \geq 0 \qquad (24)$$

where, $J = \begin{bmatrix} \frac{\partial d}{\partial f}, & \frac{\partial d}{\partial \lambda} \\ \frac{\partial p}{\partial f}, & \frac{\partial p}{\partial \lambda} \end{bmatrix}$ and p is signal control policy.

For a solution to the combined assignment signal control problem to exist and to be unique, monotonicity condition is sufficiently satisfied. Heydeker(1983) investigated a simple two-way junction with signal control policies and found that neither of the three policies(Webster, P_o and delay minimization) satisfied monotonicity condition with Webster signal function. This is the same result of Vuren,et.al(1987) and Smith, et.al(1993). This results show that the combined assignments with Webster and P_o signal policy have multiple solutions, not unique.

The monotonicity of the new two signal policies proposed in this paper are also checked with equation (24). The results of the two signal control policies are shown in ⟨Table 7⟩ and in ⟨Table 8⟩ More detailed monotonicity check with policy P_{pl} and P_{qd} are found in Appendix 2.

<Table 7> The value of $|J+J^T|$ in signal policy P_{ql}

f_i \ f_2	600	800	1000	1200	1400	1600
600	-10305030.0	-11477760.0	-12706260.0	-13970710.0	-15173820.0	-16489310.0
800	-9291802.0	-9986294.0	-10748880.0	-11442350.0	-12194300.0	-12842060.0
1000	-8664419.0	-9080871.0	-9638758.0	-9994960.0	-10496940.0	-11032130.0
1200	-7871010.0	-8237887.0	-8528027.0	-8936287.0	-9259590.0	-9598532.0
1400	-6887365.0	-7114483.0	-7271660.0	-7516528.0	-7772890.0	-7950520.0
1600	-4903535.0	-4991924.0	-5129331.0	-5224304.0	-5322098.0	-5474335.0

<Table 8> The value of $|J+J^T|$ in signal policy P_{qd}

f_1 \ f_2	600	800	1000	1200	1400	1600
600	-15378.7	-12490.8	-12122.8	-12797.7	-15378.7	-460896.2
800	-22174.6	-26374.4	-31905.8	-38029.6	-46270.3	-55837.4
1000	-81583.7	-94455.8	-110179.0	-129456.2	-148874.1	-167059.4
1200	-234476.5	-260143.4	-289350.5	-322644.4	-360664.8	-404163.8
1400	-575208.1	-621698.9	-672793.0	-729002.8	-790904.6	-859145.1
1600	-1353884.0	-1459479.0	-1535441.0	-1658531.0	-1747197.0	-1841598.0

From the ⟨Table 7⟩and ⟨Table 8⟩, all value of $|J+J^T|$ are negative. This means that the signal policies do not satisfy the monotonicity condition. This result, therefore, show that the new signal control policies also have multiple solutions like Webster and P_o policy in the case of combining with traffic assignment.

VI. CONCLUSION

This paper proposed two new signal control policies : One is based on the length of approach and the other is minimization of the queue delay in saturated condition. Four signal control policies, including two new signal policies, are assessed in saturated network condition.

As the results of evaluating the 4 signal policies with an example network, Queue-delay minimization signal policy, P_{qd} proposed in the paper, is superior to other policies with regard to total delay as a whole. But regarding to average and maximum queues, P_{qd} policy does not show promising results than expected and there are not so much different among policies except P_{qd} policy. This paper also assessed the monotonicity condition of each signal policy. No signal policies satisfy the monotonicity condition, that is, multiple solutions exist in the case of combining signal policy with assignment.

Further studies related to this research include the following issues: The first is to develop a traffic assignment model combined with the signal policies proposed in the paper and to evaluate the properties of each signal policy. The combined assignment model include the signal policies as a responsive signal control. It is expected in that case that the results may be different from those of this paper because the model can consider the drivers' behavior explicitly. The second is to develop a new signal control policy which can guarantee a unique solution. The third is to introduce a signal coordination between intersections to the model and then to determine the best green time split. Finally it also remains to apply the signal policies to real-life networks.

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[Appendix 1] Results with Koti(1991) delay function

〈Total delays of the signal policies〉

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	32.6	35.7	38.8	43.2
700	36.9	39.9	40.2	41.4
800	43.7	46.1	45.0	44.6
900	55.3	56.2	55.4	55.3
1000	75.3	75.3	75.5	75.3
1100	105.2	105.5	106.3	105.1
1200	142.5	142.2	144.4	141.9
1300	184.1	181.9	186.6	182.1
1400	228.0	223.8	230.6	223.9
1500	273.6	268.7	275.7	266.7
1600	320.7	319.3	321.7	310.7
1700	370.1	381.7	369.5	357.0
1800	424.6	473.5	422.0	409.3

〈Overflow delays of the signal policies〉

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	70.8	13.9	16.6	20.7
700	14.2	17.2	17.4	18.5
800	20.0	22.4	21.3	20.9
900	30.6	31.5	30.6	30.6
1000	49.3	49.3	49.5	49.3
1100	77.8	78.1	78.9	77.7
1200	113.6	113.3	115.6	112.9
1300	153.4	151.1	156.0	151.3
1400	195.3	190.8	198.0	190.9
1500	238.2	233.0	240.4	230.8
1600	281.8	280.3	282.9	270.6
1700	325.7	338.3	325.0	310.2
1800	369.9	423.9	366.8	349.4

〈Average overflow queues[q_o]〉

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	2.8	4.0	3.6	4.2
700	3.8	4.9	4.3	4.5
800	5.5	6.3	5.7	5.6
900	8.4	8.8	8.4	8.4
1000	13.6	13.7	13.7	13.6
1100	21.6	21.8	21.7	21.6
1200	31.5	31.9	31.5	31.6
1300	42.5	42.9	42.5	42.7
1400	54.1	54.3	54.1	54.3
1500	66.0	66.1	66.0	66.2
1600	78.1	78.1	78.1	78.3
1700	90.3	90.2	90.3	90.5
1800	102.5	102.5	102.5	102.8

〈Table 6〉 Maximum queues[N_m]

volume (f_i)	Webster	P_o	P_{ql}	P_{qd}
600	21.4	24.0	21.5	21.9
700	24.8	26.7	25.0	25.1
800	29.0	30.2	29.1	29.1
900	34.6	35.0	34.6	34.6
1000	42.5	42.5	42.6	42.5
1100	53.5	53.8	53.6	53.5
1200	66.9	67.6	66.8	67.1
1300	82.2	83.0	81.9	82.6
1400	99.0	100.0	98.7	100.0
1500	118.0	118.9	117.7	119.6
1600	140.3	140.5	140.1	143.1
1700	169.1	166.8	169.2	174.0
1800	213.9	202.8	214.9	223.4

[Appendix 2] Monotonicity Condition Check

1. Queue-length signal policy

The signal policy is $P_{ql} = \overline{d_o}\, s_i\, l_i$, where $\overline{d_o}$ is the overflow delay in expanded Webster delay function as (constant value of $\frac{9}{10}$ in Webster delay function is omitted without loss)

$$\overline{d_o} = \frac{x^*}{2\lambda s(1-x^*)} + \frac{1}{2\lambda s(1-x^*)^2} \cdot (x-x^*), \quad (x=\frac{f}{\lambda s})$$

so that,

$$P_{ql} = \overline{d_o}\, sl = \frac{lx^*}{2\lambda(1-x^*)} + \frac{l(x-x^*)}{2\lambda s(1-x^*)^2}$$

in order to calculate the Jacobian matrix, differentiating the signal policy and delay function with regard to f and λ,

① $\frac{\partial p_{ql}}{\partial f} = \frac{l}{2\lambda^2 s(1-x^*)^2}$

② $\frac{\partial p_{ql}}{\partial \lambda} = \frac{lx^*}{2\lambda^2(1-x^*)} - \frac{l(x-x^*)}{2\lambda^2(1-x^*)^2} = \frac{l(x^{*2}-x)}{2\lambda^2(1-x^*)}$

③ $\frac{\partial d}{\partial f} = \frac{C(1-\lambda)^2}{s(1-f/s)} + \frac{1}{(\lambda s-f)^2}$

④ $\frac{\partial d}{\partial \lambda} = -\frac{2C(1-\lambda)}{(1-f/s)} - \frac{s}{(\lambda s-f)^2} + \frac{1}{\lambda^2 s}$

and so, the Jacobian matrix is

$$J = \begin{bmatrix} \frac{\partial d}{\partial f}, & \frac{\partial d}{\partial \lambda} \\ -\frac{\partial p}{\partial f}, & -\frac{\partial p}{\partial \lambda} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{C(1-\lambda)^2}{s(1-f/s)} + \frac{1}{(\lambda s-f)^2}, & -\frac{2C(1-\lambda)}{(1-f/s)} - \frac{s}{(\lambda s-f)^2} + \frac{1}{\lambda^2 s} \\ -\frac{l}{2\lambda^2 s(1-x^*)^2}, & -\frac{l(x^{*2}-x)}{2\lambda^2(1-x^*)} \end{bmatrix}$$

and

$$\frac{1}{2}\,|J+J^T|$$

$$= \frac{1}{2}\left[4\left[\frac{C(1-\lambda)^2}{s(1-f/s)} + \frac{1}{(\lambda s-f)^2}\right]\left[\frac{l(x^{*2}-x)}{2\lambda^2(1-x^*)}\right] - \left[-\frac{2C(1-\lambda)}{(1-f/s)} - \frac{s}{(\lambda s-f)^2} + \frac{1}{\lambda^2 s} + \frac{l}{2\lambda^2 s(1-x^*)^2}\right]^2\right]$$

check the value of $\frac{1}{2}\,|J+J^T|$ to determine monotonicity.

2. Queue-delay Minimization signal policy(P_{qd}) is also possible to check the monotonicity condition as the same way above.