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REAL HYPERSURFACES SATISFYING $S\phi = \phi S$ IN A COMPLEX SPACE FORM

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0. Introduction

An n-dimensional complex space form $M_n(c)$ is a Kaehlerian manifold of constant holomorphic sectional curvature c As is well known, complete and simply connected complex space forms are isometric to a complex projective space P_nC, a complex Euclidean space C_n or a complex hyperbolic space H_nC according as $c > 0$, $c = 0$ or $c < 0$

Let M be a real hypersurface of $M_n(c)$. Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the Kaehlerian metric and complex structure J of $M_n(c)$. The structure vector field ξ is said to be *principal* if $A\xi = \alpha\xi$, where A is the shape operator in the direction of the unit normal C and $\alpha = \eta(A\xi)$. We denote respectively by S and h the Ricci tensor of type (1.1) on M and by trA. It is known that if ξ is principal curvature vector, then α is constant, and $dh(\xi) = 0$ [4], [9]. Takagi [10] classified all homogeneous real hypersurfaces of P_nC as six model spaces which are said to be $(A_1), (A_2), (B), (C), (D)$ and (E) and Cecil-Ryan [2] and Kimura [5] proved that they are realized as the tubes of constant radius over Kaehlerian submanifold if ξ is principal curvature vector. Namely, he [10] proved the following:

Theorem A. *Let M be a homogeneous real hypersurface of P_nC. Then M is a tube of radius r over one of the following Kaehlerian submanifolds.*

(A_1) *a hyperplane $P_{n-1}C$, where $0 < r < \pi/2$,*
(A_2) *a totally geodesic $P_kC (1 \le k \le n-2)$, where $0 < r < \pi/2$,*
(B) *a complex quadric Q_{n-1}, where $0 < r < \pi/4$,*
(C) *$P_1C \times P_{(n-1)/2}C$, where $0 < r < \pi/4$ and $n (\ge 5)$ is odd,*

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(D) *a complex Grassmann* $G_{2,5}C$ *where* $0 < r < \pi/4$ *and* $n = 9$,
(E) *a Hermitian symmetric space* $SO(10)/U(5)$, *where* $0 < r < \pi/4$ *and* $n = 15$.

Also Berndt [1] showed that all real hypersurfaces with constant principal curvatures of H_nC are realized as the tubes of constant radius over certain submanifolds when the structure vector field ξ is principal, which are said to be of type $(A_0), (A_1), (A_2)$ and (B).

On the other hand, Kimura [6] and Ki-Suh [4] proved respectively the followings:

THEOREM B. *Let M be a real hypersurface in P_nC, $n \geq 3$ on which ξ is a principal curvature vector and the mean curvature of M is constant. If $S\phi = \phi S$, then M is locally congruent to one of $(A_0), (A_1), (A_2), (B), (C), (D)$ and (E).*

THEOREM C. *Let M be a real hypersurface in H_nC, $n \geq 3$ on which ξ is a principal curvature vector. If $S\phi = \phi S$, then M is locally congruent to one of $(A_0), (A_1), (A_2)$ and (B).*

REMARK. In the proofs of Theorem A and Theorem B, they really used the condition that ξ is a principal curvature vector.

From these points of view, we prove in the present paper the following:

THEOREM. *Let M be a real hypersurface of $M_n(c)$, $c \neq 0$ satisfying $S\phi = \phi S$. If $dh(\xi) = 0$, then ξ is a principal curvature vector provided that $\alpha = \eta(A\xi)$ is constant.*

All manifolds in this paper are assumed to be connected and of class c^∞ and the real hypersurface are supposed to be orientable.

1. Structure equations of a real hypersurface

In this section, fundamental properties of a real hypersurface in a complex space form are recalled.

Let $M_n(c)$ be a real 2n-dimensional complex space form with parallel almost complex structure J and Reimannian metric tensor G which is J-Hermitian, and covered by a system of coordinate neighborhoods $\{\tilde{V}; X^A\}$.

Let M be a real (2n-1)-dimensional hypersurface of $M_n(c)$ covered by a system of coordinate neighborhoods $\{V; y^h\}$ and immersed isometrically in $M_n(c)$ by the immersion $i : M \to M_n(c)$. Throughout this paper the following convention on the range of indices are used :

$$A, B, \cdots = 1, 2, \cdots, 2n \quad ; \quad i, j, \cdots = 1, 2, \cdots, 2n-1.$$

This summation convention will be used with respect to those system of indices. We represent the immersion i locally by $x^A = x^A(y^h)$ and $B_j = (B_j^A)$ are $(2n-1)$-linearly independent local tangent vectors of M, where $B_j{}^A = \partial_j x^A$, $\partial_j = \partial/\partial y^i$. A unit normal C to M may be chosen.

Since the immersion is isometric, the induced Riemannian metric tensor g with components g_{ji} is given by

$$g_{ji} = G_{BA} B_j{}^B B_i{}^A.$$

For the tangent vectors B_i and a unit normal C to M, we can put

$$JB_i = \phi_i{}^h B_h + \xi_i C, \ JC = -\xi^i B_i$$

in each coordinate neighborhood, where we have put $\phi_{ji} = G(JB_j, B_i)$ and $\xi_i = G(JB_i, C)$, ξ^h being components of a vector field ξ associated with ξ_i and $\phi_{ji} = \phi_j{}^r g_{ir}$. We notice here that ϕ_{ji} is skew-symmetric. By the properties of the almost complex structure J, it is seen that

$$\phi_j{}^r \phi_r{}^h = -\delta_j{}^h + \xi_j \xi^h, \qquad \xi^r \phi_r{}^h = 0, \qquad \xi_r \phi_j{}^r = 0, \qquad \xi_i \xi^i = 1,$$

namely, the aggregate (ϕ, g, ξ) defined an *almost contact metric structure*.

Denoting by ∇_j the operator of van der Waerdern-Bortolotti covariant differentiation with respect to g_{ji}, equations of the Gauss and Weingarten for M are respectively obtained:

$$\nabla_j B_i = A_{ji} C, \qquad \nabla_j C = -A_j{}^r B_r,$$

where $A = (A_j{}^h)$, which is related by $A_{ji} = A_j{}^r g_{ir}$ is the shape operator derived from C.

By means of above equations, the covariant derivatives of the structure tensors are yielded :

$$\nabla_j \phi_i{}^h = -A_{ji}\xi^h + A_j{}^h \xi_i, \qquad \nabla_j \xi_i = -A_{jr}\phi_i{}^r. \tag{1.1}$$

Since the ambient space is a complex space form, equations of the Gauss and Codazzi for M are respectively given by

$$\begin{aligned} R_{kjih} = {} & \frac{c}{4}(g_{kh}g_{ji} - g_{jh}g_{ki} + \phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - 2\phi_{kj}\phi_{ih}) \\ & + A_{kh}A_{ji} - A_{ji}A_{ki}, \end{aligned} \tag{1.2}$$

$$\nabla_k A_{ji} - \nabla_j A_{ki} = \frac{c}{4}(\xi_k \phi_{ji} - \xi_j \phi_{ki} - 2\xi_i \phi_{kj}), \tag{1.3}$$

where R_{kjih} are components of the Riemannian curvature tensor of M.

To write our formulas in convention forms, we denote in the sequal by $A_{ji}{}^2 = A_{jr}A_i{}^r$, $h = t_r A = g_{ji}A^{ji}$, $\alpha = A_{ji}\xi^j\xi^i$ and $\beta = A_{ji}{}^2\xi^j\xi^i$.

If we put $U_j = \xi^r \nabla_r \xi_j$, then U is orthogonal to ξ. Hence it is, using (1.1), clear that

$$\phi_{jr}U^r = A_{jr}\xi^r - \alpha\xi_j, \tag{1.4}$$

which shows that $g(U,U) = \beta - \alpha^2$. From the second equation of (1.1) we easily see that

$$U^r \nabla_j \xi_r = A_{jr}{}^2 \xi^r - \alpha A_{jr}\xi^r. \tag{1.5}$$

Differentiating (1.4) covariantly along M and using (1.1), we find

$$\xi_j(A_{kr}U^r + \alpha_k) + \phi_{jr}\nabla_k U^r = \xi^r \nabla_k A_{jr} - A_{jr}A_{ks}\phi^{rs} + \alpha A_{kr}\phi_j{}^r, \tag{1.6}$$

which enable us to obtain

$$(1.7) \qquad (\nabla_k A_{rs})\xi^r\xi^s = 2A_{kr}U^r + \alpha_k,$$

where $\alpha_k = \partial_k\alpha$.

Transforming (1.6) by $\phi_i{}^j$ and making use of (1.1) and (1.5), we have

$$(1.8) \qquad \nabla_k U_i + \xi_i A_{kr}{}^2\xi^r + \xi^r(\nabla_k A_{sr})\phi_i{}^s = (\nabla_k\xi^r)(\nabla_r\xi_i) + \alpha A_{ki}.$$

Thus, it follows that

$$(1.9) \qquad \xi^r\nabla_r U_j = -3U^s A_{rs}\phi_j{}^r + \alpha A_{jr}\xi^r - \beta\xi_j - \phi_{jr}\alpha^r,$$

where we have used (1.1) and (1.7).

We can put

$$(1.10) \qquad A_{jr}\xi^r = \alpha\xi_j + \mu W_j,$$

where W is a unit vector field orthogonal to ξ. Then from this and (1.4) we see that $U = -\mu\phi W$ and $\mu^2 = \beta - \alpha^2$. Thus W is also orthogonal to U.

By (1.2) the Ricci tensor S with component S_{ji} of M is given by

$$(1.11) \qquad S_{ji} = \frac{c}{4}\{(2n+1)g_{ji} - 3\xi_j\xi_i\} + hA_{ji} - A_{ji}{}^2.$$

Since ξ and W are mutually orthogonal, it is, using (1.1), (1.4) and (1.10), seen that

$$\mu\xi_r\nabla_j W^r = A_{jr}U^r .tag1.12$$

REMARK. Let M be a real hypersurface of $M_n(c)$, $c \neq 0$. If the structure vector field ξ is principal, then α is constant [4], [9]. Furthermore, it is, using (1.4), (1.6), seen that $dh(\xi) = 0$, where dh is the exterior differential of h. But the converse problem as above is not yet proved

In the following, we assume that $\mu \neq 0$ on M, that is, ξ is not principal curvature vector field and we put $\Omega = \{p \in M | \mu(p) \neq 0\}$ Then Ω is an open subset of M, and from now on we discuss our argument on Ω.

2. Real hypersurfaces satisfying $S\phi = \phi S$

Let M be a real hypersurface of a complex space form $M_n(c), c \neq 0$ satisfying $S\phi = \phi S$. Then we have by (1.11)

$$(hA_{jr} - A_{jr}{}^2)\phi_i{}^r + (hA_{ir} - A_{ir}{}^2)\phi_j{}^r = 0. \tag{2.1}$$

Because of properties of the almost contact metric structure induced on M, it follows that

$$S_{jr}\xi^r = \sigma\xi_j, \tag{2.2}$$

where we have put $\sigma = g(S\xi, \xi)$, which together with (1.11) implies that

$$A_{jr}{}^2\xi^r = hA_{jr}\xi^r + (\beta - h\alpha)\xi_j, \tag{2.3}$$

where we have defined

$$\beta - h\alpha = \frac{c}{2}(n-1) - \sigma. \tag{2.4}$$

From (1.10) and (2.3), we have

$$A_{jr}W^r = \mu\xi_j + (h - \alpha)W_j. \tag{2.5}$$

because $\mu \neq 0$ on Ω, and hence

$$A_{jr}{}^2W^r = hA_{jr}\xi^r + (\beta - h\alpha)W_j. \tag{2.6}$$

Differentiating (2.5) covariantly along Ω, we obtain

$$\begin{aligned}&(\nabla_k A_{jr})W^r + A_{jr}\nabla_k W^r \\ &= \mu_k\xi_j + \mu\nabla_k\xi_j + (h_k - \alpha_k)W_j + (h - \alpha)\nabla_k W_j.\end{aligned} \tag{2.7}$$

Transvecting W^j and taking account of (1.12) and (2.5) yields

$$(\nabla_k A_{rs})W^r W^s = -2A_{kr}U^r + h_k - \alpha_k. \tag{2.8}$$

If we transvect also ξ^j to (2.7) and use (1.10), then we have

$$\mu(\nabla_k A_{rs})\xi^r W^s = (h - 2\alpha)A_{kr}U^r + \frac{1}{2}\beta_k - \alpha\alpha_k, \tag{2.9}$$

where we have used the fact that $\mu^2 = \beta - \alpha^2$.

On the other hand, taking the inner product (2.1) with μW^i and making use of (1.4), (2.5) and (2 6), we obtain

$$A_{jr}{}^2 U^r = hA_{jr}U^r + (\beta - h\alpha)U_j. \tag{2.10}$$

Now, differentiating (2.3) covariantly along Ω and using (1.1) an (2.1), we find

$$\begin{aligned}(2.11)\qquad &(\nabla_k A_{jr})A_s{}^r \xi^s + A_j{}^r(\nabla_k A_{rs})\xi^s - h(\nabla_k A_{jr})\xi^r \\ &= h_k A_{jr}\xi^r + (\beta - h\alpha)_k \xi_j + (A_{kr}{}^3 - hA_{kr}{}^2)\phi_j{}^r \\ &\quad - (\beta - h\alpha)A_{kr}\phi_j{}^r,\end{aligned}$$

which together with (1.7) gives

$$(\nabla_k A_{ts})\xi^t A_r{}^s \xi^r = hA_{kr}U^r + \frac{1}{2}\beta_k. \tag{2.12}$$

If we take the inner product (2.11) with ξ^k and make use of (1.3), (1.7), (2.10) and (2.12), then we obtain

$$\begin{aligned}(2.13)\qquad &hA_{jr}U^r + (2\beta - 2h\alpha - \frac{c}{4})U_j + A_{jr}\alpha^r + \frac{1}{2}\beta_j - h\alpha_j \\ &= dh(\xi)A_{jr}\xi^r + d(\beta - h\alpha)(\xi)\xi_j,\end{aligned}$$

where $dh(\xi) = h_t\xi^t$ and $d(\beta - h\alpha)(\xi) = \beta_t\xi^t - h\alpha_t\xi^t - \alpha h_t\xi^t$.

By the way, it is, using (1.3), (1.10), (2.8), (2.9) and (2.12), seen that

$$\begin{aligned}
&(\nabla_k A_{js})(A_r{}^s\xi^r)(A_t{}^k\xi^t) + A_j{}^s(\nabla_k A_{rs})\xi^r(A_t{}^k\xi^t)\\
=\ &(h^2 + 2h\alpha - 2\beta - \frac{c}{2})A_{jr}U^r + \{h(\beta - h\alpha) - \frac{3}{4}c\alpha\}U_j\\
&+ \alpha\beta_j - \beta\alpha_j + \frac{1}{2}A_{jr}\beta^r + (\beta - \alpha^2)h_j.
\end{aligned}$$

Thus, by transvecting $A_t{}^k\xi^t$ to (2.11), we can get

$$\begin{aligned}
&(2h\alpha - 2\beta - \frac{c}{2})A_{jr}U^r + \{h(\beta - h\alpha) - \frac{3}{4}c\alpha + \frac{c}{2}h\}U_j\\
&+ \frac{1}{2}(2\alpha - h)\beta_j - \beta\alpha_j + \frac{1}{2}A_{jr}\beta^r + (\beta - \alpha^2)h_j\\
=\ &dh(A\xi)A_{jr}\xi^r + d(\beta - h\alpha)(A\xi)\xi_j,
\end{aligned} \tag{2.14}$$

where we have used (1.3) and (2.12).

If we take the inner product with $A_s{}^j\xi^s$ and W_j to (2.14), and make use of (1 10), (2.3) and (2.5), then we obtain respectively

$$\frac{1}{2}\mu d\beta(W) - \frac{1}{2}\alpha d\beta(\xi) + \beta d\alpha(\xi) = (\beta - \alpha^2)dh(\xi), \tag{2.15}$$

$$\frac{1}{2}d\beta(W) + \mu d\alpha(\xi) - \alpha d\alpha(W) = \mu dh(\xi). \tag{2.16}$$

From the last two equations, it follows that

$$\frac{1}{2}d\beta(\xi) = \alpha d\alpha(\xi) + \mu d\alpha(W). \tag{2.17}$$

3. Main result

First of all we prove the following :

LEMMA 1. *Let M be a real hypersurface of $M_n(c)$, $c \neq 0$ satisfying $S\phi = \phi S$. Suppose that $dh(\xi) = 0$ and $\alpha = const.$ Then we have $du(\xi, X) = 0$ for any vector field X on Ω, where $u(X) = g(U, X)$ and d is denoted by the operator of exterior derivative.*

Proof. Because of (2.16) and (2.17), we have $d\beta(W) = d\beta(\xi) = 0$ by virtue of $dh(\xi) = 0$ and $\alpha = const.$ Thus (2.13) and (2.14) are reduced respectively to

$$hA_{jr}U^r + (2\beta - 2h\alpha - \frac{c}{4})U_j + \frac{1}{2}\beta_j = 0, \tag{3.1}$$

$$\begin{aligned}(2h\alpha - 2\beta - \frac{c}{2})A_{jr}U^r + \{h(\beta - h\alpha) - \frac{3}{4}c\alpha + \frac{c}{2}h\}U_j \\ + \frac{1}{2}(2\alpha - h)\beta_j + \frac{1}{2}A_{jr}\beta^r + (\beta - \alpha^2)h_j = 0,\end{aligned} \tag{3.2}$$

where we have used (1.10) Using (2 10) and (3.1), we get

$$\frac{1}{2}A_{jr}\beta^r + (h^2 + 2\beta - 2h\alpha - \frac{c}{4})A_{jr}U^r + h(\beta - h\alpha)U_j = 0.$$

Combining with the last three equations, it follows that

$$fU_j + (h\alpha - 2\beta - \frac{c}{8})\beta_j + h(\beta - \alpha^2)h_j = 0, \tag{3.3}$$

where we have put

$$f = (2h\alpha - 4\beta - \frac{c}{4})(2\beta - 2h\alpha - \frac{c}{4}) + 2h(2\alpha - h)(\beta - h\alpha) - \frac{c}{4}h(h - \alpha). \tag{3.4}$$

Differentiating (3.3) covariantly along Ω, we find

$$f_k U_j + f \nabla_k U_j + (h\alpha - 2\beta - \frac{c}{8})\nabla_k \beta_j + h(\beta - \alpha^2)\nabla_k h_j$$
$$+ (\alpha h_k - 2\beta_k)\beta_j + (\beta - \alpha^2) h_j h_k + h\beta_k h_j = 0,$$

from which, taking the skew-symmetric part,

$$f_k U_j - f_j U_k + f(\nabla_k U_j - \nabla_j U_k) + (h - \alpha)(\beta_k h_j - \beta_j h_k) = 0.$$

Thus, it is clear that

$$f\xi^k(\nabla_k U_j - \nabla_j U_k) = 0, \tag{3.5}$$

because we have used (3.4) and the fact that $d\beta(\xi) = 0$ and $dh(\xi) = 0$.

Now, let Ω_0 be a set of points in Ω such that $du(\xi, X) \neq 0$ for any vector field X on Ω and suppose that Ω_0 be not empty Then we have $f = 0$ on Ω_0 with the aid of (3.5), and hence

$$(8h\alpha - 8\beta - h^2 + \frac{c}{4})\beta_j$$
$$+ \{6\alpha\beta - 6h\alpha^2 + 2\alpha h^2 + 2(\beta - h\alpha)(\alpha - h) - \frac{c}{4}h + \frac{c}{8}\alpha\}h_j = 0.$$

Furthermore, by (3 3) we obtain

$$(h\alpha - 2\beta - \frac{c}{8})\beta_j + h(\beta - \alpha^2)h_j = 0$$

on Ω_0. From the last two equations we easily verify that β and h are both constant. Thus, from (3.1) we can see that $A_{jr}U^r = \lambda U_j$, where we have defined the function λ by

$$h\lambda + 2\beta - 2h\alpha - \frac{c}{4} = 0$$

on Ω_0 Because of (2.10) and (3.2), we also obtain respectively

$$\lambda^2 = h\lambda + \beta - h\alpha,$$

$$\lambda(2h\alpha - 2\beta - \frac{c}{2}) + h(\beta - h\alpha) - \frac{3}{4}c\alpha + \frac{c}{2}h = 0$$

on Ω_0. By means of the last three equations we can verify that $\lambda = h - \alpha$ and hence $\beta - \alpha^2 = 0$, which is a contradiction. Hence Ω_0 is empty. This completes the proof of Lemma 1.

PROOF OF THEOREM. Since α is constant, (1.9) is reduced to

$$\xi^k \nabla_k U_j = -3U^s A_{sr} \phi_j{}^r + \alpha A_{jr} \xi^r - \beta \xi_j. \tag{3.6}$$

On the other hand, using (1.5) and (2.3) we have

$$\xi^k \nabla_j U_k = (\alpha - h) A_{jr} \xi^r + (h\alpha - \beta) \xi_j$$

because ξ and U are mutually orthogonal. Thus, substituting this and (3.6) into $\xi^k (\nabla_j U_k - \nabla_k U_j) = 0$, we find

$$3U^s A_{sr} \phi_j{}^r = h(A_{jr} \xi^r - \alpha \xi_j),$$

where we have used the result of Lemma 1, which implies

$$3A_{jr} U_r = hU_j. \tag{3.7}$$

Therefore (2.10) gives

$$\beta - h\alpha = -\frac{2}{9} h^2. \tag{3.8}$$

From this we can get

$$\beta_j = (\alpha - \frac{4}{9} h) h_j. \tag{3.9}$$

Because of (3.7) and (3 8), the equation (3.1) turns out to be

$$\beta_j = (\frac{2}{9} h^2 + \frac{c}{2}) U_j, \tag{3.10}$$

which together with (3.9) yields

$$(\alpha - \frac{4}{9} h) h_j = (\frac{2}{9} h^2 + \frac{c}{2}) U_j, \tag{3.11}$$

which enable us to obtain

$$(3.12)\qquad (\alpha - \frac{4}{9}h)h_t U^t = (\frac{2}{9}h^2 + \frac{c}{2})(h\alpha - \frac{2}{9}h^2 - \alpha^2).$$

Using (3.7) and (3.10), we also have from (3.2)

$$\{\frac{1}{3}h(\beta - h\alpha) - \frac{3}{4}c\alpha + \frac{c}{3}h + (\frac{1}{9}h^2 + \frac{c}{4})(2\alpha - \frac{2}{3}h)\}U_j + (\beta - \alpha^2)h_j = 0.$$

From this and (3.8), it follows that

$$(3.13)\qquad h_t U^t = \frac{4}{27}h^3 - \frac{2}{9}\alpha h^2 + \frac{c}{4}\alpha - \frac{c}{6}h.$$

Substituting this into (3.12), we see that h is constant because h is a root of the algebraic equation with respect to h with constant coefficient. Thus β is also constant since we have (3.9). Accordingly (3.10) and (3.13) will produce a contradiction. Therefore Ω is a empty set, that is, the structure vector field ξ is principal. This completes the proof.

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