Scattering from a Periodic Array of Double-Dipole Elements over a Grounded Dielectric Slab

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CONTENTS

ABSTRACT

An analysis method of electromagnetic scattering from periodic patch array of double-dipole elements on a grounded dielectric substrate in case of oblique incident and arbitrary polarization is considered. The basis functions are chosen to be entire cosinusoidal functions covering the rectangular shaped domain in which the original dipoles are inscribed, unlike the conventional method in which basis functions are defined only for the conducting element region. To confirm the validity of the proposed analysis method, we calculate the normalized scattered power for two propagating modes and compare the results with those obtained by the previous numerical method for the double dipole elements of rectangular type and parallelogram type which have the property of frequency scanned reflection and polarizer. Good correspondence has been observed between them. Some numerical results such as variation of power and axial ratio of first-order diffracted wave by a periodic array of double-dipole elements are compared with previous results.

I. INTRODUCTION

Recently it has been proposed that the periodic grating structure can be used to construct antennas with frequency scanning properties [1]-[5]. In this structure, the first higher order diffracted wave is propagating and the specular wave is simultaneously suppressed when the periodic gratings are illuminated by an electromagnetic wave. Since the propagation direction of this diffracted wave is dependent on frequency, this diffracted wave serves as the frequency scanned beam.

F. S. Johansson [6] proposed a frequency-scanned reflection grating with polarizer properties. The grating structure consists of a periodic array of double-dipole elements etched on a grounded dielectric substrate. The two dipoles within a periodic cell are displaced and tilted approximately 90° from each other. In this case, an incident linear polarized wave will generate a first-order diffracted wave with a circular polarization or a 90°-twisted linear polarization by the difference in path lengths from the incident wave front to the diffracted wave front via the two twisted dipoles.

The purpose of this paper is to consider an analysis method of the scattering problem from periodic arrays on a grounded dielectric substrate which have double-dipole elements in each periodic cell in case of oblique incident and arbitrary polarization. The shapes of the dipole element considered are rectangle and parallelogram. The analysis method employed is the spectral-Galerkin method, which entails the representation of the surface currents in terms of a set of Fourier-transformable basis functions with unknown coefficients. Here the basis function in this analysis method is expressed to be cosinusoidal functions for the rectangular shaped domains in which each dipole element in the unit cell is inscribed respectively. To confirm the proposed analysis method, some numerical results such as variation of power and axial ratio of a first-order diffracted wave by a periodic array of double-dipole elements are compared with the results obtained by use of the subdomain rooftop basis function [7].

II. FORMULATION

The geometry under analysis is shown in Fig. 1. Double dipole elements with zero thickness are periodically arrayed on the grounded multi-layer dielectric slabs of thicknesses d_i and dielectric constants ε_{ri}. The periods in the x and y direction are T_x and T_y, respectively. A plane wave is assumed to be obliquely incident upon this double dipoles array with $(\theta,\ \phi)$ as an incident angle. The polarization angle γ (measured in the clockwise direction with respect to the incident unit vector $\overline{n}$) is the angle between the electric field vector $\overline{E}$ and the segment OA$'$ in the x-y plane as shown in Fig. 2.

The incident field with polarization angle γ is divided into transverse electric (TE)

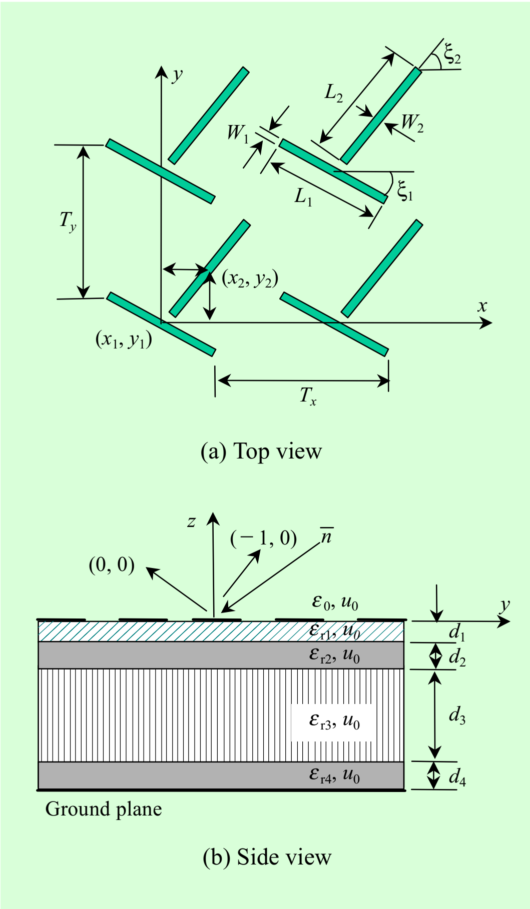


Fig. 33. Geometry of a frequency scanning reflection grating ($\varepsilon_1 = 4.5$, $\varepsilon_2 = \varepsilon_4 = 3.5$, $\varepsilon_3 = 1.07$, $d_1 = 0.3$ mm, $d_2 = d_4 = 0.2$ mm, $d_3 = 5.5$ mm).

field and transverse magnetic (TM) field to z. The TE electric component in the incident wave is given by

$$\overline{E}^i_{TE}(x,\ y,\ z) = (\hat{x}\sin\phi - \hat{y}\cos\phi) \cdot \cos\gamma E_0 e^{j(k_x^i x + k_y^i y + k_z^i z)} \tag{1}$$

and TM electric component by

$$\overline{E}^i_{TM}(x,\ y,\ z) = (-\hat{x}\cos\theta\cos\phi - \hat{y}\cos\theta\sin\phi) \cdot \sin\gamma E_0 e^{j(k_x^i x + k_y^i y + k_z^i z)} \tag{2}$$

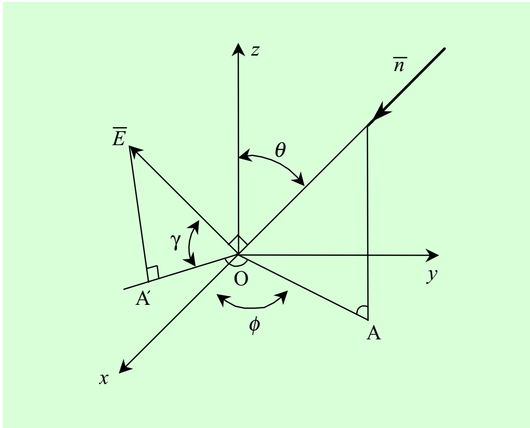


Fig. 34. Incident angle(θ, ϕ) and polarization angle γ.

in which $k_x^i = k_o\sin\theta\cos\phi$, $k_y^i = k_o\sin\theta\sin\phi$, $k_z^i = k_o\cos\theta$.

Assuming the surface current distribution as $\overline{J}_T = (J_x\hat{x} + J_y\hat{y})$ on the dipole elements, the transverse scattered electric field (to z) due to this surface current density is written as follows:

$$\overline{E}^s_T(x,\ y,\ z) = \iint \overline{\overline{G}}(x,\ y,\ z/x',\ y') \cdot \overline{J}_T(x',\ y')dx'dy'. \tag{3}$$

Here $\overline{\overline{G}}$ is the Dyadic Green's function for $z \geq 0$, which is expressed as

$$\overline{\overline{G}} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\overline{\tilde{G}}}(k_x,\ k_y)e^{-k_z z} \cdot e^{jk_x(x-x')} \cdot e^{jk_y(y-y')}dk_x dk_y, \tag{4}$$

in which tilde over $\overline{\overline{G}}$ means the usual Fourier transform such that

$$\overline{\overline{\tilde{G}}} = \hat{x}\hat{x}\,\tilde{G}_{xx} + \hat{y}\hat{x}\,\tilde{G}_{yx} + \hat{y}\hat{y}\,\tilde{G}_{yy} + \hat{x}\hat{y}\,\tilde{G}_{xy}\,. \tag{5}$$

The current distribution $\overline{J}_T$ on the dipole elements is expressed as

$$\overline{J}_T(x,\ y) = (j_x\hat{x} + j_y\hat{y})e^{j(k_x^i x + K_y^i y)}. \tag{6}$$

Since $\bar{j}_T(x,\ y)(=j_x\hat{x}+j_y\hat{y})$ is a two-dimensional periodic function, it is expanded in a Fourier series to be

$$\bar{j}_T(x,\ y)=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\hat{\bar{j}}_{pq}e^{j(k_{xp}x+k_{yq}y)}, \tag{7}$$

where

$$\hat{\bar{j}}_{pq}=\frac{1}{T_xT_y}\iint_{SP}\bar{j}_T(x',\ y')e^{-j(k_{xp}x'+k_{yq}y')}dx'dy', \tag{8}$$

in which SP denotes the area of double dipole elements in the unit cell, and $k_{xp}=2\pi p/T_x$ and $k_{yq}=2\pi q/T_y$.

Substituting (7) into (3) and interchanging the orders of summation and integration, the transverse scattered electric field in the air region ($z>0$) is obtained easily to be

$$\overline{E}_T^s=-\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{\bar{\bar{G}}}(k'_{xp},\ k'_{yq})\cdot\hat{\bar{j}}_{pq}e^{j(k'_{xp}x+k'_{yq}y-k'_{zpq}z)}, \tag{9}$$

where

$$k'_{xp}=k_{xp}+k_x^i \tag{10}$$

$$k'_{yq}=k_{yq}+k_y^i \tag{11}$$

$$k'_{zpq}=\sqrt{(k_o^2-k'^2_{xp}-k'^2_{yq})}, \tag{12}$$

and $\tilde{\bar{\bar{G}}}$ is given by

$$\begin{bmatrix}\tilde{G}_{xx} & \tilde{G}_{xy}\\ \tilde{G}_{yx} & \tilde{G}_{yy}\end{bmatrix}=\begin{bmatrix}\tilde{Z}^e\cos^2\theta+\tilde{Z}^h\sin^2\theta & (\tilde{Z}^e-\tilde{Z}^h)\sin\theta\cos\theta\\ (\tilde{Z}^e-\tilde{Z}^h)\sin\theta\cos\theta & \tilde{Z}^e\sin^2\theta+\tilde{Z}^h\cos^2\theta\end{bmatrix} \tag{13}$$

Here

$$\tilde{Z}^{e,h}=\frac{1}{Y^{+e,h}+Y^{-e,h}}, \tag{14}$$

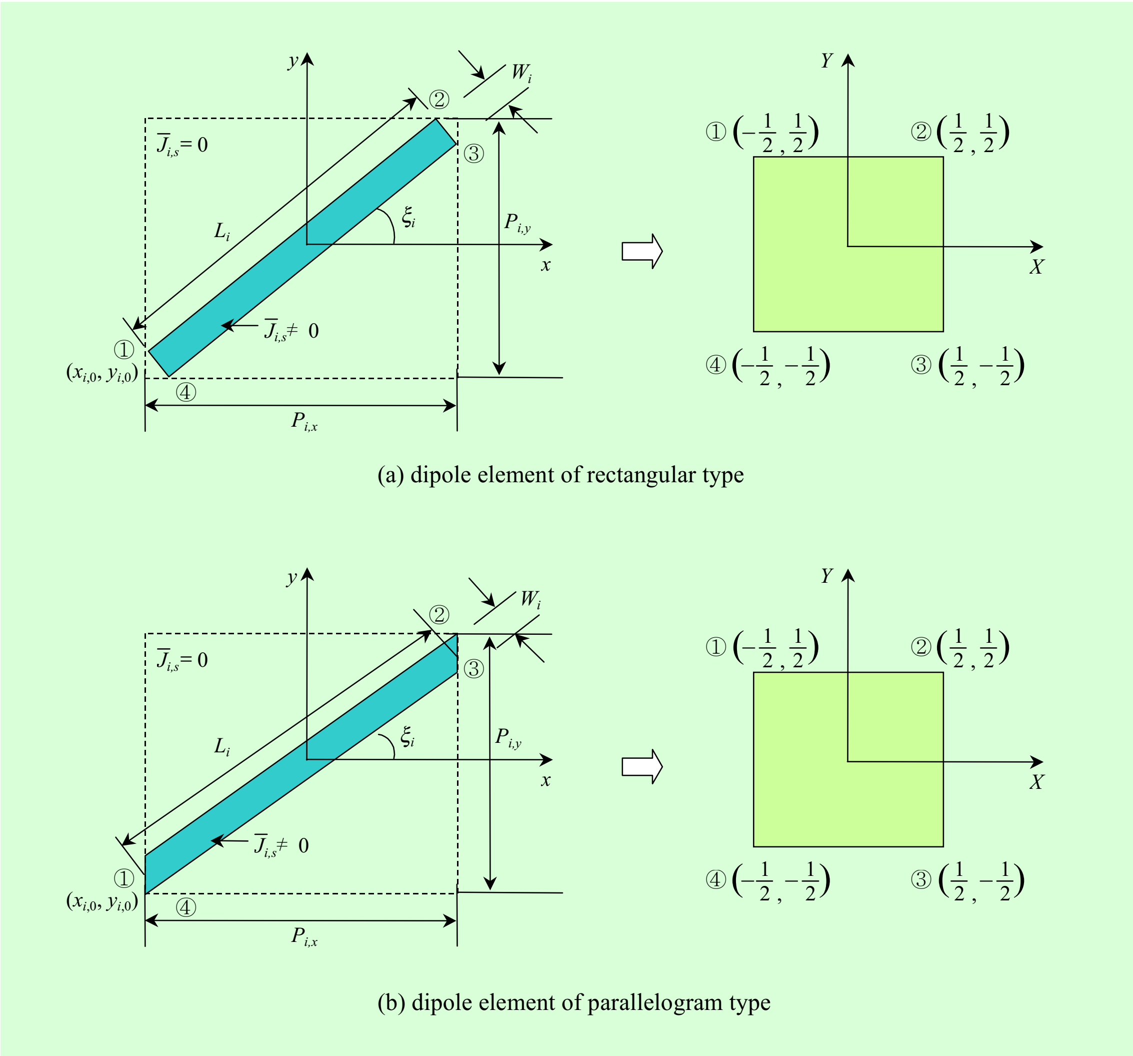
$$\cos\theta=\frac{k'_{xp}}{(k'^2_{xp}+k'^2_{yq})^{1/2}}, \tag{15}$$

$$\sin\theta=\frac{k'_{yq}}{(k'^2_{xp}+k'^2_{yq})^{1/2}}. \tag{16}$$

In (13) and (14), $+(-)$ denotes the direction of z-axis for $z>0$ ($z<0$), $Y^{\pm}$ are the field admittance on the corresponding side of the double dipoles array, and $e(h)$ means TM(TE) mode [7].

From now, we will treat the surface current density induced on the dipole elements. When a dipole element takes the arbitrary shape of a parallelogram (not rectangular), it is impossible to use the entire domain basis function. So some sub-domain basis function expansion applicable to such geometries should be used. In order to describe the arbitrary shape of the dipole elements with good resolution to solve the scattering problem accurately, it is usually necessary to employ a large number of subdomains. This results in the large size of the moment method matrix.

In order to reduce the size of the moment method matrix for the subdomain basis expression approach, a simple method for the surface current on the patch is considered as follows. First, we draw a rectangle(dash line) in which the dipole element is inscribed as shown in Fig. 3. The unknown surface current distribution on the dipole element can be expanded into cosinusoidal functions for the rectangle (not on



(a) dipole element of rectangular type

(b) dipole element of parallelogram type

Fig. 35. Geometry of a dipole element and mapping xy-plane into XY-plane.

the dipole region) as follows:

$$j_{i,x}(x,y) = \sum_m \sum_n A_{i,xmn} \sin \frac{m\pi}{P_{i,x}}(x - x_{i,o}) \cdot \cos \frac{n\pi}{P_{i,y}}(y - y_{i,o}) \tag{17}$$

$$j_{i,y}(x,y) = \sum_m \sum_n A_{i,ymn} \cos \frac{m\pi}{P_{i,x}}(x - x_{i,o}) \cdot \sin \frac{n\pi}{P_{i,y}}(y - y_{i,o}), \tag{18}$$

where i means the dipole element 1 and 2.

The next step is to integrate (8) which is the Fourier transform of the current distribution. This is readily integrated by use of the mapping technique as shown in Fig. 3, in which the xy-plane is mapped into the XY-plane by the following functions:

$$x = f_x(X,\ Y) = aX + bY + c \tag{19}$$

$$y = f_y(X,\ Y) = dX + eY + f, \tag{20}$$

By applying (19) and (20) into (8), the component of the transform of the current densities $\hat{\bar{j}}_{pq}$ is expressed as

$$\hat{\bar{j}}_{pq} = \frac{1}{T_x T_y} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \bar{j}_T(f_x(X,\ Y), f_y(X,\ Y)) \cdot e^{-j(k_{xp} f_x(X,Y) + k_{yq} f_y(X,Y))} J_{aco} dX dY, \tag{21}$$

in which J_{aco} means the Jacobian coefficient, $J_{aco} = |ae - bd|$. Substituting (17) and (18) into (21), (21) is calculated to be after algebra

$$\hat{\bar{j}}_{i,pq} = \sum_m \sum_n A_{i,xmn} \, \widetilde{B}^*_{i,xmnpq} \, \hat{x} + \sum_m \sum_n A_{i,ymn} \, \widetilde{B}^*_{i,ymnpq} \, \hat{y}, \tag{22}$$

where

$$\begin{aligned}\widetilde{B}^*_{i,xmnpq} = & \frac{J_{aco}}{T_x T_y} \frac{e^{-j(k_{xp} c + k_{yq} f)}}{4j} \\ & \cdot \{ e^{j\gamma_1} \sin c(h_1 - g_1) \cdot \sin c(h_2 - g_2) \\ & - e^{-j\gamma_1} \sin c(h_1 + g_1) \cdot \sin c(h_2 + g_2) \\ & + e^{j\gamma_2} \sin c(h_3 - g_1) \cdot \sin c(h_4 - g_2) \\ & - e^{-j\gamma_2} \sin c(h_3 + g_1) \cdot \sin c(h_4 + g_2) \},\end{aligned} \tag{23}$$

$$\begin{aligned}\widetilde{B}^*_{i,ymnpq} = & \frac{J_{aco}}{T_x T_y} \frac{e^{-j(k_{xp} c + k_{yq} f)}}{4j} \\ & \cdot \{ e^{j\gamma_1} \sin c(h_1 - g_1) \cdot \sin c(h_2 - g_2) \\ & - e^{-j\gamma_1} \sin c(h_1 + g_1) \cdot \sin c(h_2 + g_2) \\ & - e^{j\gamma_2} \sin c(h_3 - g_1) \cdot \sin c(h_4 - g_2) \\ & + e^{-j\gamma_2} \sin c(h_3 + g_1) \cdot \sin c(h_4 + g_2) \}.\end{aligned} \tag{24}$$

Here

$$\begin{aligned}\gamma_1 &= \pi m (c - x_{i,o})/P_{i,x} + \pi n (f - y_{i,o})/P_{i,y} \\ \gamma_2 &= \pi m (c - x_{i,o})/P_{i,x} - \pi n (f - y_{i,o})/P_{i,y} \\ h_1 &= (\pi m \ a/P_{i,x} + \pi n \ d/P_{i,y})/2 \\ h_2 &= (\pi m \ b/P_{i,x} + \pi n \ e/P_{i,y})/2 \\ h_3 &= (\pi m \ a/P_{i,x} - \pi n \ d/P_{i,y})/2 \\ h_4 &= (\pi m \ b/P_{i,x} - \pi n \ e/P_{i,y})/2 \\ g_1 &= \pi p \ a/T_x + \pi q \ d/T_y \\ g_2 &= \pi p \ b/T_x + \pi q \ e/T_y \\ \sin c &= \sin(x)/x\end{aligned}$$

and the asterisk becomes the complex conjugate.

Last, substituting (22) into (9), and applying the boundary condition that the total tangential electric field should vanish on the surface of dipole elements and employing Galerkin method, the final matrix equation is obtained to be

$$\sum_p \sum_q \widetilde{\widetilde{G}} \cdot (\hat{\bar{j}}_{1,pq} + \hat{\bar{j}}_{2,pq}) \, \widetilde{B}_{i,stpq} = [\overline{E}^i_T + \overline{E}^r_T] \, \widetilde{B}_{i,st00}, \ i = 1, 2 \ , \tag{25}$$

where $\overline{E}^i_T$ denotes the transverse component of the incident field and $\overline{E}^r_T$ corresponds to the transverse component of the reflected field by the grounded dielectric slab in the absence of the conducting dipole elements. Once unknown surface current distribution is computed, the scattered field from the grating is calculated by use of (9).

III. NUMERICAL RESULTS

In order to confirm this analysis method, we calculate the scattered power and compare the results for the normalized scattered power of two propagating mode, (0, 0) and (−1, 0), by this present analysis method with those obtained by the numerical method (rooftop basis function) for the periodic array of each double dipole type for the case that the linear TE-polarized plane wave (with the electric field parallel to y axis) is incident upon the periodic array plane (z=0) at the incident angle of $\theta = 45°$ and $\phi = 0°$. Here, the number of basis functions for the present case is (m, n) = (5, 5), and the number of small rectangular cells along the x and y directions for the numerical method (rooftop basis function) is (M, N) = (80, 80). Comparisons show good agreements in Fig. 4, where it is observed that, while the first-order diffracted mode (−1, 0) is propagating, the specular mode (0, 0) is suppressed. This is a kind of off-Bragg blazing phenomena. What is more, the diffracted wave becomes a left-hand circular polarized (LHCP) wave. The conversion loss to the circular polarized diffracted wave is less than 0.1 dB over the frequency band 8.8∼10.8 GHz for the rectangular double dipole type in Fig. 4(a). On the other hand, same performance is achieved over the frequency 9.0∼11.75 GHz for parallelogram double dipole type in Fig. 4(b). Over these frequency ranges, the axial ratio is observed to less than 0.85 dB and 0.95 dB, respectively, and the frequency scan angle $\theta_{-1,0}$ ranges from 41.8° to 23.3° and 39.4° to 18.7°, respectively, for the rectangular case and the parallelogram case.

Figure 5 shows the comparison between the calculated results for the normalized scattered power of (−1, 0) spectral order

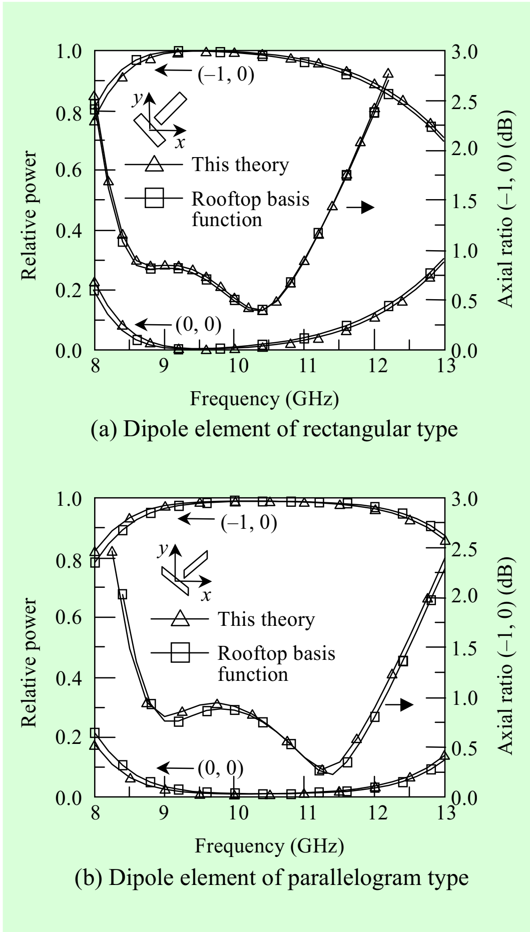
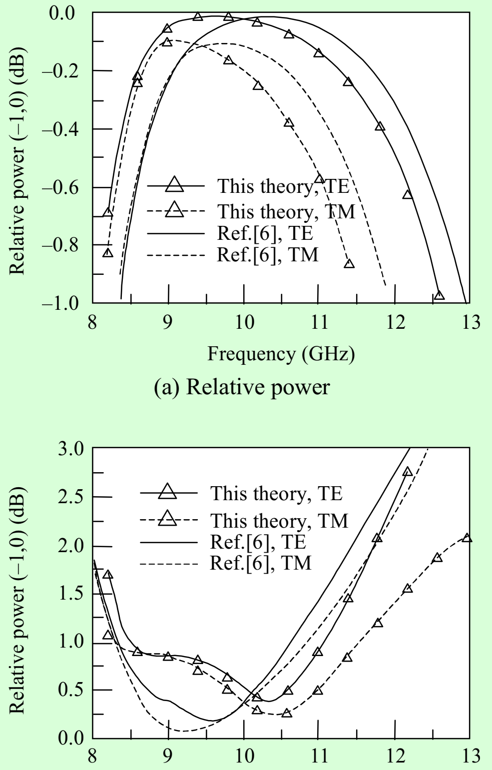


Fig. 36. Computed relative power of the specular wave and the diffracted wave of periodic arrays: (a) T_x=24.8 mm, T_y=16.0 mm, $L_1 = L_2$=10.6 mm, $W_1 = W_2 = 1.5$ mm, $\xi_2 = -\xi_1 = 38°$, $x_2 = 6.2$ mm, y_2=5.4 mm, $\theta = 45°, \phi = 0°, \gamma = 0°$, (b) T_x=24.8 mm, T_y=16.0 mm, $L_1 = L_2$=10.3 mm, $W_1 = W_2$ =1.8 mm, $\xi_2 = -\xi_1 = 40°, x_2 = 6.2$ mm, $y_2 = 5.4$ mm, $\theta = 45°, \phi = 0°, \gamma = 0°$.

and the axial ratio and the previous results [6]. As shown in Fig. 5(a), some discrepancy is observed; that is, the curve for the present results is shifted leftward roughly



(a) Relative power

(b) Axial ratio

Fig. 37. Comparison between the present results and the previous results of the rectangular dipole arrays as shown in Fig. 4(a).

to the extent of 0.4 GHz as a whole, in comparison with that for the previous results [6], retaining almost the same shape to that for previous results. This, in turn, gives some discrepancy between the present results and previous results for the axial ratio as shown in Fig. 5(b). These discrepancies are thought to be mainly due to the fact that only the current densities of axial components (of the dipole elements) have been taken into account in the previous work, while, in the present work, the current densities of normal components to the dipole axis as well as the current densities of axial components have been taken into account.

IV. CONCLUSION

An analysis method of the scattering problem from the periodic array of double dipole elements over a grounded dielectric substrate in case of oblique incident and arbitrary polarization is considered. Some numerical results for the normalized scattered powers, under the condition that only the two space harmonics, (0, 0) and (−1, 0), are propagating, are calculated and compared with those by use of the moment method whose basis function is of rooftop type. Good correspondence has been observed between them. The power conservation relationship has also been checked. The present method will be useful in the design of a periodic array of parallelogram dipoles and quadrilateral patches as well as the typical rectangular dipoles.

The method presented here will considerably reduce memory space in comparison with the rooftop sub-domain and it will be comparable to the rooftop sub-domain method along with FFT algorithm from the viewpoint of solution convergence [7]. However, which solutions for the current density distribution, between the present method and the previous rooftop basis expansion method, can satisfy the boundary condition more accurately along the edge of the dipole element, in an average sense, still remains to be investigated further.

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