부분지지되고 개구부를 갖는 적층복합판의 동적해석

Transient Analysis of Partially Supported Laminated Composite Plates With Cutouts

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요 약 : 본 연구에서는 부분지지되어 있고 사각형의 개구부를 갖는 적층복합판의 과도해석을 유한요소법을 이용하여 연구하였다. 수학적 지배방정식은 변분과 일차전단변형이론을 고려하여 유도하였고 적층판의 폭-두께비, 재료의 특성, 적층각도, 지지조건이 중앙점의 동적응답에 미치는 영향에 대하여 연구하였다. 수치적인 결과는 그래프와 표로 나타내었으며 참고문헌의 결과와 비교 분석하였다.

ABSTRACT : The transient analysis of partially supported laminated plates with rectangular holes under uniformly distributed transverse load is studied using finite element method. The first-order shear deformation theory and the variational energy method are employed in mathematical formulation. The effects on central deflection by plate thickness ratio, material modulus ratio, ply lamination geometry and boundary conditions are investigated. Numerical results are presented and comparisons of the results by the present method with those given in the literature are made.

핵 심 용 어 : 적층복합판, 동적해석, 전단변형, 부분지지, 유한요소법

KEYWORDS : laminated composite plates, transient analysis, shear deformation, partial support, finite element method.

1. Introduction

It is well known that composite materials have been in great usage in modern structures. For lightweight consideration, the composite laminates used in modern structures are usually very thin. Owing to the anisotropic and nonhomogeneous properties of the composite laminate, its transient response is different from that of the traditional material, so it is necessary to pay attention to the transient response of composite materials. Most structures, whether they are used in land, sea or air, are subjected to dynamic loads during their operation. Therefore, there exists a need for assessing the transient response of laminated plates.

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The transient response of laminated composite plates has been studied under a variety of loading and boundary conditions by both analytical and numerical techniques, such as finite element method.[1)-(5] These studies have concentrated mainly on square/rectangular plates with either simply supported or clamped boundary condition. In practical situations, however, because most civil and industrial structures consist of pipes and instruments, structural engineers may quite often encounter partially supported square plates with cutouts. Therefore, opening plate is necessary for pipes containing their sevicability passing through the plate.

In the present paper, transient analysis of layered, anisotropic, partially supported composite plates with cutouts is investigated using a shear deformable finite element method.

2. Governing Equations

The laminated plate with constant thickness h is composed of orthotropic laminae stacking symmetrically or anti-symmetrically about the middle surface of plate. Rectangular cartesian coordinates (x, y, z) are used for the plate coordinates where the x-y plane coincides with the middle surface of plate, as shown in Fig. 1.

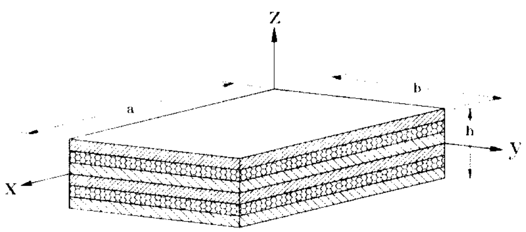

Fig. 1 Coordinate system for laminated plate

It has been shown that transverse shear must be taken into account for a plate made of advanced composites that feature a relatively low shear modulus.

The constitutive equations for an orthotropic layer can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

where $\overline{Q}_{ij}$ are the reduced stiffnesses, which can be expressed in terms of the elastic constants C_{ij} by coordinate transformations.[6]

A first-order shear deformation theory[7] is employed in this study, thus the displacement field is assumed to be of the form:

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\phi_x(x,y,t) \\ u_2(x,y,z,t) &= v(x,y,t) + z\phi_y(x,y,t) \quad (2) \\ u_3(x,y,z,t) &= w(x,y,t) \end{aligned}$$

where u, v and w denote the displacement of any point on the middle surface; and ϕ_x and ϕ_y are the rotations of normals to midplane about the y and x axes, respectively. Linear strain-displacement relations are used for the derivation of governing equations. The total energy of a laminated plate under static loading is defined as

$$\Pi = U_b + U_s + V \tag{3}$$

where U_b the strain energy due to plate bending, U_s the strain energy due to transverse shear, and V the potential energy by external loadings. They are given by

$$U_b = \frac{1}{2}\int_v [\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_{xy}\varepsilon_{xy}]\,dxdydz$$

$$U_s = \frac{1}{2}\int_v [\tau_{yz}\varepsilon_{yz} + \tau_{xz}\varepsilon_{xz}]\,dxdydz$$

$$V = -\int_R q\,w\,dxdy$$

The transverse load, q , defined by

$$q = q_0 H(t - t_0)$$

where q_0 is the density of the load, $H(t)$ denotes the Heavyside step function and t_0 denotes the time removing load.

The kinetic energy of laminated plate is defined by

$$T = \int_v \rho[(u_{,t})^2 + (v_{,t})^2 + (w_{,t})^2]\,dxdydz \tag{4}$$

where ρ is the density of the material. Applying the theorem of stationary total potential energy[8] and the variational method[9] in conjunction with the stress-strain and strain-displacement relations, one may obtain, from Equation (3) and (4), the following equilibrium equations.

$$\begin{aligned}
N_{x,x} + N_{xy,y} &= Pu_{,tt} + R\phi_{x,tt} \\
N_{xy,x} + N_{y,y} &= Pv_{,tt} + R\phi_{y,tt} \\
Q_{1,x} + Q_{2,y} &= Pw_{,tt} + q(x, y, t) \\
M_{x,x} + M_{xy,y} - Q_1 &= I\phi_{x,tt} + Ru_{,tt} \\
M_{xy,x} + M_{y,y} - Q_2 &= I\phi_{y,tt} + Rv_{,tt}
\end{aligned} \tag{5}$$

where P, R and I are the normal, coupled normal-rotary and rotary inertia coefficients, respectively,

$$(P, R, I) = \int_{-h/2}^{h/2} (1, z, z^2)\rho\,dz$$

and N_i, Q_i and M_i are the stress and moment resultants defined by

$$(N_i,\ M_i) = \int_{-h/2}^{h/2} \sigma_i(1, z)\,dz,\ (i = x,\ y,\ xy)$$

$$Q_1,\ Q_2 = \int_{-h/2}^{h/2} (\tau_{yz},\ \tau_{xz})\,dz\ .$$

3. Finite Element Formulation

The shear deformable theory of laminated composite plates involves five dependent unknowns $(u,\ v,\ w,\ \phi_x,\ \phi_y)$. Here, we develop the finite-element model of the theory using the variational statement in the theorem of stationary total potential energy.

Suppose that the midplane R of the plate is subdivided into a finite number of elements, $R_e(e=1,2,\ldots.)$. Over each element, R, the generalized displacement U is interpolated by expressions of the form[10]

$$U = \sum_{i}^{r} U_i(t)\,\phi_i(x,y) \tag{6}$$

where U_i is the value of U at node i at time t, ϕ_i is the finite element interpolation functions at node i and r is the number of nodes in the element. For simplicity, we use the same interpolation for each of the generalized displacements (u, v, w, ϕ_x, ϕ_y) Substituting Eq. (6) into Eq. (5), we obtain the element equations

$$[M]\{\ddot{\triangle}\} + [K]\{\triangle\} = \{F\} \tag{7}$$

where $\{\ddot{\triangle}\}$ is the column vector of the nodal values of the generalized displacements, $[K]$ is the matrix of stiffness coefficients, $[M]$ is the matrix of mass coefficients, and $\{F\}$ is the column vector containing the boundary and body force contributions.

Equation (7) can be reduced to appropriate forms depending on the type of analysis. For static analysis, $\{\ddot{\triangle}\}$ is set to zero. To complete the discretization, we must now approximate the time derivatives appearing in equation (7). Here we use the Newmark's direct integration method,[11] in which the vectors $\{\triangle\}$ and $\{\dot{\triangle}\}$ at the end of a time step Δt are expressed in the form

$$\{\dot{\triangle}\}_{n+1} = \{\dot{\triangle}\}_n + [(1-\alpha)\{\ddot{\triangle}\}_n + \alpha\{\ddot{\triangle}\}_{n+1}]\Delta t$$

$$\{\triangle\}_{n+1} = \{\triangle\}_n + \{\dot{\triangle}\}_n \Delta t + [(\frac{1}{2} - \beta)\{\ddot{\triangle}\}_n + \beta\{\ddot{\triangle}\}_{n+1}]\,(\Delta t)^2 \tag{8}$$

where α and β are parameters that control the accuracy and stability of the scheme, and the subscript 'n' indicates that solution is evaluated at the n-th time step (i.e. at time, $t = n\Delta t$). The choice $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{4}$ (which corresponds to the Constant Average Acceleration method) is known to give unconditionally stable solutions (in linear problems).

Rearranging equation(7) and (8), we arrive at

$$[\hat{K}]\{\triangle\}_{n+1} = \{\hat{F}\} \tag{9}$$

where

$$[\hat{K}] = [K] + a_0[M]$$

$$\{\hat{F}\} = \{F\}_{n+1} + [M](a_0\{\triangle\}_n + a_1\{\dot{\triangle}\}_n + a_2\{\ddot{\triangle}\}_n)$$

$$a_0 = 1/(\beta\,\Delta t^2),\; a_1 = a_0\Delta t,\; a_2 = \frac{1}{2\beta} - 1$$

Once the solution $\{\triangle\}$ is known at $t_{n+1} = (n+1)\Delta t$, the first and second derivatives (velocity and accelerations) of $\{\triangle\}$ at t_{n+1} can be computed , rearranging the expressions in equation(8), as

$$\{\ddot{\triangle}\}_{n+1} = a_0(\{\triangle\}_{n+1} - \{\triangle\}_n) + a_1\{\dot{\triangle}\}_n + a_2\{\ddot{\triangle}\}_n$$

$$\{\dot{\Delta}\}_{n+1} = \{\dot{\Delta}\}_n + a_3\{\ddot{\Delta}\}_n + a_4\{\ddot{\Delta}\}_{n+1} \quad (10)$$

where $a_3 = (1-\alpha)\Delta t$, $a_4 = \alpha\Delta t$.

All of the operations indicated above, except for equation(8), can be obtained for the whole problem. The equation is then solved for the global solution vector at time $t = t_{n+1}$.

4. Numerical Result and Discussion

In the present study nine-node isoparametric element was employed. Since the finite element accounts for the transverse shear strains, reduced integration[12] was employed to evaluate the shear terms numerically. That is, the 2×2 Gaussian rule was used to integrate the shear related terms while the 3×3 Gaussian rule was used to integrate the bending terms. The Young's moduli, shear moduli, Poisson's ratio, material density, load density, time step, length and width of the rectangular laminated plate, and thickness of the laminate are assumed to be.[13], [14]

$$E_1/E_2 = 25,\ G_{12}/E_2 = G_{13}/E_2 = G_{23}/E_2 = 0.5,$$

$$\nu_{12} = 0.25 \quad a = b = 25\,cm,\ h = 5\,cm$$

$$\rho = 8\times10^{-6} \quad N\sec^2/\,cm^4$$

$$q_0 = 10\,N/\,cm^2,$$

$$E_2 = 2.1\times10^6 \quad N/\,cm^2,\quad \Delta t = 5\,\mu\sec .$$

To validate the derived equations, the obtained deflections of simply supported isotropic plates under suddenly applied uniformly distributed pulse loading are compared with those of Reddy[14] in Table 1. They are in excellent agreement.

Table 1. Comparison of center deflection for isotropic and orthotropic plates under uniform pluse loading.

time μsec	time step	Deflection, $w\times10^3$ (cm)			
		Isotropic plate		Orthotropic plate	
		Reddy (ref.14)	present	Reddy (ref.14)	present
10	2	0.0079	0.00792	0.0079	0.00796
20	4	0.0399	0.03992	0.0398	0.03985
40	8	0.1855	0.18547	0.1939	0.19389
60	12	0.5339	0.53392	0.4303	0.43033
80	16	0.9249	0.92493	0.5531	0.55311
100	20	1.2278	1.22781	0.5264	0.52642
120	24	1.4591	1.45907	0.3705	0.37046
140	28	1.6537	1.65369	0.1779	0.17787
160	32	1.6667	1.66675	0.0353	0.35330
180	36	1.4604	1.46037	-0.0395	-0.39463
200	40	1.1728	1.17276	0.1105	0.11046

To show the effect of the coupling between the inplane displacements (u, v) and bending displacements (w, ϕ_x, ϕ_y) on the transverse central deflection, cross-ply and angle-ply plates, subjected to suddenly applied uniform pulse loading, were analyzed and the results are shown in Fig. 2. From this figure one can see

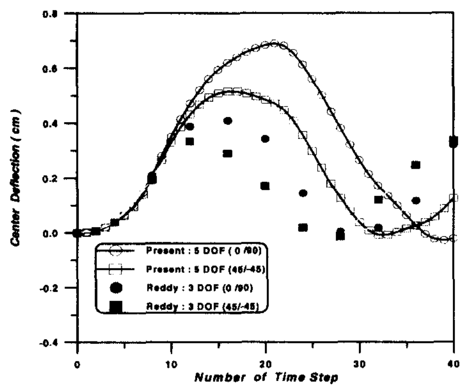

Fig. 2 Comparison of center deflection for two-layer composite laminated plate under uniform loading

Table 2. Deflection of simply supported cross-ply and angle-ply square plates ($\bar{w} = wE_2h^3/qa^4$, a/h = 5)

Lamination angle	FSDT(present)			CLPT(ref. 13)		
	Transient	Static	Ratio	Transient	Static	Ratio
0/90	3.990	1.947	2.049	3.421	1.695	2.018
0/90/0/90	2.131	1.061	2.008	-	-	-
45/-45	2.611	1.279	2.041	2.120	1.028	2.062
45/-45/ 45/-45	1.394	0.692	2.014	-	-	-

FSDT : First-order Shear Deformation Theory
CLPT : Classical Plate Theory

that the coupling has a noticeable influence on the response of the plate. Table 2 shows the ratio of maximum transient deflection to static deflection. The maximum transient deflection for the two-layer cross-ply laminated composite plate is 3.990 and it is about 2.049 times that of the static deflection. The effect of transverse shear is greater on transient response than static response in the two-layer cross-ply plate but the effect of transverse shear is less on transient response than static response in the two-layer angle-ply plates.

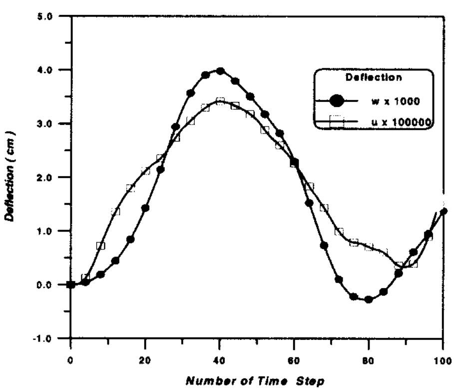

Fig. 3 Deflection and axial displacement for cross-ply partially supported laminated plate with center cutout

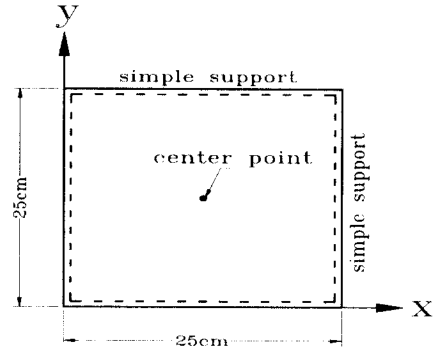

Fig. 4 simple supported plate

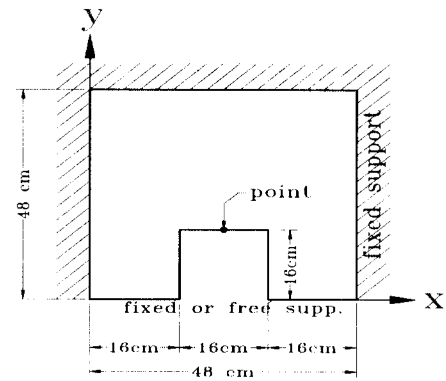

Fig. 5 Bottom-middle cutout plate sample

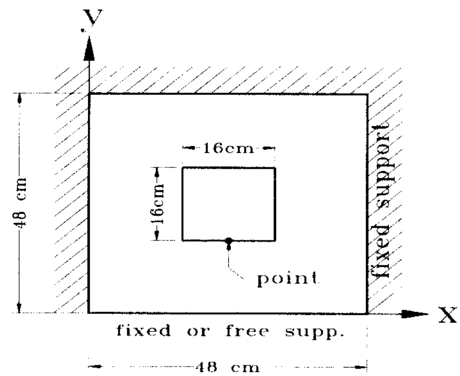

Fig. 6 Center cutout plate sample

The coordinate system and the boundary conditions used for laminated plate with cutouts are shown in Fig. 3-6.

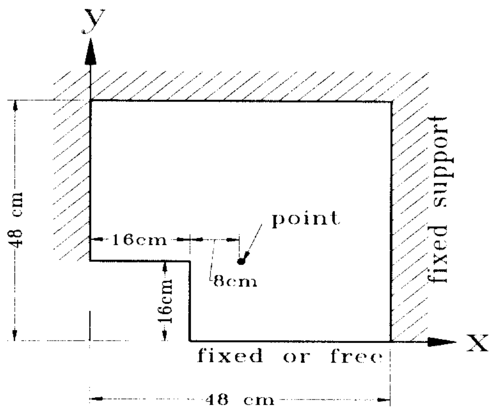

Fig. 7 Bottom-left corner cutout plate sample

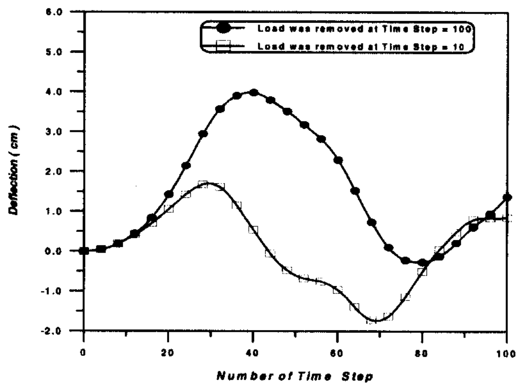

Fig. 8 Deflection for partially supported laminated plate with cutout (middle) under heavyside step loading

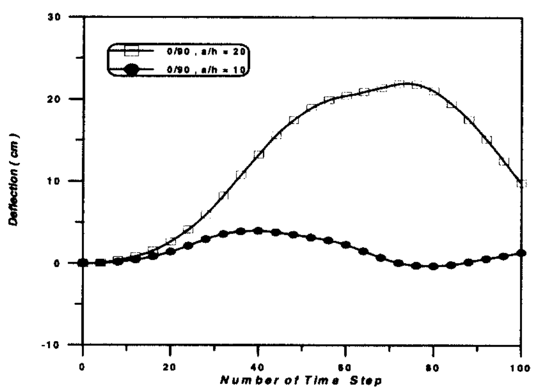

Fig. 9 Effect of plate side to thickness ratio

Fig.7-Fig.9 shows transverse deflections (at $x=24$ cm and $y=16$ cm). In Fig. 3, the axial displacement u(at $x=44$ cm and $y=40$ cm) for the partially supported laminated plate with cutouts as function of time is also presented.

Fig. 8 shows the transient responses for the partially supported laminated plate with cutouts when the applied load was removed at time step=10. Since no damping is accounted for in the present model, the solutions do not decay with time. Fig. 9 shows the transient response of a two-layer cross-ply partially supported laminated plate with cutouts. It can be seen that the period decreases with increasing values of thickness of the plate. Fig. 10 shows the transient response of a two-layers lamination angle(0/90) and all edge fixed supported laminated plate with cutouts. It can be seen that the maximum transverse deflections (at $x=24$ cm and $y=16$ cm) of bottom-middle cutout plate is nearly two times that of center cutout plate. The position of cutout is significant effect on

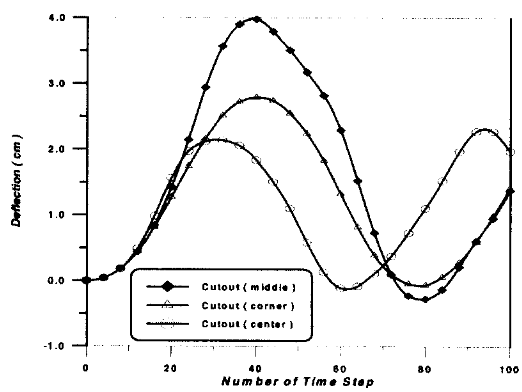

Fig. 10 Deflection of plate [0/90] with three type cutouts

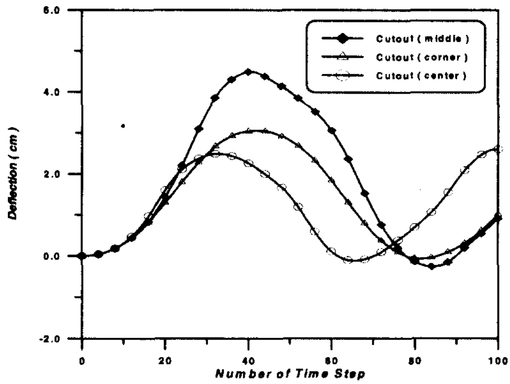

Fig. 11 Deflection of plate (45/-45) with three type cutouts

maximum transverse deflections.

Fig. 11 shows the transient response of a two-layer antisymmetric angle-ply (45/-45) and all edge fixed supported laminated plate with cutouts. The maximum transverse deflections (at x=24 cm and y=16 cm) of bottom-middle cutout plate is high by 80 percent for a center cutout plate. As noted previously, the position of cutout is extremely important in maximum transverse deflections.

Fig. 12 shows the transient response of a two-layers lamination angle(0/90) and (45/-45), all edge fixed supported laminated plate with center cutouts.

It can be seen that the maximum transverse deflections (at x=24 cm and y=16 cm) of (45/-45) lamination is high by 16 percent that of (0/90) lamination angle plate and (45/-45) lamination plate has a period of vibration 6.5 percent larger than the (0/90) lamination plate. Nevertheless, it is apparent that angle of lamination is significant effect on maximum transverse deflections and period of vibration.

Fig. 13 shows the transient response of different layer numbers in lamination angle(0/90) with all edge fixed supported laminated plate with center cutouts. The maximum transverse deflections (at x=24 cm and y=16 cm) of two layers plate is higher by 56 percent that of eight layers plate and period of vibration is lower by 17 percent. The effect of coupling between bending and extension on the deflection is significant effect in two layers plate and the bending-extension coupling effect dies out rapidly as the number of layers increases.

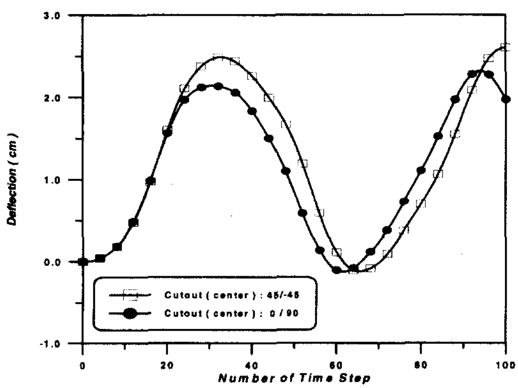

Fig. 12 Deflection of two layers plate different lamination angle

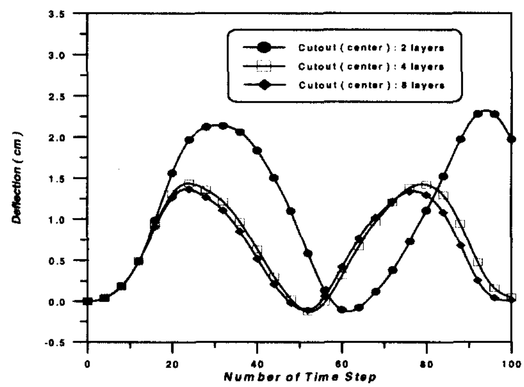

Fig. 13 Deflection of center cutouts plate by layer numbers.

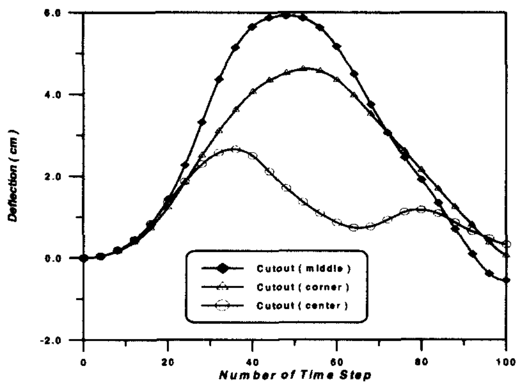

Fig. 14 Deflection of 8-layers plate[0/90] with three type cutouts

Fig. 14 shows the transient response of a eight-layers lamination angle (0/90/0/90/0/90/0/90) with three fixed edge and one free edge laminated plate with cutouts.

The maximum transverse deflections (at x=24 cm and y=16 cm) of bottom-middle cutout plate is high by 122 percent that of center cutout plate. As noted previously, the position of cutout is extremely important in transient response.

5. Conclusions

Transient responses of partially supported laminated composite plates with cutouts have been estimated by employing the first-order shear deformation theory and finite element technique. The present results for isotropic plates are shown to be very close agreement to the other finite element solutions available in the literature.

The position of cutout is extremely important in transient responses behavior.

The coupling between the extension and bending has a noticeable influence on the response of the laminated composite plate and the period decreases with increasing values of thickness of the laminated composite plate and the effect of coupling is significant effect at two layers laminated plate and the bending-extension coupling effect dies out rapidly as the number of layers increases.

The influence of cutouts on transient response as a function of thickness ratio is more significant for thinner plate. Current and future investigations on this subject should be directed to forced vibration and impulse loadings in composite plates with damping included.

The information presented should be useful to composite-structure designers, to researchers seeking to obtain better correlation between theory and experiment and to numerical analysts in checking out their programs.

감사의 말

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