KIGAS Vol.2, No.4, December, 1998
(Journal of the Korean Institute of Gas)

수정된 유사정상상태 해법을 이용한 폐쇄 가스저류층의 장기 거동 해석

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(1998년 11월 9일 접수, 1999년 1월 16일 채택)

Modified Pseudosteady-State Approach to Calculate Long-Time Performance of Closed Gas Reservoirs

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(Received 9 November 1998; Accepted 16 January 1999)

요 약

폐쇄된 고갈성 저류층의 장기 거동을 직접 계산할 수 있는 유사정상상태 해법을 수정하여 실제가스 유동의 해석에 응용하고 그 적용성을 검토하였다. 이 방법은 정규화된 유사시간을 포함하는 선형 가스 확산 방정식과 물질수지 방정식을 결합하여 유도되었다. 본 해법을 서로 다른 유량으로 장기간 생산하는 다수의 정들로 이루어진 실제 문제의 수치 시뮬레이션에 이용하였다. 각 유동 기간의 영향을 고려하기 위하여 유사정상상태 방정식을 연속적으로 적용하였다. 연구 결과로부터 본 해법은 고갈 조건하에 있는 가스 저류층의 장기거동을 빠르고 정확하게 모델링할 수 있는 유용한 방법임을 보여주었다.

Abstract - This paper considers the applicability of a pseudosteady-state approach to the long-time behavior of real gas flow in a closed reservoir. The method involves a combination of a linearized gas diffusivity equation using a normalized pseudotime and a material balance equation. For the simulation of field-scale problems with multiple wells of differing production rates over extended production periods, the pseudosteady-state equation was solved successively for each flow period. Results from this study show that the approach provides a fast and accurate method for modeling the long-time behavior of gas reservoirs under depletion conditions.

Key words : pseudosteady-state, gas reservoir, pseudotime, diffusivity equation

Introduction

One or more wells, produced at constant rate in a closed reservoir, will exhibit monotonic pressure decline. After producing for a time sufficiently long to cause significant pressure effects at all reservoir boundaries and discontinuities, wells approach a pseudosteady-state flow regime. This flow behavior is also sometimes referred to as stabilized. For most reservoirs, stabilized time begins at times that are much shorter than the life of the reservoir, so that the day-to-day operation of wells in depletion reservoirs is governed by relationships during this time.

Previous work concerned with performance predictions during pseudosteady state

generally calculated the long-time solution to the diffusivity equation. Results are mostly limited to geometrically regular drainage shapes with homogeneous rock properties (Dietz, 1965; Earlougher *et al.*, 1968; Earlougher and Ramey, 1973; Yaxley, 1987). Several methods for predicting inflow performance of horizontal wells have been discussed in the literature (Babu and Odeh, 1989; Suprunowics and Butler, 1992). To directly calculate long-term behavior of closed reservoirs without solving the full time-dependent form of diffusivity equation, Lee *et al.* (1996) developed a pseudosteady-state approach. For single phase liquid flow, this approach has shown to be an effective tool for analyzing long-time performance of a bounded reservoir with wells producing at constant.

However, there appears to be a lack of information regarding extension of the approach to gas flow problems mainly due to nonlinearities in the diffusion equation. The main objective of this study, therefore, is to develop a simple but general method for determining long-time behavior of bounded gas reservoirs by extending the pseudosteady-state approach to single phase gas flow. As a first step to extend the approach, a linearized form of gas diffusivity equation was derived. The derivations are created by combining a normalized pseudotime and a material balance equation. The validity and computational efficiency of the approach for the analysis of gas reservoirs has then been confirmed.

A direct solution for the pseudosteady-state behavior which avoids calculation of the infinite-acting and transition period results would allow rapid calculation of pressures in a closed gas reservoir during a period that is of primary importance for long-term reservoir behavior.

Mathematical Formulation

To apply pseudosteady-state approach to gas flow in a closed reservoir, one has to derive a linearized diffusivity equation. As is well known, the flow of gas through a permeable medium is represented by the diffusivity equation using pseudopressure, *pp*, suggested by Al-Hussainy and Ramey (1966).

$$\nabla^2 p_p = \frac{\phi \mu c_t}{k} \frac{\partial p_p}{\partial t} \tag{1}$$

$$p_p = \int_0^p \frac{2p}{\mu z} dp \tag{2}$$

Because the diffusivity equation for gas flow in terms of the pseudopressure involves the least number of assumptions, it is considered to be the most rigorous of the various treatments. The nonlinearity in the diffusivity equation, however, has long been recognized as one of the problems in dealing with gas flow. In this paper, a linearized diffusivity equation is derived by introducing a pseudotime. The linearized diffusivity equation allows one to apply pseudosteady-state method and estimate late-time behavior.

The derivation of the linearized gas diffusivity equation starts by using pseudopressure and normalized pseudotime. Pseudotime, which is an empirical function, has been proposed by many authors like Agarwal (1979), Meunier *et al.* (1987), Reynolds *et al.* (1987), Finjord (1989), and Ding *et al.* (1990).

In order to correlate the real gas pseudopressure solution with analogous liquid solution during boundary-dominated flow, we normalized Agarwals pseudotime through the following equation:

$$t_{pn} = \mu_i c_{ti} \int_0^t \frac{dt}{\mu c_t} \tag{3}$$

Use of pseudopressure and normalized pseudotime produces a theoretical basis for deriving a linearized gas diffusivity equation:

$$\nabla^2 p_p = \frac{\phi \mu_i c_{ti}}{k} \frac{\partial p_p}{\partial t_{pn}} \tag{4}$$

The dimesionless pseudovariables can be defined as

$$p_{pD} = \frac{\pi k h T_{sc}}{q_{sc} p_{sc} T} \left(p_{pw} - p_p \right) \tag{5}$$

$$t_{pnD} = \frac{kt_{pn}}{(\phi\mu c_t)_i r_w^2} \qquad (6)$$

$$t_{pnDA} = \frac{kt_{pn}}{(\phi\mu c_t)_i A} \qquad (7)$$

Use of Eqs. 5 and 6 in the gas diffusivity Eq. 4 results in an equivalent liquid equation:

$$\nabla^2 p_{pD} = \frac{\partial p_{pD}}{\partial t_{pnD}} \qquad (8)$$

The clost_{pnDA}. For constant rate production, the long-time performance of a gas reservoir can be expressed ase analogy between the diffusivity equation of liquid flow and the linearized real gas flow suggests that the liquid and gas solutions can be correlated at late times provided that we graph the gas solution in terms of

$$\bar{p}_{pD} = 2\pi t_{pnDA} \qquad (9)$$

and

$$\bar{p}_{pD} - p_{pwD} = -\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} \qquad (10)$$

In this case, the long-time condition refers to constant decline rate of pseudopressure with respect to normalized pseudotime. Therefore, pseudopressure is a linear function of normalized pseudotime, as shown in Eq. 9.

Because the pseudotime approach requires information on the average reservoir pressure and fluid properties, it is impossible to obtain an expression for average pseudopressure in a closed form. For practical purpose, the average real gas potential in our approach is evaluated from overall system material balance considerations. For a single-phase gas reservoir, material balance can be expressed in terms of hydrocarbon pore volume (Dake, 1978) :

$$\frac{\int_0^t q(\tau)d\tau}{G} = 1 - \frac{\overline{(p/z)}}{(p/z)_i} \qquad (11)$$

To inves$\bar{p}$ $\bar{p}_p$ can be computed from the *pp* versus *p* table. *pw* can then be computed from the rearrangement of Eq. 10 and the *pp* versus *p* table.is known from Eq. 10 as function of time, tigate this approach, a table of gas properties and real gas pseudopressure as function of pressure for any given gas and reservoir conditions is generated. Once

Numerical Simulation Study

This section provides a comparison of the pseudosteady-state solutions with results generated using a conventional finite-difference commercial reservoir simulator. The pseudosteady-state runs were made using FIDAP (Fluid Dynamics International, 1993) on a Cray Y-MP. The commercial simulator was the VIP model (Western Atlas International, Inc., 1993) and was run on an IBM-590. For comparison purpose, the CPU time consumed for preprocessing and postprocessing was estimated and excluded from time comparison. Speed ratios for both machines were also determined. Though the two programs use different numerical scheme and are run on different machines, the following results can be viewed as a guideline.

Table 1. Input data for simulation of field-scale gas reservoir

Reservoir Configuration	Length(a) (miles) Width(b) (miles) Thickness(h) (ft)	3 4 148(58/50/40)
Reservoir Properties	Permeability (k) (md) Porosity(ϕ) Compressibility (c_r)(psi^{-1}) Temperature(T_r)(°F)	25/10/25 0.35/0.25/0.30 1×10^{-6} 100
Fluid Properties	Gas gravity(γ_g) Viscosity(μ_{gi}) Compressibility(c_{gi})(psi^{-1})	0.7 0.026 121.1×10^{-6}
Conditions	Initial pressure (P_i)(psia)	2,000
Run Parameters	Max. press. change(psia) Max. time step size (day) Min. time step size (day)	50 5-25 0.001

To provide an example for predicting the long-time performance of a field-scale reservoir problem, a multiwell, multilayer simulation study was conducted. The model is based on reservoir parameters of the Hugoton

field in southern Oklahoma, as shown in Fig. 1 (Fetkovich *et al.*, 1994). Table 1 shows reservoir properties and run parameters. The flowrate for each well hypothetically changed twice during a 800-day simulation period as indicated in Table 2.

Table 2. Flowrate (Mscf/day) of each well of field-scale gas reservoir.

Well #	Period 1 (0-300)	Period 2 (301-500)	Period 3 (501-800)
1	5,000	2,000	4,000
2	4,500	2,000	3,600
3	4,000	2,000	3,200
4	4,000	1,800	4,000
5	5,000	1,600	4,000
6	5,000	1,800	3,200
7	4,500	1,600	3,200
8	4,000	1,800	3,600
9	5,000	2,000	4,000
10	4,500	1,800	3,200
11	4,000	1,600	3,600
12	4,500	1,600	3,600

Table 3. Comparison of results for field-scale gas reservoir (Number in parentheses indicates flow period)

Reservoir Simulation		Pseudosteady-state Approach	
No. of grid blocks	2916	No. of nodes	3004
No. of time steps	41	No. of time steps	3
Equivalent well radius	116.2ft	Well radius	200
Computer time	310sec	Equivalent computer tiem	33sec
$\overline{P}_{pD} - \overline{P}_{pwfD}$ (1)	-7.3848	$\overline{P}_{pD} - \overline{P}_{pwfD}$ (1)	-7.6437
$\overline{P}_{pD} - \overline{P}_{pwfD}$ (2)	-7.6925	$\overline{P}_{pD} - \overline{P}_{pwfD}$ (2)	-7.9886
$\overline{P}_{pD} - \overline{P}_{pwfD}$ (3)	-7.5962	$\overline{P}_{pD} - \overline{P}_{pwfD}$ (3)	-7.8764

Table 3 compares results from both approaches for three different flow periods. Compared with results from the reservoir simulator, the pseudosteady-state approach provides reasonable pseudopressure drawdown values. with much less computational effort. Fig. 2. shows the comparison of wellbore pressures from both approaches. The quality of the match between two solutions is excellent. The results of Fig. 2 indicate that the pseudosteady-state approach is correct within 2% for all time including infinite-acting and transient periods. Relatively large differences at the early time of each period are due to the difference between transient and pseudosteady-state behavior. Negligible differences at late times confirm the accuracy of pseudosteady-state approach.

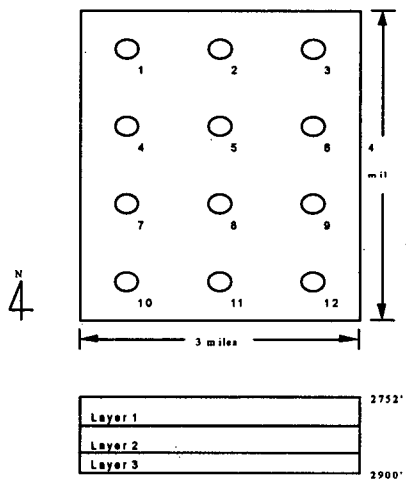

Fig. 1. Reservoir model of Oklahoma Hugoton study area (Ffetkovich *et al.*, 1994)

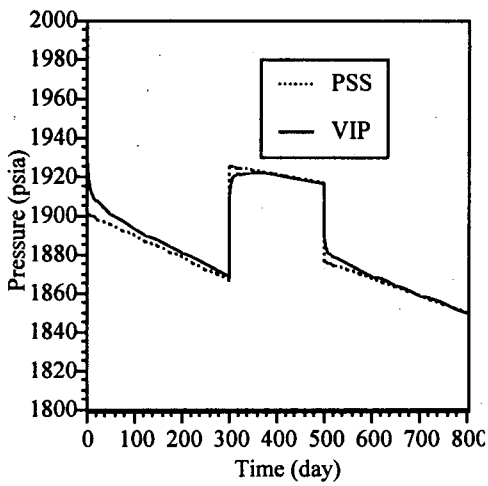

Fig. 2. Comparison of well flowing pressure of Well #1

Conclusion

Pseudosteady-state approach has been applied to model the flow behavior of closed gas reservoirs with wells producing at constant flowrate during boundary-dominated flow. The derivations are based on the linearized gas diffusivity equation that is created by combining a normalized pseudotime and a material balance equation. Comparison with a commercial simulator for reservoirs considered in this study showed that highly accurate values of pseudopressure drawdown and well pressure are obtained by the pseudosteady-state approach with much less computational effort. Such a model can be used for situations where large numbers of runs are desirable, for example, for stochastic simulation or risk analysis.

Nomenclature

a	=	length of rectangular reservoir in x direction [m, L]
A	=	area [m^2, L^2]
b	=	length of rectangular reservoir in y direction [m, L]
c	=	compressibility [Pa^{-1}, Lt^2m^{-1}]
C_A	=	shape factor
G	=	initial gas-in-place [std m^3, std L^3]
h	=	formation thickness [m, L]
k	=	permeability [m^2, L^2]
p	=	pressure [Pa, $mL^{-1}t^2$]
$\bar{p}$	=	average pressure [Pa, $mL^{-1}t^2$]
p_p	=	real gas potential [$Pa^2(Pa\text{-}sec)^{-1}$, $mL^{-1}t^{-3}$]
q	=	flow rate [m^3/sec, L^3t^{-1}]
r	=	radial coordinate
t	=	time [sec, t]
T	=	temperature [K, T]
z	=	real gas deviation factor

Greek Symbols

γ	=	Euler's constant (
μ	=	viscosity [Pa-sec, $mL^{-1}t^{-1}$]
ϕ	=	porosity (fraction), dimensionless
ρ	=	fluid density [kg/m^3, mL^{-3}]

Subscripts

D	=	dimensionless variable
DA	=	dimensionless variable based on area
g	=	gas
i	=	initial condition
o	=	reference condition
p	=	pseudovariables
pn	=	normalized pseudovariables
r	=	rock
sc	=	standard condition
t	=	total
w	=	well

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