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DENSITY OF SEMIMARTINGALE IN CANONICAL STOCHASTIC DIFFERENTIAL EQUATION

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Abstract. The existence and the smoothness of densities of random variables, which are generated by the canonical stochastic differential equation, can be proved by the Malliavin - Bismut method.

0. Introduction

It was pasted a little long time after J.M.Bismut(c.f. [1]) studied the applications of Malliavin calculus in the stochastic differential equation(SDE) for the jump-type processes. But we can not meet many papers for that problem yet, and in general, it is known as a difficult problems to study the existence and the regularity of random variables of jump-type semimartingales. Therefore, we want to study more this problem for some SDE.

In the previous paper[5], we used a SDE which is defined by the same vector fields for the continuous part and the jump part of semimartingales. But in this paper, we will deal the SDE which is defined by another vector fields for the continuous part and the jump part, respectively. Indeed, in [5], we studied the conditions of the existence and the smoothness of densities of random variables, which are generated by the canonical SDE of the form;

$$\xi_t(x) = x + \sum_{j=1}^{m} \int_0^t v_j(\xi_s(x))dW_s^j + \int_0^t \mathcal{L}(\xi_{s-}(x))ds + \int_0^t \int_{E_\alpha} \mathbf{c}_\alpha(x,z)\tilde{N}_\alpha(ds,dz),$$

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where $v_1, v_2, \cdots, v_m$ are C^∞-vector fields, $\tilde{N}_\alpha$ is a compensated Poisson point process, $W_s = (W_s^1, W_s^2, \cdots, W_s^m)$ is a Brownian motion and $\mathcal{L}$ is a generator of semigroup of probability.

In this paper, we will think the same problems for another SDE of the form;

$$
\begin{aligned}
\xi_t(x) &= x + \sum_{j=1}^m \int_0^t v_j(\xi_s(x))dW_s^j + \int_0^t \mathcal{L}(\xi_{s-}(x))ds \\
&+ \int_0^t \int_{E_\alpha} [exp(\sum_{j=1}^m z^j \bar{v}_j)(\xi_{s-}(x)) - \xi_{s-}(x)]\tilde{N}_\alpha(ds, dz),
\end{aligned} \tag{1}
$$

where $v_1, v_2, \cdots, v_m, \bar{v}_1, \bar{v}_2, \cdots, \bar{v}_m$ are C^∞-vector fields, $\tilde{N}_\alpha$ is a compensated Poisson point process, $W_s = (W_s^1, W_s^2, \cdots, W_s^m)$ is a Brownian motion and $\mathcal{L}$ is a generator of the form;

$$
\mathcal{L}(x) = \mathcal{A}(x) + \int_{E_\alpha} [exp(\sum_{j=1}^m z^j \bar{v}_j)(x) - x - \sum_{j=1}^m z^j \bar{v}_j(x)]G_\alpha(dz),
$$

and

$$
\mathcal{A}(x) = (1/2) \sum_{j=1}^m v_j^2(x) + v_0(x).
$$

Let $\bar{\mathbb{B}}(x)$ and $\bar{\mathbb{C}}_\alpha(x, z)$ be the matrices defined by the coefficients of noise part and jum-part, respectively. We put as

$$
\bar{\mathbf{c}}_\alpha(x, z) = exp(\sum_{j=1}^m z^j \bar{v}_j)(x) - x,
$$

and

$$
\tilde{\bar{\mathbf{c}}}_\alpha(x, z) = \bar{\mathbf{c}}_\alpha(x, z) + x.
$$

For the function $\tilde{\bar{\mathbf{c}}}_\alpha(x, z)$, if there exist two constants $\zeta, \theta > 0$ such that

$$
|\tilde{\bar{\mathbf{c}}}_\alpha(x, z)| \leq \zeta(1 + |x|^\theta),
$$

for all $x \in \mathbb{R}^d$ and $z \in E_\alpha$, and there exists a Borel set $\Gamma_\alpha \subset \mathbb{R}^d \times E_\alpha$ such that for any $y \in \mathbb{R}^d$ and for the x-section $\Gamma_{\alpha,x} \subset \Gamma_\alpha$,

$$
(\cup_{z \in \Gamma_{\alpha,x}} \{y | \bar{\mathbb{C}}_\alpha y = 0\}) \cap \{y | \mathbb{B} y = 0\} = \{0\},
$$

then the solution $\xi_t(x)$ of SDE (1) has a density $y \to p_t(x, y)$ for all $x \in \mathbb{R}^d$ and $t \in (0, T]$.

Furthermore, for $x, y \in \mathbb{R}^d$, there exist two constants $\delta \geq 0, \epsilon > 0$ and two functions $f_\alpha(z)$ and $\rho_\alpha(z)$ which are defined by some conditions with a constant γ such that

$$y^t \bar{\mathbb{C}}_\alpha(x, z) y \rho_\alpha(z) \geq f_\alpha(z) \frac{|y|^2 \epsilon}{1 + |x|^\delta}$$

for all $z \in E_\alpha$, then the solution $\xi_t(x)$ of SDE (1) has a smooth density $y \to p_t(x, y)$.

Therefore, it is also good if we look this paper as a kind of generalization and continuation of [5]. Further, we would like say that, for simplicity, all of the terminologies and the notations of this paper are same as [5] also.

1. Canonical stochastic differential equations

Let us think a canonical SDE;

$$\text{(I-1)} \qquad d\xi_t(x) = X(\xi_t(x), \diamond dt)$$

driven by the vector fields valued semimartingale of the form;

(I-2)

$$X_t(x) = \sum_{j=1}^m v_j(x) W_t^j + v_0(x) t + \int_{E_\alpha} (\sum_{j=1}^m z^j \bar{v}_j)(x) \tilde{N}_\alpha((0, t], dz),$$

where W_t is a Brownian motion, and $v_0, v_1, \cdots, v_m, \bar{v}_1, \bar{v}_2, \cdots, \bar{v}_m$ are all smooth complete vector fields on $\mathbb{R}^d$. Then, by the solution of (I-1), we will think the process $\{\xi_t, t \geq 0\}$ satisfying

(I-3)

$$\begin{aligned} \xi_t(x) = x &+ \sum_{j=1}^m \int_0^t v_j(\xi_s(x)) dW_s^j + \int_0^t \mathcal{L}(\xi_{s-}(x)) ds \\ &+ \int_0^t \int_{E_\alpha} [exp(\sum_{j=1}^m z^j \bar{v}_j)(\xi_{s-}(x)) - \xi_{s-}(x)] \tilde{N}_\alpha(ds, dz), \end{aligned}$$

where

$$\mathcal{L}(x) = \mathcal{A}(x) + \int_{E_\alpha} [exp(\sum_{j=1}^{m} z^j \bar{v}_j)(x) - x - \sum_{j=1}^{m} z^j \bar{v}_j(x)] G_\alpha(dz),$$
$$\mathcal{A}(x) = (1/2) \sum_{j=1}^{m} v_j^2(x) + v_0(x),$$

and $G_\alpha(\cdot)$ is a Lebesgue measure. From the equation (I-3), we put as

$$\bar{\mathbf{c}}_\alpha(x, z) = exp(\sum_{j=1}^{m} z^j \bar{v}_j)(x) - x,$$

and

$$\tilde{\bar{\mathbf{c}}}_\alpha(x, z) = \bar{\mathbf{c}}_\alpha(x, z) + x.$$

Then, form the Proposition III-1 of [5], $D_x \tilde{\bar{\mathbf{c}}}_\alpha(x, z)$ is also invertible.

If we put

$$(a^{ik}(x))_{d\times d} = \sigma_{d\times m}(x)(\sigma_{d\times m}(x))^t,$$

and

$$(\bar{a}^{ik}(x))_{d\times d} = \bar{\sigma}_{d\times m}(x)(\bar{\sigma}_{d\times m}(x))^t,$$

where

$$\sigma_{d\times m}(x) = \begin{pmatrix} v_1^1(x) & v_2^1(x) & \cdots & v_m^1(x) \\ v_1^2(x) & v_2^2(x) & \cdots & v_m^2(x) \\ \cdots & & & \\ v_1^d(x) & v_2^d(x) & \cdots & v_m^d(x) \end{pmatrix}_{d\times m},$$
$$\bar{\sigma}_{d\times m}(x) = \begin{pmatrix} \bar{v}_1^1(x) & \bar{v}_2^1(x) & \cdots & \bar{v}_m^1(x) \\ \bar{v}_1^2(x) & \bar{v}_2^2(x) & \cdots & \bar{v}_m^2(x) \\ \cdots & & & \\ \bar{v}_1^d(x) & \bar{v}_2^d(x) & \cdots & \bar{v}_m^d(x) \end{pmatrix}_{d\times m},$$

and $\bar{v}_j^i(x)$ are the component functions of vector fields $\bar{v}_j$, and we put

$$\mathbb{B}(x) = (a^{ik}(x))_{d\times d},$$

and

$$\bar{\mathbb{B}}(x) = (\bar{a}^{ik}(x))_{d\times d},$$

then by the similar calculation with (III-7) and $\mathbb{B}(\tilde{\mathbf{c}}_\alpha(x,z))$ of [5], we see that

$$\bar{\mathbb{B}}(\tilde{\bar{\mathbf{c}}}_\alpha(x,z)) = (D_z\bar{\mathbf{c}}_\alpha(x,z))(D_z\bar{\mathbf{c}}_\alpha(x,z))^t.$$

We put as following;

$$\text{(I-4)} \qquad \bar{\mathbb{C}}_\alpha(x,z) = (D_x\tilde{\bar{\mathbf{c}}}_\alpha)^{-1}\bar{\mathbb{B}}(\tilde{\bar{\mathbf{c}}}_\alpha)[(D_x\tilde{\bar{\mathbf{c}}}_\alpha{}^{-1}]^t,$$

and make two assumptions;

Assumption ($\bar{\mathbb{A}}$). There exists two constants $\zeta, \theta > 0$ such that

$$|\tilde{\bar{\mathbf{c}}}_\alpha(x,z)| \le \zeta(1+|x|^\theta)$$

for all $x \in \mathbb{R}^d$ and $z \in E_\alpha$.

Assumption ($\bar{\mathbb{B}}$). There is a Borel subset $\Gamma_\alpha = \{(x,z)\} \subset \mathbb{R}^d \times E_\alpha$ such that for all $x \in \mathbb{R}^d$ and for the x-section $\Gamma_{\alpha,x} \subset \Gamma_\alpha$, if $G_\alpha(\Gamma_{\alpha,x}) = \infty$,

$$(\cup_{z\in\Gamma_{\alpha,x}}\{y|\bar{\mathbb{C}}_\alpha(x,z)y=0\}) \cap \{y|\mathbb{B}(x)y=0\} = \{0\},$$

if $G_\alpha(\Gamma_{\alpha,x}) < \infty$,

$$\mathbb{R}^d \cap \{y|\mathbb{B}(x)y=0\} = \{0\}.$$

Then we get the following result.

Theorem I-1. *Under ($\bar{\mathbb{A}}$) and ($\bar{\mathbb{B}}$), the solution $\xi_t(x)$ of (I-3) has a density $y \to p_t(x,y)$ for all $x \in \mathbb{R}^d$ and $t \in (0,T]$.*

Proof. Since $v_0, v_1, \cdots, v_m, \bar{v}_1, \bar{v}_2, \cdots, \bar{v}_m$ are all smooth complete vector fields on $\mathbb{R}^d$, the coefficients of the second part and the third part of the right hand side of (I-3) are r-times differentiable with bounded derivatives.

By the same method with the proof of Theorem III-1 of [5], we can choose $\zeta', \theta' > 0$ such that

$$|D^k_{z^k}\bar{\mathbf{c}}_\alpha(x,z)| \le \zeta'(1+|x|^{\theta'}) \text{ for } k \ge 1.$$

Further, we can choose a function $\eta \in \cap_{2\le p<\infty} L^p(E_\alpha, G_\alpha)$ such that

$$|D^n_{x^n}\bar{\mathbf{c}}_\alpha(x,z)| \le \zeta|\eta(z)|(1+|x|^\theta), n \ge 1,$$

for $\zeta, \theta > 0$. Thus we can get,

$$|D^{n+k}_{x^n z^k} \bar{\mathbf{c}}_\alpha(x,z)| \le \zeta(1+|x|^\theta), n+k \ge 1, k \ge 1.$$

Therefore, from this, the condition (ii) of Assumption (A-r) of [5] are satisfied. Also, from the putting (I-4), we see that $(\bar{\mathbb{B}})$ imply

$$\cap_\alpha(\cup_{z\in\Gamma_{\alpha,x}}\{y|\bar{\mathbb{C}}_\alpha(x,z)y=0\}) \cap \{y|\mathbb{B}(x)y=0\} = \{0\},$$

which is the main part of Assumption (B) of [5]. □ □

This Theorem also has some subcases. We will explain these by using the following Corollaries.

Corollary I-1. *(c.f.[4]) If $Rank\mathbb{B}(x) = d$, or $Rank\bar{\mathbb{B}}(x) = d$, then the solution $\xi_t(x)$ of (I-3) has a density $y \to p_t(x,y)$.*

Proof. If $Rank\bar{\mathbb{B}}(x) = d$, then from the putting $\bar{\mathbb{C}}_\alpha(x,z)$ as (I-4), we see that

$$\cup_{z\in\Gamma_{\alpha,x}}\{y|\bar{\mathbb{C}}_\alpha(x,z)y=0\} = \{0\},$$

because $D_x\tilde{\bar{\mathbf{c}}}_\alpha(x,z)$ is also invertible. Therefore, the condition of $(\bar{\mathbb{B}})$ is satisfied. If $Rank\mathbb{B}(x) = d$, then

$$\{y|\mathbb{B}(x)y=0\} = \{0\}.$$

Thus, $(\bar{\mathbb{B}})$ is satisfied also. □ □

Corollary I-2. *If $Rank\mathbb{B}(x) = 0(or, Rank\bar{\mathbb{B}}(x) = 0)$, then to get the existence of density of $\xi_t(x)$ of (I-3), it must be held that $Rank\bar{\mathbb{B}}(x) = d(or Rank\mathbb{B}(x) = d)$, respectively.*

Proof. If $Rank\mathbb{B}(x) = 0$, then we get that for any $y \in \mathbb{R}^d$,

$$\{y|\mathbb{B}(x)y=0\} = \mathbb{R}^d.$$

Therefore, to satisfy the condition of $(\bar{\mathbb{B}})$, it is needed that

$$\{y|\bar{\mathbb{C}}_\alpha(x,z)y=0\} = \{0\},$$

for some $z \in \Gamma_{\alpha,x}$, which is equivalent to $Rank\bar{\mathbb{C}}_\alpha(x,z) = d$. But, since $D_x\tilde{\bar{\mathbf{c}}}_\alpha(x,z)$ is invertible, it is equivalent to $Rank\bar{\mathbb{B}}(x) = d$.

If $Rank\bar{\mathbb{B}}(x) = 0$, we can prove by the similar method. □ □

COROLLARY I-3. *If* $0 < Rank\mathbb{B}(x)($ *or* $Rank\bar{\mathbb{B}}(x)) = k < d$, *then to get the existence of density of* $\xi_t(x)$ *of (I-3), it is needed at least that* $0 < Rank\bar{\mathbb{B}}(x)($ *or* $Rank\mathbb{B}(x)) = l < d$ *and* $k + l = d$, *respectively.*

Proof. Let $0 < Rank\mathbb{B}(x) = k < d$. Then we get

$$\{y|\mathbb{B}(x)y = 0\} = \{\mathbf{y} = (y_1, y_2, \cdots, y_d) \in \mathbb{R}^d | d-k \text{ numbers of } y_i \text{ are } 0\}.$$

Therefore, to satisfy the condition of $(\bar{\mathbb{B}})$, it must be held;

(i) $0 < Rank\bar{\mathbb{C}}_\alpha(x, z) = l < d$ and $l + k = d$, because of

$$Rank\bar{\mathbb{C}}_\alpha(x, z) = Rank(\bar{\mathbb{B}}(x)),$$

(ii) the set $\{\mathbf{y}' = (y'_1, y'_2, \cdots, y'_d) \in \mathbb{R}^d\}$ such that

$$\{y|\bar{\mathbb{C}}_\alpha(x, z)y = 0\}$$
$$= \{\mathbf{y}' = (y'_1, y'_2, \cdots, y'_d) \in \mathbb{R}^d | d - l \text{ numbers of } y'_i \text{ are } 0\}$$

is disjoint with $\{\mathbf{y} = (y_1, y_2, \cdots, y_d) \in \mathbb{R}^d\} \setminus \{0\}$, i.e.,

$$\{\mathbf{y} = (y_1, y_2, \cdots, y_d) | \mathbf{y} \in \mathbb{R}^d\} \cap \{\mathbf{y}' | \mathbf{y}' \in \mathbb{R}^d\} = \{0\}.$$

But, from the condition of this Corollary, we see that (i) and (ii) are satisfied. Therefore, we get the result.

If $0 < Rank\bar{\mathbb{B}}(x) = k < d$, then we can prove similarly. □ □

EXAMPLE. In $\mathbb{R}^2 = \{\mathbf{x} = (x^1, x^2)\}$, let us think the vector fields valued Lèvy process of the type;

$$X_t(\mathbf{x}) = v_1(\mathbf{x})W_t + \int_{E_\alpha} (zv_2)(\mathbf{x})\tilde{N}_\alpha((0, t], dz),$$

where W_t is an 1-dimensional Brownian motion. Then by the solution of a canonical SDE;

$$d\xi_t(\mathbf{x}) = X(\xi_t(\mathbf{x}), \diamond dt),$$

we get the process $\xi_t(\mathbf{x})$ satisfying;

$$\text{(I-5)}\qquad \begin{aligned}\xi_t(\mathbf{x}) &= \mathbf{x} + \int_0^t v_1(\xi_s(\mathbf{x}))dW_t + \int_0^t \mathcal{L}(\xi_{s-}(\mathbf{x}))ds \\ &+ \int_0^t \int_{E_\alpha} [exp(zv_2)(\xi_{s-}(\mathbf{x})) - \xi_{s-}(\mathbf{x})]\tilde{N}_\alpha(ds, dz),\end{aligned}$$

where

$$\mathcal{L}(\mathbf{x}) = \mathcal{A}(\mathbf{x}) + \int_0^t \int_{E_\alpha} [exp(zv_2)(\mathbf{x}) - \mathbf{x} - zv_2(\mathbf{x})]G_\alpha(dz),$$

and

$$\mathcal{A}(\mathbf{x}) = (1/2)v_1^2(\mathbf{x}).$$

If we represent SDE (I-5) in the component form, we get;

$$\xi_t^1(\mathbf{x}) = x^1 + \int_0^t v_1(\xi_s(\mathbf{x}))dW_s + \int_0^t \mathcal{L}^{(1)}(\xi_{s-}(\mathbf{x}))ds,$$

and

$$\begin{aligned}\xi_t^2(\mathbf{x}) &= x^2 + \int_0^t \mathcal{L}^{(2)}(\xi_{s-}(\mathbf{x}))ds \\ &+ \int_0^t \int_{E_\alpha} [exp(zv_2)(\xi_{s-}(\mathbf{x})) - \xi_{s-}^2(\mathbf{x})]\tilde{N}_\alpha(ds, dz),\end{aligned}$$

where

$$\mathcal{L}^{(1)}(\mathbf{x}) = \mathcal{A}(\mathbf{x}),$$

and

$$\mathcal{L}^{(2)}(\mathbf{x}) = \int_{E_\alpha} [exp(zv_2)(\mathbf{x}) - x^2 - (zv_2)(\mathbf{x})]G_\alpha(dz).$$

(i) If $Rank\bar{\mathbb{B}}(x) = 2$, then for any $y \in \mathbb{R}^2$,

$$\{y|\bar{\mathbb{C}}_\alpha(x, z)y = 0\} = \{0\},$$

for some $z \in \Gamma_{\alpha,x}$. Therefore, from Theorem I-1, the solution $\xi_t(\mathbf{x})$ of (I-5) has a density. If $Rank\mathbb{R}(x) = 2$, then

$$\{y|\mathbb{B}(x)y = 0\} = \{0\}.$$

Thus it is held also.

(ii) If $Rank\bar{\mathbb{B}}(x) = 0$, then we get

$$\{y|\bar{\mathbb{C}}_\alpha(x,z)y = 0\} = \mathbb{R}^2$$

for any $z \in \Gamma_{\alpha,x}$. Therefore, to satisfy the conditions of $(\bar{\mathbb{B}})$, we need that

$$\{y|\mathbb{B}(x)y = 0\} = \{0\},$$

which is equivalent to $Rank\mathbb{B}(x) = d$. If $Rank\mathbb{B}(x) = 0$, then we get also;

$$\{y|\mathbb{B}(x)y = 0\} = \mathbb{R}^d.$$

Therefore, we need that

$$\{y|\bar{\mathbb{C}}_\alpha(x,z)y = 0\} = \{0\}$$

for all $z \in \Gamma_{\alpha,x}$ which is equivalent to $Rank\bar{\mathbb{B}}(x) = d$.

(iii) Let $0 < Rank\mathbb{B}(x) = k < d$. Then, because of $d = 2$ and $k = 1$, if $Rank\mathbb{B}(x) = 1$, then

$$\{y|\mathbb{B}(x)y = 0\} = \{(0,y_2)\} \text{ or } \{(y_1,0)\}.$$

Therefore, if

$$\{y|\bar{\mathbb{C}}_\alpha(x,z)y = 0\} = \{(y_1,0)\} \text{ or } \{(0,y_2)\}$$

for all $z \in \Gamma_{\alpha,z}$, respectively, then we get

$$\{y|\mathbb{B}(x)y = 0\} \cap \{y|\bar{\mathbb{C}}_\alpha(x,z)y = 0\} = \{0\}.$$

Therefore, the conditions of $(\bar{\mathbb{B}})$ are satisfied.

Theorem I-2. *Assume* ($\bar{\mathbb{A}}$). *Further, for* $x, y \in \mathbb{R}^d$, *if there exist* $\epsilon_1 > 0$ *and* $\delta_1 \geq 0$ *such that*

$$y^t \mathbb{B}(x) y \geq |y|^2 \epsilon_1 \geq \frac{\epsilon_1 |y|^2}{1 + |x|^{\delta_1}}, \tag{I-6}$$

and for $x, y \in \mathbb{R}^d$ *and* $f_\alpha(z) \in L^1(E_\alpha, G_\alpha)$ *which is satisfying;*

$$\int_0^\infty s^{\zeta - 1} exp[-\theta \int_{E_\alpha} (1 - e^{-sf(z)}) dz] ds < \infty,$$

where ζ *and* θ *are two positive numbers, and if there exist* $\epsilon_2 > 0, \delta_2 \geq 0$ *and a function* $\rho_\alpha(z) : E_\alpha \to [0, \infty)$ *having the following properties;*

(i) $\rho \in C_b^\infty$,
(ii) $\rho_\alpha(z) \to 0$ *as* $z \to \partial(E_\alpha)$ *(boundary of* E_α*),*
(iii) $|D_{z^r}^r \rho_\alpha| \in L^1(E_\alpha, G_\alpha)$ *for all* $r \in \mathbb{N}$ *such that*

$$y^t \bar{\mathbb{C}}_\alpha(x, z) y \rho_\alpha(z) \geq f_\alpha(z) \frac{|y|^2 \epsilon_2}{1 + |x|^{\delta_2}} \tag{I-7}$$

for all $z \in E_\alpha$,

then we get that $\xi_t(x)$ *of (I-3) has a smooth* (C^∞) *density* $y \to p_t(x, y)$.

Proof. From ($\bar{\mathbb{A}}$), we see that $\bar{\mathbb{B}}(x)$ and $\bar{\mathbb{C}}_\alpha(x, z)$ are $d \times d$ - symmetric and nonnegative matrices, and that $D_x \bar{\mathbf{c}}_\alpha(x, z)$ is also invertible. Thus there exist a constant $\zeta > 0$ such that

$$|det(I + D_x \bar{\mathbf{c}}_\alpha(x, z))| \geq \zeta,$$

identically, and from (I-6) and (I-7) we can choose $\epsilon > 0, \delta \geq 0$ and $\rho_\alpha(z)$ satisfying

$$y^t \mathbb{B}(x) y + inf_z \frac{\rho_\alpha(z)}{f_\alpha(z)} y^t \bar{\mathbb{C}}_\alpha(x, z) y \geq \frac{|y|^2 \epsilon}{1 + |x|^\delta},$$

for all $z \in E_\alpha$, which is the main inequality implying the regularity.□□

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