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Improving Thinking in Children with Low Mathematics Achievement[1]

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Many primary school children struggle with mathematics and have low self-esteem in their own abilities. They know that the subject is important but they cannot cope, get left behind in their work and begin to hate mathematics. This paper reports the efforts to encourage and help a group of seventeen low achievers in mathematics prepare for their "primary six" public examination. The children were lacking in many thinking skills, but with encouragement, guidance and practice, thirteen of them (76.5%) showed improvements in their mathematical thinking and passed this important examination. This paper discusses these children's thinking in mathematics and how improvements were made.

INTRODUCTION

Often mathematics teaching was undiscerning and non-differentiating teacher-centered methods that are incapable of motivating or effecting learning among low-achievers. Yet mathematics is a compulsory subject in many public examinations. These children who are weak in mathematics need nurturing and assistance to pass this 'killer' subject that they both fear and hate.

Five weeks before the "primary six" public examination, a group of seventeen low achievers in mathematics from three classes were identified by their teachers as needing assistance in their preparations. The children were given remedial instruction in selected basic topics by the first author in separate groups of eleven and six. These small classes were conducted once a week for five weeks. The bigger group was allotted one and a half hours while the smaller group had an hour each week. The children were found to be lacking in many thinking skills and were given guidance and practice in developing the necessary skills.

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During the lessons, assessment was made of the children's ideas about important basic skills and concepts in mathematics — the meanings they hold and what they have managed to learn and not been able to understand and learn. We need to continually seek to understand what a student can do and how the student is able to do it. The teacher can then use this information to guide instruction.

This study sought to elicit information that reveals pupil cognition and metacognition. Questions were asked that focus on pupil's solution strategy rather than on the answer. Questions asked that can reveal how pupils think about problems are:

> 'How did you solve this problem?'
> 'Did anyone else use the same method?'
> 'Did anyone use a different method?'
> 'Can anyone think of another way?'
> 'Susilawaty, what do you think about Saiful's working?

Evidence of pupil thinking was also seen from their written work and from manipulation of objects that accompanies their solutions. When the evidence is inadequate, probing questions were asked. Chambers (1993), and Thompson and Briars (1993) reported that children solve problems in well-identified ways. We can learn to recognize how they solve problems, and learn to use that knowledge to make good instructional decisions.

Nagasaki and Becker (1993) reported that many Japanese teachers strive for classroom teaching that draws on students' different ways of thinking in order to raise the level of the mass understanding as a whole. The focus is on how students think in their individual natural ways instead of making them think in the way that we do which often times they cannot make sense of.

Six Aspects of Thinking Skills

Thinking has diverse meanings for different people. Do we want children to come out with creative new ideas, focus on a task, make connections with past knowledge, explore consequences, or simply guess what is in our head? According to Costa (1990) students should become more aware of their own thought processes, better able to use different strategies in different classroom situations to solve problems, and be able to apply such strategies to a broad range of problems that they will encounter in real life.

The processes of thinking, decision making, and conceptualization should be described and graphically represented by teachers to show what goes on in our heads when we think. Throughout the remedial lessons, the children were assisted in understanding, monitoring and taking charge of their own thinking abilities. Such transparent mental interactions proved to have an effect on their mental processes even within a relatively short period of time. They were able to help each other, acquire a wider range of problem-solving processes and gain a sense of efficacy, empowerment, and command of their own thought processes.

The six aspects of thinking skills advocated by Clarke (1990) were used in this experiment to improve the children's thinking in mathematics. According to Clarke, the patterns of thinking used by the mind to manage information keep the same basic structure although the content grows from simple to complex forms. The children were challenged with cognitive tasks in which they could experience success. When the task gets too complex, such as, when the area of a square is given as $25cm^2$, and they have to find the perimeter, they just tune out, become dis-interested and inattentive. However, if cognitive tasks are too easy, they also lose interest. It is important to find their level of capability and to make the lesson challenging with opportunities for success.

The six aspects of thinking skills that were developed in the children were:

Scanning and Focusing
Using Categories
Proposing Relationships
Using Concept Network
Predicting Effects
Planning Procedures

Areas of Difficulty Revisited

Number Operations

All the children could add and subtract even four digit numbers including decimals. However some of them relied on counting on their fingers and made careless mistakes. They were also able to multiply together single small digit numbers, but faced difficulties with bigger digits; and they were careless, especially when trying to work out the sums mentally or with their fingers.

The children were given practice **in using their concept networks** of addition, subtraction and multiplication, and the inter connections among them. They could then check their addition with subtraction and vice versa. They were encouraged not to use their fingers but to use the work space on the question paper to work out operations on bigger digits. Instead of trying to remember their basic multiplication facts, they were encouraged to work out and write down the unfamiliar ones in the work space to avoid mistakes. These procedure were readily picked up by the children and in subsequent lessons they were seen to be using them on their own.

Multiplication with Decimals

The children could not do multiplication with decimals. They lined up the decimal point as in addition and subtraction, not counting the number of decimal places. This new relationship had to be revisited and had to be inferred by the children from **predicting effects** of multiplying or estimating with whole numbers. Subsequently, the children were encouraged to always check their decimal multiplications by estimation. However, many of them were still unable to predict such estimations when it was taken up again the following week.

They probably need more time. As suggested by Vygotsky (1986) a cycle in which induction and deduction support each other provides a simple framework for teaching thinking in the content areas. He explained that we have to discover some ideas inductively, whereas other ideas must occur first in our minds and then slowly gather specific examples.

Division

The next topic examined, division, was a difficult one for many children. Test items include division of 1, 2, 3 and 4-digit numbers by a 1-digit number without remainder. Problems of omiting ‘0’ in the quotient and of ‘further dividing’ were addressed by peer tutelage and example, making their own sums, and checking by multiplying the quotient with the divisor.

Examples of these very common errors are:

```
        32                  15051
   5 ) 1510            4 ) 6024
       15                  4
       ----                ----
         10                20
         10                20
       ----                ----
          0                  24
                             20
                             ---
                              4
                              4
                             ---
                              0
```

The skill of **predicting effects**, estimating and checking the ‘reasonableness’ of the answer was practised. They would then be reminded to check if a zero was missing. In the case of dividing 24 by 4 first and then 1, they were further encouraged to write down the needed multiplication table in the work space.

The children were encouraged by their own successes and were given take home assignments which they could check with family members or friends. They were attentive and motivated by these cognitive tasks which are all included in the public examination and appropriate for their level of ability. In their usual classes, remedial work would involve more difficult sums catering to all students.

As recommended by Dolan (1993) connections were made between mathematics and language. The children made up situations that go with the division problems to link mathematical notations to their concept of mathematics as applied in the real world. Also, answers were given and they had to supply the question.

This allowed them to explain many different situations and problems. It cannot be over-emphasised that students must be able to link conceptual and procedural knowledge; recognise relationships among different topics in mathematics; see mathematics as an integrated whole; and apply mathematical thinking and modeling to solve problems in the subject area as well as in other disciplines. The skills of proposing relationships and using concept network were reinforced.

Measurement (Conversion of *kg-g*, ℓ -mℓ, *km-m-cm*, hr-min-sec involving + and – operations)

The children did not know by hand

$$35 + \_\_ = 100 \quad \text{or} \quad 350 + \_\_\_ = 1000.$$

Each time they had to was a standard algorithm. Some did not know

$$\text{one dollar} = 100 \text{ cents.}$$

They were given practice in using categories, such as number pairs that make five, ten, twenty, fifty, hundred, thousand, and so on. They need to develop categories and concepts so that their minds can work more efficiently.

All the children were able to perform correctly the addition operation on certain measurement units mechanically without even knowing the equivalent units such as that 1 *km* = 1000 *m*. The standard algorithm works for such units as seen in the example.

	kilometre	metre	Remarks:
	12	400	Operation performed
	23	500	mechanically from right to left.
+	32	600	Carrying over from metre to km
			as a ten.
	68	500	

As suspected, the children did not know equivalent units and had difficulty with word problems and conversion of hr-min-sec using base 60 as seen in the example.

hour	minute	second	Remarks:
2	50	40	All 17 pupils using
3	50	40	standard algorithm from the right.
			Pupils shown how to subtract 60
6	00	80	to make minute and hour.

The thinking skills **of using categories** (e. g., that time is different from the other units); **proposing relationships** of different measurement units, and **using a concept network** of what they know were practised. Yet, the pupils knew that an hour is 60 minutes and that a minute is 60 seconds.

We need to develop in children mathematics reasoning, involving them in activities that call on them to reason and communicate their reasoning rather than to reproduce memorised procedures and rules (Garofalo & Mtetwa 1993). This was achieved by asking them the meaning of 80 seconds. Clarke (1990) stressed that experts, as compared with novices, see problems in terms of organising patterns. Remarkable feats of memory and skill in problem solving may rely in part on our ability to ‘chunk’ information and to use a perceptual pattern or general schema to hold the information. These low achievers, more than anything, need assistance in seeing patterns and bigger ‘chunks’ of information. They showed they were able to do it if given the right guidance.

Algebra

Many were weak in algebra, unable to answer questions like 2r + r – r = _____. The skill of **proposing relationships** was taught so that the pupils could see that r is the same as 1r. This worked well for the children because they could then relate to whole number operations. They were then given practice at drawing pictures of different objects and labelling their algebraic representations.

Time (*24 hour clock*)

A few of the children were not able to relate time to the 24-hour clock. They needed to develop and **use this new category** of telling time and to relate this to their previous knowledge of telling time, whether a.m. or p.m. and **proposing relationships**.

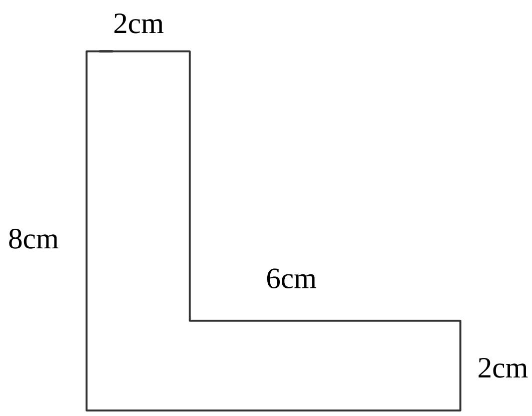
This 'filling-in-the-table' activity showing 1 a.m. to 12 a.m. and their corresponding 0100 to 2400 names proved interesting and useful. They were then successfully taken to a higher level with more difficult times such as 1525 and 1850.

1 am	0100
2 am	
3 am	
. . .	
	2300
12 am	

0130	
	0245
	0325
	. . .
	2315
0015	

Perimeter

The children knew how to work out perimeter and area problems if all the sides are given. In this example, they only add up the given sides.



Why are children careless with perimeter if not all the sides of the shape are labelled? **Scanning, focusing, designing** procedures were taught and practised. These helped the children to focus and concentrate on the concept and procedures needed to resolve the cognitive task at which they succeeded after a short session.

The reteaching of simple cognitive tasks commonly asked in the public examination, emphasising development of their own thinking skills, raised the performance of the group of 17 children. Thirteen of them (76.5%) showed improvements in their mathematical thinking and passed the important terminal examination for primary schools for promotion to the secondary level.

Conclusion

Many children process their reality in ways that may seem to be awkward and inefficient. By observing and interacting with them, we can better appreciate, mediate and enhance the productiveness of their thinking because they have shown to some extent their thoughts. Through exchanges of thought processes especially with the teacher, they can emulate how we think and solve problems. Such cognitive-metacognitive strategies can be learned and developed as seen by the improvements shown by the children in this study.

The thinking we seek to teach is nonalgorithmic; complex; yields multiple solutions; involves uncertainty, conflicting criteria, and self-regulation of the thinking process, not regulation by others. This small-scale study shows that even low-achievers in mathematics can taste success, which builds self-confidence, when we teach them how to think for themselves.

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