

Children's Representations of Numbers¹

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We discuss some aspects of mathematics for teachers such as algebra for teachers, geometry for teachers, statistics for teachers, etc., which can be taught in teacher preparation courses. Mathematics for teachers should consider the followings:

- (a) Various solutions for a problem,
- (b) The dynamics of a problem introduced by change of condition,
- (c) Relationship of mathematics to real life,
- (d) Mathematics history and historical issues,
- (e) The difference between pure mathematics and pedagogical mathematics,
- (f) Understanding of the theoretical backgrounds, and
- (g) Understanding advanced mathematics.

1. INTRODUCTION

When we discuss about mathematics education, we assume that the mathematics teachers know the concepts and principles of mathematics well enough to teach. However, this assumption is not correct as far as the Korean situation goes.

Let us illustrate it according to what the author heard from various sources. The first case was brought to the author's attention at a committee meeting of mathematics teachers in Seoul several years ago. One member of the committee told us the following experience.

He was a teacher of the eighth grade (the second year in the junior high school). In the 8th grade mathematics textbook, the following table is presented.

X	x_1	x_2	\dots	x_k	\dots	x_n	Total
f	f_1	f_2	\dots	f_k	\dots	f_n	N

To find the mode of central tendency in the above frequency table, let

¹ Paper presented at TG 2: Mathematics Curriculum in Teacher Preparation, ICMI-China Regional Conference on Mathematical Education, Shanghai, China, August 16-20, 1994.

$$f_k = \text{Max} (f_1, f_2, \dots, f_n).$$

Then, the class mark x_k , corresponding to the frequency f_k , is the mode. That is,

$$M_O = x_k$$

The teacher presented the lesson in the same way as the textbook. Then he gave the following table for an exercise and asked students to find the mode.

X	40	50	60	70	80	90	Total
f	1	4	6	9	7	3	30

To find the mode, one particular student used a formula, which he may have learned from a private tutor or from an advanced mathematics textbook.

He solved the problem in the following manner:

$$M_O = 60 + \frac{7}{6+7} \times 10 = 70.4$$

The teacher did not know that there exists such a formula, and he confessed that he did not understand the formula.

Another case has happened in a model presentation for the mathematics classroom in a secondary school in a rural area.

In the 9th grade class, the teacher presented a lesson about *proposition*. Using the Venn diagram, he explained that when a proposition is true, the contraposition is also true, i.e.,

$$\text{If is } p \rightarrow q \text{ true, then } \bar{q} \rightarrow \bar{p} \text{ is true.}$$

Then a student raised his hand and gave a counterexample. He said that even if a proposition may be true, the contraposition may not always be true. As an illustration, he gave an example:

“If we do not use medicine, then we cannot heal the sickness.”

The proposition may be true, but the contraposition

“If we heal the sickness, then we use medicine”

is not true. Of course, the teacher was perplexed and did not know what to do.

As we illustrated in the above two cases, the mathematics teacher must understand completely the contents of the subject that he or she is teaching. After that the teacher can consider the methodology of presentation.

In this paper, we will discuss what kind of mathematics that the teacher must

comprehend. In other words, we will talk about the teachers' mathematics.

2. DISCUSSION

The mathematics curriculum of one country may differ from another. Therefore, we cannot present the teachers' mathematics applicable to all countries, but the basic concepts are very similar.

In this discussion, we are going to use examples of junior high school mathematics.

1. Various solutions for a problem

The individual students are all different. Therefore, one method of teaching may not be so effective for all students, and one kind of solutions may not be understood by every one. Hence, the teacher has to know various methods of solutions for a problem.

Example 1: Many ways to get the total degrees of the interior angles of a triangle.

Example 2: Many ways to get the quadratic formula.

2. The dynamics of a problem by the change of the condition

When the variable or condition varies, the result also varies. The students must be trained to respond to the dynamic situation by changing the variables and the conditions. The teacher must be prepared for the dynamic aspects of the problem.

Example 1: Fuzzy set

Example 2: Intuitive probability

3. Relationship of mathematics in real life

We usually say that mathematics is the foundation of science and of other branches of learning. However, if we do not provide concrete and detailed applications, the students do not understand the relationships between them. Therefore, the mathematics teacher must understand many examples of real life and present them in meaningful ways, so that the students may understand the relationship of the mathematical problems to science and other branches of learning.

Example 1: Verification of horizontal orientation and the properties of isosceles triangles.

Example 2: Proportional representation system and characteristics of the equilateral triangle.

4. Mathematics history and historical issues

There are many things that the teacher should know about the historical issues of mathematics.

Example 1: The trial and error of d' Alembert's calculation of probability.

Example 2: The history of the symbol of root ($\sqrt{\quad}$).

5. The difference between pure mathematics and pedagogical mathematics

There are some differences between pure mathematics and pedagogical mathematics in schools. Pedagogical mathematics is a simplified version of pure mathematics and the teacher should understand the differences.

Example 1: The difference between the mode and the crude mode.

Example 2: The definition of similar terms.

6. Understanding of the theoretical background

In the textbook, only the results of some theory may be presented. However, the teacher should understand the proofs and background of the theory in depth.

Example 1: The algorithm of extraction of a square root.

Example 2: The arithmetic mean.

7. Understanding the advanced mathematics

The teacher should understand the advanced materials and the new trend of mathematics so that the teacher can lead and direct the students to cope with their further studies and future demands.

Example 1: The solutions of the cubic and the quartic equation.

Example 2: The adjusted mean value.

3. CONCLUSION

So far, we have discussed some aspects of mathematics for teachers. These suggestions should be reflected in pedagogical mathematics, such as

Mathematics for Teachers,
Algebra for Teachers,
Geometry for Teachers,

Statistics for Teachers, etc.,

which can be taught in teacher preparation courses.

The mathematics teachers should have knowledge beyond the above subjects. However, the individual who does not understand these thoroughly, is not qualified to be certificated as a mathematics teacher. The ideal mathematics teacher should have deep understanding of students and the educational process in addition to the listed subjects.

Furthermore, certified mathematics teachers are supposed to have knowledge in depth of a specific field of mathematics, so that they may communicate with mathematicians and continue to expand their knowledge.

REFERENCES

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