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Combinatorial Approach for Solving The Layout Design Problem

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Abstract

자동생산화를 위한 공장설계에 있어서 설비배치에 관한 연구는 제조 유연성 요소의 통합과 이용을 적절하게 수행함으로써 가능하다. 특히 현실적인 설비배치의 특성은 그룹테크놀로지와 물자흐름의 전략을 파악하고 그들의 방법을 조합함으로써 기존의 연구에서 이론적으로 치우치는 경향을 몇 가지의 방법을 통합함으로 실질적인 응용에 그 목적을 두고있다. 본 연구는 그래프이론과 수학적인 모형을 개발하여 통합적인 접근방법을 전개한다. 또한 설비배치 디자인에 대한 평가를 정량적인 방법으로 나타내고 있으며 경영전략에 있어 제조설비능력을 제고하는데 그 응용성을 보여주고 있다. 그것은 자동화 생산환경에 있어 각 시스템의 응용성과 목적관계 그리고 물자흐름관계 등을 정확하게 반영하는데 이바지한다. 현대 제조산업에 있어 고려할 수 있는 모든 각각의 제조요소가 제 특성을 수행하기 위해서는 우선적으로 설비배치 디자인의 중요함을 예를 들어 보여준다.

1. Introduction

A flexible manufacturing system starts from the installation of automated facilities. Concurrent use of manufacturing resources, alternative process plans, flexible routings, and efficient scheduling system can only work together when the facility layout is designed well. The practical layout design requires the integration of not only group technology concepts but also material handling strategies. No matter how well a product is designed, the manufacturing process will not perform well if layout design is not balanced.

The overall objective in plant layout is to choose an arrangement of facilities which minimize incremental costs. The most obvious of these incremental costs is due to material handling and thus most literature considers only total distance traveled. However, defining the

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boundaries on a plant layout design is no easy undertaking. In fact, the boundaries are different from one problem to the next. There is also a tremendous variety in the types of layout problems one may encounter [14].

A graph theoretic formulation opening up new possibilities for improved solution technique was developed [13]. If two facilities are adjacent, their locations are joined by the line which intersects only their common boundary. This results in a drawing of a graph which depicts the adjacency structure of the layout. The aim is to design a system so that the sum of the ratings of adjacent pairs is maximized as this represents travel saved.

The graph theoretic approach seems to be more suitable for the design of a new layout rather than the modification of an existing one [2]. Optimal design of the physical layout is one of the most important issues that must be resolved in the early stages of the system design. Good solutions to these problems provide a necessary foundation for effective utilization of the system. This is particularly true for FMSs [16].

Since most of existing cell formation approaches usually do not consider flow directions and volumes, most layout and handling strategies have been neglected. However, the concurrent design concept has not been applied to the layout design problem. Therefore, this research introduces a concurrent approach for layout which integrates group technology and material handling strategies. A systematic

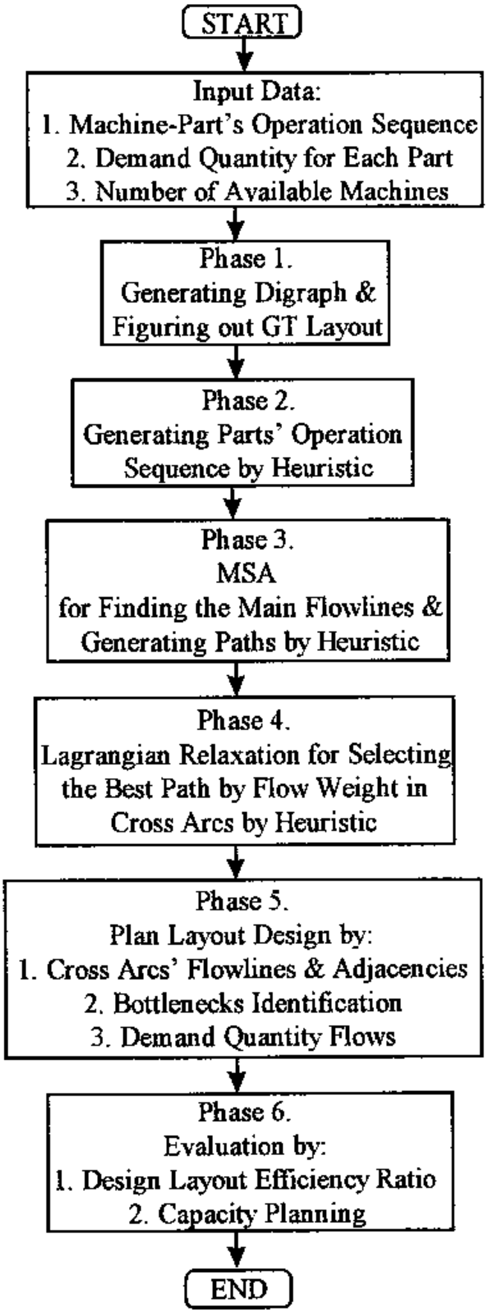

Figure 1. Summary of Solution Procedure Flow

approach solving for the layout design problem is proposed as Figure 1 shows.

2. Methodology

2.1 Phase 1

The group technology layout with input data such as machine-parts operation sequence, demand quantity for each part, and the number of available machines can be drawn. However, this group technology layout only indicates the layout design without considering the material flow lines. The final layout design will be compared to the original group technology layout. Hence, the concept of the intercell and intracell flow movement is the key factor for developing the proposed layout design.

If intercell layout must be incorporated with intracell layout, an additional type of flow is created when the cells have common machine requirements. Flows may occur between any pair of flowline if the machine loads were such that an integer number of machines could not be assigned to each cell [15].

This fact might reduce the number of machines available within a cell for duplication at two or more stations to reduce intracell handling times [12]. Hence, the problems of machine utilization and integer allocations of shared machines at the intracell and intercell levels are interrelated.

The related problems of designing intracell and intercell layouts can be solved using machine-specific data, such as the flow information captured in a travel chart. The method of partitioning the flow network captured in a travel chart enables the bottleneck machine problem to be viewed as it would appear when the intracell and intercell layouts are designed.

In the pioneering work of researchers on unidirectional flowline design, the flows in the travel chart and operation sequences used as input data have been classified as in-sequence, bypass, or backtrack [4,5]. This classification is sufficient since they consider the optimum layout for a single group of machines only.

The intracell layout design will be performed by a block diagonal heuristic algorithm using the travel chart as an input data. The intercell layout must be designed so as to locate cells with common machines close to each other, and cells which may have several machines in common should be placed adjacent to each other. This will facilitate intercell material handling and minimize intercell flow delays.

2.2 Phase 2

The following notations are needed in order to proceed following given phases.

T=Maximum spanning arborescence generated in the travel chart.

C, S=Common root (or raw material store) and sink nodes of the digraph D (or travel chart) occurring in the operation sequences of all parts.

Q_k=Batch quantity for part k.

S_k=Operation sequence of part k, represented as $\{C, \langle 1\rangle, \langle 2\rangle, \cdots, \langle n\text{-}1\rangle, \langle n\rangle, S\}$, where $\langle i\rangle$ is the machine required for the ith operation on the part.

$f_{ij} = \Sigma\ Q_k\ \forall\ S_k \mid \alpha_{ij} \in S_k,\ i \neq S,\ j \neq C$ i.e. the sum of the batch quantities of all parts whose operation sequences contain machine i and j consecutively.

TC_{ij}=Travel chart, where the numbers in the travel charts are f_{ij}.

m_{ij}=The arc representing precedence relations, where $m_{ii} = m_{jj} = *$.

P=Number of paths corresponding to the leaf in T, which is generated from the phase 3, where path is a set of machine cells given by MSA.

W_t=Non-negative flow weight assigned to a machine common to paths P_i and P_j.

pi=The number of arcs existing node i, i = 1 to n.

qi=The number of arcs incident to node i, i = 1 to n.

Xi=Position in the linear sequence to which P_i is assigned.

TF_{PiPj}=Intercell flows among the cross arcs, saying the sum of flow volumes of arcs that connect paths P_i and P_j, i.e. either the head or tail node of each arc lies on P_i and P_j, $\Sigma_t \Sigma_k$ fkl $\forall k \notin P_i \in P_j, \forall l \in P_i \notin P_j,\ \alpha_{kl} \in C$.

$$R_{ij} = \begin{cases} \text{Path index difference between paths Pi} \\ \quad \text{and Pj, if Pi is to the left of Pj} \\ 0\text{, otherwise} \end{cases}$$

$$L_{ij} = \begin{cases} \text{Path index difference between paths Pi} \\ \quad \text{and Pj, if Pi is to the right of Pj} \\ 0\text{, otherwise} \end{cases}$$

$$A_{ik} = \begin{cases} 1\text{, if Pi is assigned to position k in the} \\ \quad \text{optimal sequence of pendent nodes} \\ 0\text{, otherwise} \end{cases}$$

λ_{ij}=Lagrangian multiplier

Concurrent design of complex systems may involve a large number of design activities [11]. The requirements for the design of a complex system are diverse and often conflicting. Complex designs may involve many activity variables defining a product or system, how it is made, and how it behaves.

The precedence considerations may prevent same or similar activities from being performed twice in the design of different products or systems. In order to perform activities concurrently that are interdependent, negotiation among specialists might be required [3]. Potentially many engineers from various disciplines must be involved in the complex decision process [21].

Design activities are represented with a graph and the corresponding incidence matrix [18]. A developed algorithm transforms an incidence matrix into a triangular form. The initial stage in the approach to sequencing of design is defining the precedence relationship between machines.

The objective of this algorithm is to transform an unstructured travel chart representing sequences between machines into a structured upper diagonal matrix. As soon as each machine is selected and placed in the structured submatrix, all its required predecessors would be to the right or left of the diagonal. The following heuristic algorithm is proposed for comparing to the next phase.

The element * to refer to any nonblank element in the machine to machine precedence matrix. Namely any integers in the block can be assumed as *. Whenever machines are reordered, the same reordering of the rows and columns are proceeded. If the machines could be reordered so that the travel chart is upper triangular, i.e., all elements are either on or upper the diagonal, then proceeding in this order, the sequences of machines could be determined one at a time.

As each machines is determined, all its required predecessors would be to the right of the diagonal and thus already known. If two machines occur in the same block, they are put into the same block, otherwise, they must be put into different blocks.

A travel chart helps to integrated the identification of machine groups, flowline layout for each group and the overall layout of these flowline. It also eliminates the need for pairwise comparison of the operation sequences of part, especially when many sequences are usually identical or subsets of other sequence.

Step 0. Begin with the travel chart (TC_{ij}).

Step 1. Convert the travel chart (TC_{ij}) into activity matrix (m_{ij}) where, if f_{ij} is nonzero then express *, otherwise leave blank.

Step 2. Place starting node, if any, in the upper left corner of the matrix $m_{ij}^{(0)}$, and place the ending node, if any, in the lower right corner of the matrix $m_{ij}^{(0)}$, where $m_{ij}^{(0)}$ represents the original matrix from step 1.

Step 3. IF there are diagonal elements $m_{ii}=*$ with the corresponding pi=1, except starting and ending node(s),
THEN place them in the upper left corner of the structured $m_{ij}^{(1)}$, where $m_{ij}^{(1)}$ represents the remaining matrix with unclustered elements from the previous step.
ELSE go to step 4.

Step 4. IF there are diagonal elements mii with $q_i=1$ from the previous matrix,
THEN place them in the most right lower corner of the structured $m_{ij}^{(2)}$,
ELSE go to step 3 until there is no more diagonal element with pi=1 or mii with qi=1 from the previous matrix $m_{ij}^{(n-1)}$.

Step 5. Group the unselected element(s) m_{ij} in one matrix.

Step 6. Sequence the unclustered elements beginning with the smallest pi,
AND place them in the most upper left corner of the structured $m_{ij}^{(n-1)}$.
IF there are more than two elements with identical pi values,
THEN place them arbitrarily in the structured $m_{ij}^{(n-1)}$.

Step 7. IF all elements are clustered within

```
        blocks, and makes a triangular matrix,
        THEN stop.
        ELSE go to step 4.
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2.3 Phase 3

An arborescence (or directed tree) is a connected graph which contains no circuits. A spanning arborescence of a digraph is an arborescence which is a partial graph. The theoretical classification of the flows in a travel chart on dominators in acyclic digraphs [1], strong components in a digraph [22], and optimum branching in a digraph has been shown in computer science literature [23].

A new graph structure, maximum spanning arborescence (MSA), was preferred over several undirected graph structures such as cut tree, maximum spanning tress and maximal planar graph which have been used to solve the more general problem of facilities [6], which is defined as a tree in which no two arcs are directed into same vertex [8]. A maximum arborescence of graph is any arborescence of graph with the largest possible weight [24].

The maximum branching algorithm constructs a maximum branching for any graph, and can be also used to find minimum branching, maximum spanning arborescence, minimum spanning arborescence, and minimum spanning arborescence rooted at a special node.

The maximum branching algorithm finds a maximum spanning arborescence, and a maximum spanning arborescence (MSA) has the clustering property of the maximum spanning tree. In addition, it gives the flowline layout for the group of machines in each path. Since nodes in the MSA can have outdegree greater than one, this combines the concepts of functional layout with a tree layout for the shop when flowline sharing machines are merged.

Due to its directed property, it conforms to the in-sequence, bypass and backtrack (for intracell flows) and crossover (for intercell flows) classification of flows in the travel chart for the shop. In a line layout there will inevitably be bypassing or backtracking of jobs as they pass down the line. Of these two imperfections it would seem that bypass is more desirable than backtrack and that the number of operations bypassed or backtracked has some significance, if only in terms of distance moved [20]. The following heuristic algorithm is to find the maximum spanning arborescence, which generates the paths.

```
Step 0. Begin with the travel chart ($TC_{ij}$).
Step 1. Select the starting node I arbitrarily.
Step 2. Select the largest value $f_{ij}$ of the
        corresponding row I, where $f_{ij}$ is the
        total of batch quantity flow in arc (i,
        j).
Step 3. Select the corresponding column j as
        the to-incident node.
Step 4. Set i=j.
Step 5. Repeat step 2 and 3 until all nodes are
        processed.
Step 6. IF there is no node available,
        THEN put operation sequence of
```

selected nodes as one machine cell, AND stop.

2.4 Phase 4

The presented model is a reduced form of Love's model for two-dimensional location in a Euclidean plane adapted to solve the one-dimensional quadratic assignment problem, and relaxed by a Lagrangian method [19]. It describes the integer programming model which finds the permutation of the branches of T to minimize intercell flow distance [17].

Minimize $\Sigma_{i=1}^{p-1} \Sigma_{j=i+1}^{p} \{TFp_ip_j + \Sigma_{k \in pi \cap pj} W_k\} (R_{ij}+L_{ij})$

Subject to $\Sigma_i A_{ij} = 1 \quad \forall j = 1, \cdots, p$ (a)

$\Sigma_i A_{ij} = \quad \forall i = 1, \cdots, p$ (b)

$Aik \ \{0, 1\} \ \forall i, k$ (c)

$0 \leq R_{ij} \leq P$ (d)

$0 \leq L_{ij} \leq P$ (e)

$R_{ij} - L_{ij} = (\Sigma_k kA_{ik}) - (\Sigma_k kA_{jk}) \ \forall i \langle j$ (f)

Each path becomes a node in the linear layout describing the left-to-right order of the paths in T. The R_{ij}'s and L_{ij}'s yield (n^2-n) variables and the A_{ik}'s are another (n^2) variables. The X_i's are dummy variables which describe the position in the linear arrangement to which a path is assigned. The formulation to select the ordering of the paths to minimize the weighted flow distances in the cross arcs is as follows:

Constraints (a) and (b) ensure that each position is assigned a unique path, and each path is assigned a unique position, respectively. Constraint (c) ensures that only integer value can be assigned. Constraint (d) and (e) limit the maximum feasible values, and constraint (f) relates the assignments of paths to position on the line to the distance (R_{ij} or L_{ij}).

Inspection of this mathematical programming model leads to the observation that constraint (f) are complicating, while the problem reduces to two relatively easy problems if this set of constraints were ignored. That is, the problem would separate into a standard assignment problem to choose the set of variable A, while choice of the variables R and L are restricted only by their upper and lower bounds.

The number of applications of Lagrangian Relaxation has grown to include over a dozen of the most infamous combinatorial optimization problems [9]. For most of these problems, Lagrangian Relaxation has provided the best existing algorithm for the problem and has enabled the solution of problems of practical size [10]. Hence, Lagrangian Relaxation is an important new computational technique in the management scientists arsenal.

By relaxing constraints $R_{ij}-L_{ij}=(X_{ik}-X_{jk})$, where $X_{ik}=\Sigma_k kA_{ik}$ and $X_{jk}=\Sigma_k kA_{jk}$, a new Lagrangian objective function is then constructed: $\Sigma_{i=1}^{p-1} \Sigma_{j=i+1}^{p} \{TFp_ip_j + \Sigma_{k \in pi \cap pj} W_k\} (R_{ij}+L_{ij}) - \Sigma_{i=1}^{p-1} \Sigma_{j=i+1}^{p} - \lambda ij(R_{ij}-L_{ij}-(\Sigma_k kA_{ik})+(\Sigma_k kA_{jk}))$, where the infeasibility in each relaxed constraint (f) is multiplied by a multiplier, and subtracted from the original cost function. This Lagrangian function can be rewritten as:

$$\Sigma_{i=1}^{p-1} \ \Sigma_{j=i+1}^{p} \ \{TFp_ip_j + \Sigma_{k \in pi \cap pj} W_k - \lambda_{ij}\} \ R_{ij} +$$
$$\Sigma_{i=1}^{p-1} \ \Sigma_{j=i+1}^{p} \ \{TFp_ip_j + \Sigma_{k \in pi \cap pj} W_k + \lambda_{ij}\} \ L_{ij} +$$
$$\Sigma_{i=1}^{p} \ \Sigma_{k=1}^{p} \ K[\Sigma_{j=1}^{i-1} \lambda_{ij} - \Sigma_{j=i+1}^{p} \lambda_{ij}] \ A_{ik}.$$

This Lagrangian function is then minimized subject to all constraints except (f). The problem in turn separates into three subproblems:

Minimize $\Sigma_{i=1}^{p-1} \Sigma_{j=i+1}^{p} \{TFp_ip_j + \Sigma_{k \in pi \cap pj} Wk - \lambda_{ij}\} \ R_{ij}$
Subject to $0 \leq R_{ij} \leq P$,

which is solved by setting each R_{ij} equals to 0 if its coefficient is positive, and P if its coefficient is negative.

Minimize $\Sigma_{i=1}^{p-1} \Sigma_{j=i+1}^{p} \{TFp_ip_j + \Sigma_{k \in pi \cap pj} W_k + \lambda_{ij}\} \ L_{ij}$
Subject to $0 \leq L_{ij} \leq P$,

which is solved by the same procedures as that used to assign values R_{ij}, and

Minimize $\Sigma_{i=1}^{p} \Sigma_{k=1}^{p} K[\Sigma_{j=1}^{i-1} \lambda_{ij} - \Sigma_{j=i+1}^{p} \lambda_{ij}] \ A_{ik}$
Subject to $\Sigma_i \ A_{ij} = 1 \qquad \forall k$
$\Sigma_k \ A_{ij} = \qquad \forall i$
$A_{ik} \quad \{0, 1\}$,
R_{ij}, L_{ij} integer for all $i \langle j$,

which is solved by any method for the classical linear assignment problem, e.g., the well-known Hungarian method. The sum of the optimal costs of these three problems is therefore the optimal cost of the relaxed problem. The minimum objective value of the relaxed problem is clearly dependent upon the choice of the Lagrangian multipliers ij, and it can be proved that it is a lower bound for the minimum value of the original function in the original problem.

In order to maximize the lower bound provided by the Lagrangian Relaxation, the Lagrangian multipliers ij can be adjusted by the subgradient optimization method [10]. The maximum value of the lower bound can then be used in a branch-and-bound method to get the optimal variables, R_{ij}, L_{ij}, and A_{ij}, etc.

2.5 Phase 5

As Figure 1 shows the methodology solution procedures, layout can be designed by the cross arcs flowline and adjacencies, bottlenecks identification, and demand quantity flows. Especially, the layout design efficiency has been traditionally measured as total traveling distance multiplies to the flow quantity of each parts.

However, there are many ways to measure the layout design efficiency based on such considerations as the total cost, group technology concept, material handling strategy, and adjacency ratio, etc. All the described efficiency measurements can be performed depending on what kind of variables are relevant to the particular purpose.

Cross arcs in a maximum spanning arborescence are only considered. Since the forward and backward arcs relate to intracell flows among nonadjacent machines, layout and handling strategies can eliminate the machine requirements for these arcs.

Further, the number of paths in the arborescence, the average number of nodes in each path and the number of bottleneck machines in the arborescence can quickly indicate whether independent cells can be designed or not. Since, an arborescence contains only (n-1) arcs ignoring up to (n^2-2n+1) additional arcs which can influence machine duplication, intracell or intercell layout decisions.

2.6 Phase 6

The comparison of the final layout by the proposed method with the original group technology applied layout without considering material flows should be emphasized. Hence, the proposed approach determines how well the layout is designed in a different way following definition, since most of all traditional layout design measurements has been same, but the proposed one depends on the how many material flows are backward between each levels of the final layout design.

$$\left(1 - \frac{\text{Total penalty weight}}{\text{Total number of movement between levels}}\right) \times 100(\%)$$, where the total penalty weight includes:

a. No penalty for forward movement from level i to level i + k, k=1, .., n - 1.

b. Penalty equal to n for backtracking movements from level i to level i - k.

c. No penalty for cross movement from different paths in the same level i.

3. Empirical Example

The proposed method will be explained using an empirical example from a non-ferrous metal manufacturing company, which produces primarily copper and copper alloy strip products [7].

Table 2 lists the alloys which meet the end uses described in Table 1. Note that each alloy in the product list in Table 2 has a job routing based on its own mechanical properties. Mechanical properties include tensile strength, hardness, grain size, temper, yield, elongation, coil weight, gauge, and packing methods. etc. It specifies the sequence of machine centers beginning with the production operational sequences. Table 3 describes the machine center lists and capacity.

Phase 1. The original group technology layout without considering products to be process oriented as Figure 2 shows. This group technology layout neglects the material flow lines, which will be compared to the final layout.

Phase 2. Heuristic algorithm generates the machine centers sequence as H-T-(W-E-P-L-C-D-B)-S.

Phase 3. Heuristic algorithm generates the maximum spanning arborescence. It indicates three main flowline such H-T-W-C-D-S as path 1, H-T-E-P as path 2, and H-T-E-B-L as path 3. Machine types T and E are two branching nodes. Figure 4 shows only the cross arcs, while one forward arc (W, D), and two back

Table 1. End Uses for Primary Brass Mill Shipment-Strip, Sheet & Plate

Unit : Thousand of Pounds

Serial No.	End Uses	Demand / Year	%
100	Building Products	37,156	3.7
200	Household Products	13,331	1.3
300	Transportation Equipment	116,795	12.3
400	Electrical & Electronics	126,610	13.5
500	Industrial Machinery & Equipment	5,043	0.4
600	Fasteners & Closures	13,010	1.6
700	Other Identifiable Uses	146,974	14.8
800	Components Parts	35,303	3.5
900	Unidentified End Uses	486,056	48.9
	Total	983,276	100.0

Source: Copper Development Association Inc., June 1994

Table 2. Copper and Copper Alloy Product List

Unit: Ten thousand pound per month

Part No.	Alloy No.	Chemical Composition (%)	Application & Specification	Operation Sequence	Demand Quantity
1	102	Cu 99.95	Wire for electric purpose	H-T-E-P-D-S	10
2	104	Cu 99.95, Ag 0.05	Oxygen-free with Ag. strip	H-T-W-E-D-P-B-D-S	2
3	110	Cu 99.90, O 0.05	Tough pitch copper strip	H-T-W-D-B-D-S	5
4	122	Cu 99.90, P 0.01	Phosphorous deoxidized copper	H-T-E-P-B-D-S	1
5	210	Cu 95.00, Zn 5.00	Gilding metal	H-T-E-W-B-D-S	4
6	220	Cu 90.00, Zn 10.00	Commercial bronze	H-T-W-L-B-C-D-S	2
7	230	Cu 85.00, Zn 15.00	Red brass strip	H-W-L-B-C-D-S	5
8	240	Cu 80.00, Zn 20.00	80/20 brass strip	H-T-W-L-B-C-D-S	2
9	260	Cu 70.00, Zn 30.00	Cartridge brass strip	H-T-W-C-D-S	25
10	268	Cu 65.00, Zn 35.00	Yellow brass strip	H-T-E-W-C-D-S	13
11	353	Cu 62.0, Zn 32.5, Pb 2.25	Leaded brass strip	H-T-E-P-E-C-D-S	7
12	705	Cu 95.00, Ni 5.00	Copper-Nickel bonding	H-T-E-C-D-S	9
13	713	Cu 75.00, Ni 25.00	Cupro-Nickel strip	H-T-E-B-L-D-S	15

arcs (P, E), (L, B) are also required (but not show in the figure).

If machines T and E are duplicated, then the total cross arcs intercell flows will be decreased, however, the main objective of the layout in this case is not to duplicate any machine cell. The number of unique permutations or ordering of the paths are three such as 1-2-3, 2-1-3, and 1-3-2 respectively in Phase 4. These three permutation can easily be explicitly enumerated and evaluated. Bottleneck machines T and E should be more closely examined regarding the

Table 3. Machine Center List

Unit of capacity: Ten thousand pound per machine per month

Center Symbol	Center Description	Center Function	Available machine	Production Capacity
H	Hot steckel rolling	Reheat metal pliable for thinner rolling	1	100
T	Tandem rolling mill	Reduce metal to thinner gauge to 0.040"	1	100
W	Welding / Trimming line	Weld and trim the coil edges by cutter	2	30
E	Ebner bell furnace anneal	Recrystallize metals' grain size for bright surface	6	10
P	Pickling machine	Lubricate while maintaining metals' high luster	2	10
C	Continuous anneal pickle	Shield by air for making shape unmarred	2	30
D	Degreasing machine	Degrease oils on metals surface	2	50
L	Leveling tension machine	Control grain size for making tension target	1	40
B	Bonding mill machine	Bond two different alloy strip to one strip	1	40
S	Slitter & Packing line	Slit coils and pack as customer requirements	1	80

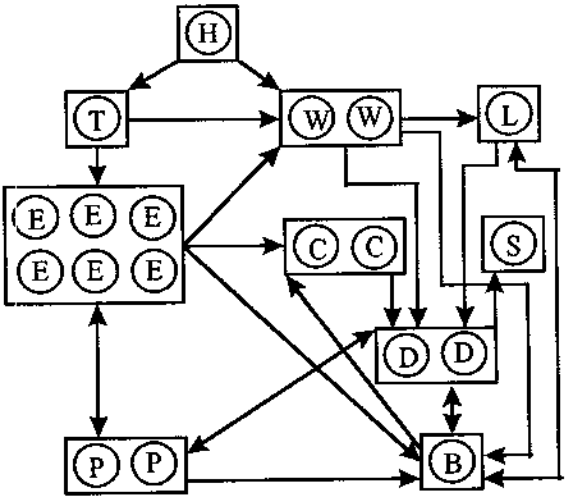


Figure 2. Group Technology Layout

$$
\begin{array}{c|cccccccccc}
 & H & T & W & E & P & L & C & D & B & S \\
\hline
H & * & * & * & & & & & & & \\
T & & * & * & * & & & & & & \\
W & & & * & & & * & * & * & * & \\
E & & & * & * & * & & * & & * & \\
P & & & & * & * & & & * & * & \\
L & & & & & & * & & * & * & \\
C & & & & & & & * & * & & \\
D & & & & & * & & & * & * & * \\
B & & & & & & * & * & * & * & \\
S & & & & & & & & & & *
\end{array}
$$

Figure 3. Final Matrix by Block Diagonal Algorithm

flow weight since those two machines are branching nodes.

Phase 4. Lagrangian Relaxation model shows no machine duplication yields the ordering path.

If part mix or demand distribution change over time, the system layout must be generated from several travel chart, corresponding to the flow patterns for several production periods. None of the existing cell formation methods have this capability. Hence, a cluster analysis of arborescence must be performed to minimize intercell flows and machine duplication.

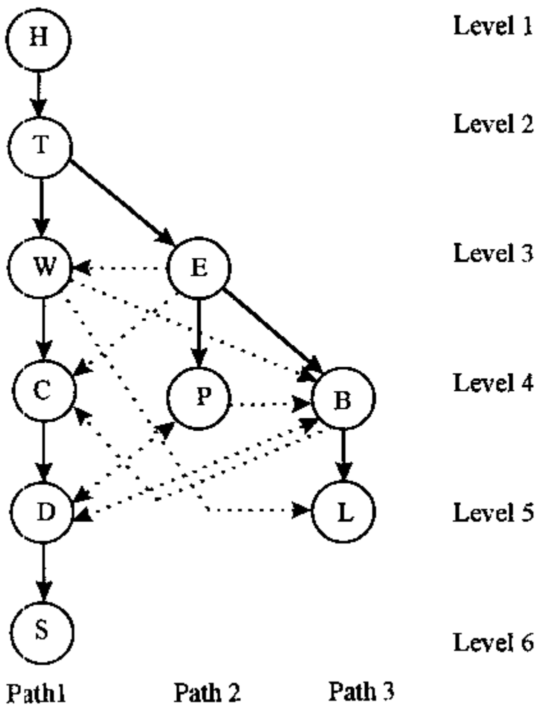


Figure 4. Cross Arcs in the Arborescence

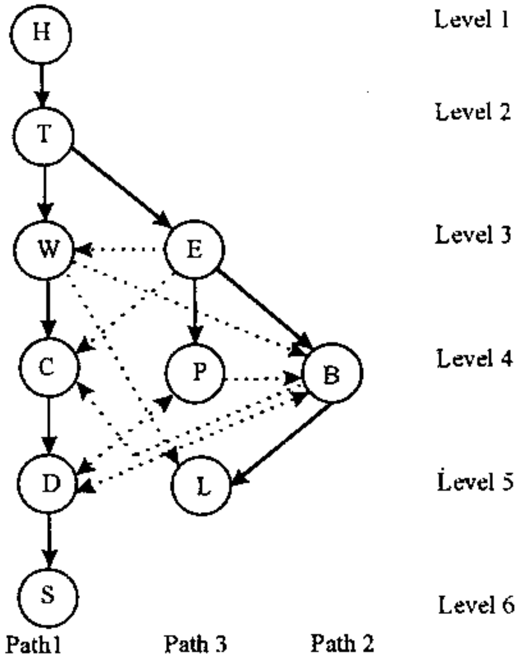


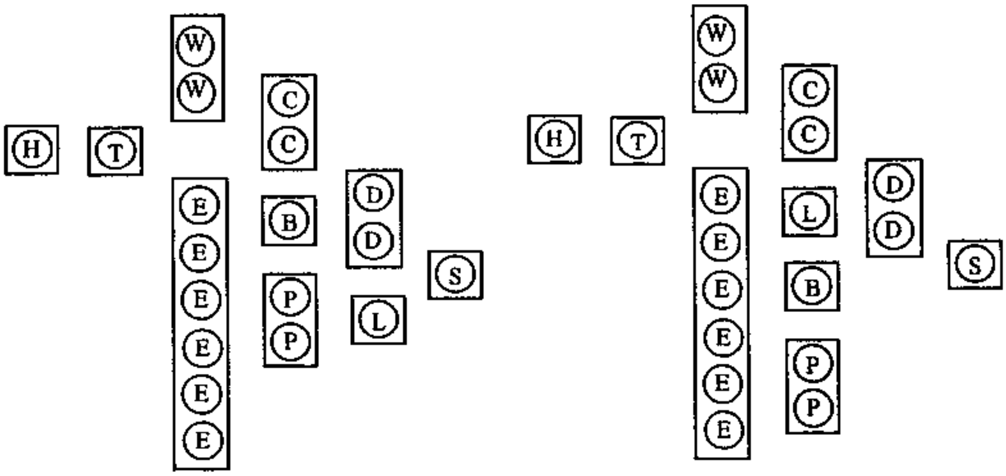
Figure 5. Arborescence without Machine Duplication by Lagrangian Relaxation Model

In phase 4, Note the ending nodes of each path. Therefore, one pickling machine could be placed in another path (path 1 or 3) in order to eliminate cross arcs (P, D), or (P, B) respectively. In order to avoid the duplication of pickling lines, path 2 could be adjacent to path 1 and/or path 3.

Phase 5. Finally two different layout designs are proposed as Figure 6 shows. In this empirical example, the number of unique permutations, or ordering, of the paths is quite small, namely 1-2-3, 2-1-3, and 1-3-2 (not including the reversal of these permutations 3-2-1, 3-1-2, and 2-3-1 respectively, which may be considered as equivalent).

These three permutations can easily be explicitly enumerated and evaluated without the need for the Lagrangian relaxation method proposed. Bottleneck machines T and E should be more closely examined regarding the flow weight since those two machines are branching nodes.

Weighted flow for the bottleneck machine centers T and E are assumed to be 135 since max. $(TFp_1p_2) = 45$ and $p=3$ which yields $45*3=135$, where weighted flow between each path is defined as cross weight between paths multiply by number of paths. Hence, Figure 5 is generated the paths as path 1- path 3 - path 2 without machine duplication.



Design I **Design II**

Figure 6. Suggested Machine Centers Layout Design

As path 4 seeks to place cells with high flow interaction adjacent to each other, the head and tail nodes of any cross arc will belong to different paths, since phase 4 uses a symmetric intercell flow matrix, the directions of these arcs are ignored.

Replacing the directed cross arcs by undirected edges, the matrix of intercell flows between the three paths in the arborescence, i.e., the sum of the flows in the cross arcs can be calculated. Notice that the deviation from planar graph theory applied to facilities design, since arcs which cross each other are also considered.

For each path, the machine type with the least number of machine available is chosen. Using this number as a bound, as many copies as possible of that path are created parallel to each other. The process is repeated for the other paths. However, they must be located based upon flow directions and practical considerations for each machine type. Adjacencies can be determined by the two machines included in each cross arc. A part can be inspected immediately after machining or assembly before it is sent to the next machine in its operation sequence. This decision is supported by one of the major benefits of group technology cells.

Therefore, figure 6 suggests the layout designs. Compared to the original layout for the shop, machine locations in both layouts create identical machine adjacencies, although the original layout has unnecessary machine duplication. The suggested layouts have been designed based on the levels that are shown in the final arborescence without duplication of machine centers. However, if machine center L is moved from the current level 5 to the previous level 4 for considering minimizing the travel distance, then travel distance between corresponding nodes would be reduced.

The movement between machines in machine centers are the forward arc (W, D) in path 1, and backward arcs (P, E) in path 2, and (L, B) in path 3. Those arcs are entirely within the paths which include those nodes, which means there is no forward or backward arcs between different paths.

Phase 6. Four bottleneck machines E, C, D, and S are identified as Table 4 shows. Therefore it may be required to add other machines. The most significant bottleneck is in D. It indicates that if machine E capacity is increased, then the number of machines may be reduced since the number of succeeding machine centers is comparatively small only if the succeeding capacity is also small.

Although machine B has sufficient capacity to handle all incoming flow from the previous machine centers, machine B has slack capacity so that its efficiency is critical if it is to avoid becoming another bottleneck. For example, from the machine load capacity list in Table 4, even though six Ebner machines are available at this moment, they are not enough to meet the current demand pattern.

Bottleneck machines within a cell are accommodated by the intercell handling system. An intercell layout of parallel flowline allows crossover for some adjacent lines involving all bottleneck machines which are common among them. Machine duplication for some intercell flow is further avoided by permuting those flowline to make them adjacent to each other across intermediate aisles. This leaves only those parts and machines involved in intercell flows among non-adjacent flowline.

According to the proposed layout design efficiency (LDE) shows that original group technology layout is 54%, while the suggested design I is 83%, and design II is 87.5%. Note that a layout design efficiency equals to 100% means that the final layout does not allow any backtracking machine movements in any level. Although 100% LDE can not be achieved and the perfect layout design does not exist, the greater the layout design efficiency, the better

Table 4. Machine Load Capacity Planning

	H	T	W	E	P	C	D	L	B	S
Each machine capacity	100	100	30	10	10	30	50	40	40	80
# of available machines	1	1	2	6	2	2	2	1	1	1
Machine cell capacity	100	100	60	60	20	60	100	40	40	80
Machine cell loading	100	95	58	66	20	63	122	24	36	100
Shortage for required machineloading				6		3	22			20
# of activity succeeding machines		12	8	8	4	7	15	4	8	13
# of succeeding machines		1	3	2	2	3	5	2	5	1

Table 5. Research Table

	Proposed Research	Previous Works
1	More input data possible Machine capacity consideration	Restrict number of input Simplifying assumptions made
2	Eliminates the need for pairwise comparison of the operation sequence of parts, using a travel chart, especially as many sequences are usually identical or subsets of other sequence	It does not evaluate the flow for each part to check whether its arcs connected machines contained in only a pair of adjacent parts in an arborescence
3	No assumptions of the constraints	Ignoring a clearly defined cell for a part family and predetermined lower & upper limits on the number of cells and machine duplication
4	Variety of input data identifies machine bottleneck situation in flowline	Assumption that each machine node in any arborescence, ignoring additional arcs
5	Performance measurement can consider the flexibility factors in a certain threshold value	Cross arcs weight should be dependent on demand quantity, variety of parts, number of operations, similarity of operation sequence
6	Alternative analysis, e.g., number of required AGVs or machine adjacencies can be applied	Assumption that arborescence has a work center exists each of its node, if there is more than one machine, final design is incomplete
7	N/A	Neglects rectilinear / Euclidean distance
8	Exploits use of handling system to avoid risk of having to change the machine composition	Exact machine duplication decision isn't made as lack of routing information details on each part
9	Possible input data, e.g., setup or run times, alternative machine selection can be applied	No clear cut solution strategy is known to connected pairs of machines for adjacent flows
10	Using cross arcs, the part whose operation sequence causes intercell flows are identified	Accurate part family without using data on each operation sequence of parts is impossible
11	Design criteria is set up, and cluster analysis is performed to minimize the intercell flows, machine intercell handling delays	No capability for changeable factors, it should be generated from several travel charts corresponding to the flow patterns in period
12	Generating parts operation sequence by developing heuristic algorithm	N/A
13	Heuristic algorithm for finding the MSA is less computational burden	Integer programming for finding MSA which lacks guarantee of optimality
14	Generating optimal paths by Lagrangian Relaxation model for simplicity	Linear programming from the existing Love and Wong's model
15	Computation efficiency to known problems	N/A
16	Applicable to real industry data: identification of bottleneck work center for better facility plan	Theoretical approach only to the flowline & batch quantity
17	Solution to optimal work centers layout design problem	Difficulty to understand solutions required to separate machines from machine centers
18	Graph and mathematical approach	Similarity approach
19	Consolidate management decision support	N/A
20	Design performance is criterized : Layout design efficiency and capacity planning application	N/A

the layout for the purpose of material flows. The described efficiency measurement can be performed depending on variables to relevant to the particular purpose. The variables here uses parts operation sequence and required batch quantity. If the relevant variables were either total distance or required layout square feet, then another appropriate efficiency measurement could be easily found.

4. Discussion

One of the meaningful evaluation of the proposed approach relative to the traditional way in cell formation methodology lies in the use of graph structure which simultaneously integrate the machine grouping and cell layout problem. Good layout design for making parts operation efficient would be highly effective for reducing cycle time in flexible manufacturing system.

Moreover, direction of flow and locations of individual machines are important for making layout and machine duplication decisions. Table 5 shows the difference and contributions between the proposed research and the previous works [18].

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