BOUNDED MOVEMENT OF GROUP ACTIONS

PAN SOO KIM

Abstract. Suppose that G is a group of permutations of a set Ω. For a finite subset Γ of Ω, the *movement* of Γ under the action of G is defined as

$$\text{move}(\Gamma) := \max_{g \in G} |\Gamma^g \setminus \Gamma|,$$

and Γ will be said to have *restricted movement* if $\text{move}(\Gamma) < |\Gamma|$. Moreover if, for an infinite subset Γ of Ω, the sets $|\Gamma^g \setminus \Gamma|$ are finite and bounded as g runs over all elements of G, then we may define $\text{move}(\Gamma)$ in the same way as for finite subsets.

If $\text{move}(\Gamma) \leq m$ for all $\Gamma \subseteq \Omega$, then G is said to have *bounded movement* and the *movement of* G $\text{move}(G)$ is defined as the maximum of $\text{move}(\Gamma)$ over all subsets Γ of Ω. Having bounded movement is a very strong restriction on a group, but it is natural to ask just which permutation groups have bounded movement m. If $\text{move}(G) = m$ then clearly we may assume that G has no fixed points in Ω, and with this assumption it was shown in [4, Theorem 1] that the number t of G-orbits is at most $2m-1$, each G-orbit has length at most $3m$, and moreover $|\Omega| \leq 3m + t - 1 \leq 5m - 2$. Moreover it has recently been shown by P. S. Kim, J. R. Cho and C. E. Praeger in [1] that essentially the only examples with as many as $2m-1$ orbits are elementary abelian 2-groups, and by A. Gardiner, A. Mann and C. E. Praeger in [2,3] that essentially the only transitive examples on a set of maximal size, namely $3m$, are groups of exponent 3. (The only exceptions to these general statements occur for small values of m and are known explicitly.) Motivated by these results, we would decide *what role if any is played by primes other than* 2 *and* 3 *for describing the structure of groups of bounded movement.*

REFERENCES

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