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Expert Opinion Elicitation Process Using a Fuzzy Probability

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Abstract

This study presents a new approach for expert opinion elicitation process to assess an uncertainty inherent in accident management. The need to work with rare events and limited data in accident management leads analysts to use expert opinions extensively. Unlike the conventional approach using point-valued probabilities, the study proposes the concept of fuzzy probability to represent expert opinion. The use of fuzzy probability has an advantage over the conventional approach when an expert's judgment is used under limited data and imprecise knowledge. The study demonstrates a method of combining and propagating fuzzy probabilities. Finally, the proposed methodology is applied to the evaluation of the probability of a bottom head failure for the flooded case in the Peach Bottom BWR nuclear power plant.

1. Introduction

Treatment of uncertain information elicited from experts has encountered increasing interest among various investigators in recent years. Especially, expert opinion plays an important role in the assessment of probabilities related to complicated physical phenomena for the PSA (Probabilistic Safety Assessment) such as seismic frequencies or the occurrence of steam explosions.

Roughly speaking, there are three basic issues when we consider the treatment of expert opinion :

a) How to represent the uncertain information elicited from experts,
b) How to combine the uncertain information obtained from different experts, and
c) How to propagate the information through a system.

Up to now, conventional approaches using point-estimate probabilities have been extensively used for expert opinion. However, the problem occurs because experts frequently express difficulty in estimating point-valued probabilities associated with complicated physical phrnomena. This is due to the fact that expert opinion under incomplete knowledge and limited data is inherently imprecise and uncertain.

The fact that expert opinion elicitation process involves an uncertainty to be explicitly evaluated needs to first of all distinguish between two major types of uncertainty :

a) Uncertainty due to stochastic variability, and
b) Uncertainty due to lack of knowledge.

The first type of uncertainty is due to the actual random behavior in some physically measurable quantity. Examples of the stochastic variability are variations in weather, variations in component failure times from one observation to another, and variations in consequences from one accident to another.

The second type of uncertainty is quite different from the stochastic variability. It is vagueness or imprecision in an analysis, or stated value. The uncertainty exists because of a lack of knowledge ; if we gained more information and more knowledge, the uncertainty would decrease or would not exist. Examples of the uncertainty are uncertainties associated with an estimated value of a value, or uncertainties in the appropriateness of an consequence model.

The first case assesses the uncertainty when the end point is an unknown distribution of values. The assessment end point is a true but unknown distribution of values representing random variability in the parameters or measured data used in the model. On the other hand, the second involves a case when the end point is a fixed but unknown value due to the imprecision in the analysists knowledge about models, their parameters, and/or their predictions. The subjective confidence interval can be used for the unknown value in this case. The distribution used represents a range of "degrees of belief" that the but unknown value is equal to or less than any value selected from the distribution.

Statistics generally deal with the first type of uncertainty. Recently an approach using Monte Calro method has been developed for handling the second type of uncertainty[1]. On the other hand, a theory termed fuzzy set theory has undergone rapid development in the past several years. Fuzzy set theory attempts to address the uncertainty due to lack of knowledge which is not addressed by conventional approaches.

The purpose of this paper is to present a new approach to expert opinion elicitation using fuzzy set theory. The study uses three steps. First, it introduces a fuzzy probability to model expert opinion. The concept of fuzzy probability can account for an expert's judgment which uses limited data and imprecise knowledge and in which events are frequently complicated and ill-defined. Second, for the aggregation of uncertain information issued from different knowledge sources, it demonstrates a method of combining fuzzy probabilities in a manner consistent with the Dempster-Shafer's Theory (DST). This method provides an important measure which indicates the validity of the result obtained. Finally, it introduces the computational algorithm based on fuzzy logic, which is as easy to implement as that of random variables. Hence, the fuzzy probabilities can easily be propagated through a system to obtain the final results. A case study is presented as the evaluation of the probability of a vessel failure for the flooded case in the Peach Bottom BWR nuclear power plant.

2. Method

2.1. Fuzzy Probability

Point-valued probabilities can be used for representing expert opinion. However, due to very limited data and knowledge, it is often difficult to quantify exact values for the probabilities regarding events in severe accident. In this case, the analysis should handle uncertain and imprecise values as discussed in the previous section. One of the natural ways of quantifying these probabilities is to use interval values. It is possible to choose a best-estimate value from the interval values by applying reasonable engineering judgement. The following is an example :

"Due to limited knowledge, the value of containment failure probability due to an ex-vessel steam explosion may lie between 0.2 and 0.4. However, based on reasonable engineering judgement, the value of 0.3 could be highly preferred as a best estimate."

This statement accounts for the subjective degrees of belief about quantity that is fixed but unknown. To handle the uncertainty due to lack of knowledge, the study introduces the concept of fuzzy probability instead of a unique value of probability. Fuzzy probability is called the possibility distribution of probability, which represents an imprecise probability by means of subjective possibility measures associated with judgmental uncertainty. This can simultaneously model the probability and its degree of possibility expressed by an expert.

To use the concept, a consistent way of assigning values for the degree of possibility should be developed[2]. This can be done by introducing a membership function or possibility distribution. The membership function plays a central role in fuzzy set theory and represents the numerical degree to an element which belongs to a set. This means that a small value of the membership function represents a low degree of judgement about the element, and a high value represents a high degree of judgement. Also, the concept is consistent with the classical approach using the classical probability theory. In other words, the arithmetic operations of fuzzy probabilities such as addition, multiplication, and joint and marginal probability are equivalent to those of the classical approach.

The concept of a membership function or possibility distribution is the cornerstone of the fuzzy set theory. However, up to now, it is not yet clear what their natural meaning is, and how to derive them. The assignment of a membership function or possibility distribution is quite a matter of subjective opinion. Since the possibility distributions are determined from an expert's experience and intuition, it can be generally considered a satisfactory approximation if a membership function or possibility distribution has a good shape.

This study uses a triangular representation of the possibility distribution, which is the most popular membership function. This representation can be defined by a triplet (a_1, a_2, a_3) where the definition of the membership function is shown below.

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases} \qquad (1)$$

A graphical representation is shown in Figure 1. In this representation, the modal value (i. e., a_2) is interpreted as the most possible value (possibility is one)

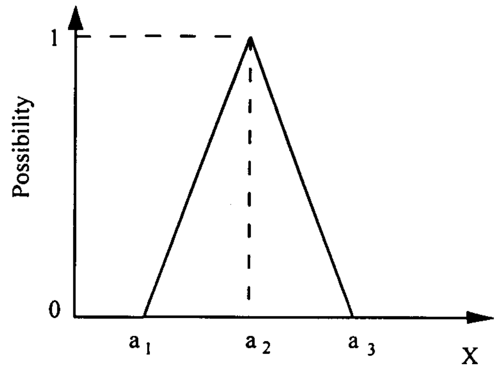

Fig. 1. Triangular Representation of a Fuzzy Probability

and the two extremes (i. e., a_1 and a_3) are the least possible values (possibility is zero). This gives a rational approximation with an appropriate bound for an uncertain probability. In addition, it has a simple form and therefore is easy to handle. Also, it has a relatively small number of parameters for estimation. These properties of the representation provide a good basis for combining information when available information is limited.

2. 2 Combination of Opinion

Among the three issues mentioned previously, the second one, i. e., how to combine the opinion of different experts is a big concern regarding risk assessment. The problem refers to aggregation of uncertain probabilities elicited from different experts dealing with the same issue.

Conventional methods such as an arithmetic-average or weighted-average using point-estimate probabilities have been widely used in PSA. Expert opinion is, however, inherently uncertain and imprecise under incomplete knowledge and limited data. In this case, the formalism using point-estimate probabilities may not adequately reflect this kind of imprecision in human judgement. Also, the methods above may not produce a meaningful result under a certain circumstance. For example, when experts disagree considerably (i. e., the data elicited from the experts are highly conflicting), the result obtained usually shows a central tendency, which may not satisfy each expert.

This section describes a methodology regarding the combination of fuzzy probabilities in a manner consistent with the Dempster-Shafer's Theory (DST). Detailed explanations regarding the DST can be shown in Reference[3]. It also demonstrates the extension of the proposed method incorporating experts' credibility weighting.

Combining Fuzzy Probabilities

To use Dempster's rule for combining fuzzy probabilities, a formal connection between the DST and possibility theory is required. Mathematically, a possibility measure, $\Pi(A)$, is one of the plausibilities, and is defined in terms of the basic belief masses (bbms) as follows :

$$\Pi(A) = \sum_{A \cap B \neq \emptyset} m(B) \quad (2)$$

where $m(B)$ is a bbm in the DST. Hence, the possibility measure, $\Pi(A)$, is the sum of all values of m following the intersection operation of the subset A and B.

Even though Eq. (2) represents the relationship between possibility measure and bbms in the DST, it is not, however, unique to find masses given the possibility distribution. One of the consistent ways to accomplish this conversion is to use *α-level cut* representation of possibility distribution. Using the property of consonant subset, we can derive basic belief masses (bbms) from possibility distribution as follows :

$$m(X(\alpha_j)) = \alpha_j - \alpha_{j-1} \qquad j=1, n \quad (3)$$

where X is the set element between α_j and α_{j-1}. The α_i is the cut value associated with the possibility measure on the possibility distribution (See Figure 2).

If we combine two continuous mass functions this way, the mass function resulting from Dempster's rule will be extremely complex and will not possess any simple intuitive meaning. In particular, it will be a highly non-consonant. This violates closure property. In other words, this operation does not produce consonant possibility measures.

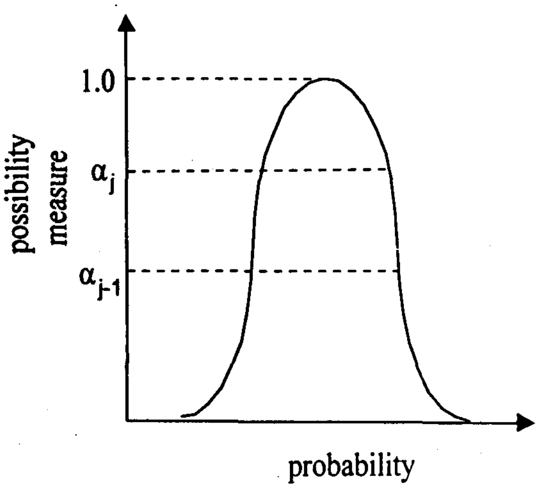

Fig. 2. The α-Level Cut on Possibility Measure

Hence, to avoid the difficulty, Fua has proposed a simple and statistically sound way, which is to multiply fuzzy probabilities point-wise, and normalize the product[4]. This work assumed that the possibility distributions are uni-modal, strictly monotonically decrease about their maximum, differentiable, and have zero value on the boundary. The method provides a simple and sound way of combining fuzzy probabilities in a manner consistent with the DST.

Checking Conflict

Hence, the Dempster's combination rule can be used to compute a new fuzzy probability as described previously. When combining two bbms, the theory have to normalize the results in order to obtain m_{result}(for all cases) = 1. The normalization factor is defined as follows.

$$K = \sum_{X \cap Y = \emptyset} m_1(X) m_2(Y) \quad (4)$$

This normalization seems to be natural. However, it has been seriously criticized by Zadeh[5] and Wu et al[6] since the method discards dissonant of information and renormalizes only a consonant part of the knowledge when two opinions are strongly conflict. In order to solve the problem, a new interpretation associated with the normalization factor in DST has been proposed[7]. The study has proposed that the normalization factor should be interpreted as a contradiction factor since the more conflicting informations are given, the larger K is. If there is no conflict, K=0. If K=1, the bodies of evidence represented by $m_1(X)$ and $m_2(Y)$ are totally contradictory and their combination is not defined. This case yields a total ignorance.

In order to apply this concept to possibility distributions, the calculational method associated with the value of K should be developed. It is desirable to develop a calculational method without computing the mass functions derived from possibility distributions described in the previous section. Fua[3] has developed the direct way to calculate the value K given two possibility distributions. In this case, the amount of agreement, i. e., A=1−K, can be calculated as follows :

$$A = 1 - K = poss_2(x_1) + \int_{x_1}^{x_2} poss_2'(x)\, poss_1(x)\, dx \qquad (5)$$

where poss′(x) is a derivative of possibility distribution with respect to its argument and, x_1 and x_2 are the peaks, i. e., modal values of each possibility distribution.

The value of A means the amount of agreement between two possibility distributions. The value A can be used for an indicator about the validity of combined result. In other words, it represent a reliability of result. As the value increases, result is more reliable since two experts reach a considerable agreement about the issue that they deal with. When the value A is equal to one, two experts agree completely. On the other hand, a low value means a considerable disagreement between experts. Reliability here refers to the information content of message. Hence, if a message about the truth being contained in a set contains very little information, then it is unreliable and would receive a relatively low value. In a sense, this is a Shannon's definition of uncertainty, where the message is a communication, and information is that which reduces uncertainty. Therefore, the value A could be a good criterion about how much agreement between two experts exists when they consider a problem. Also, a decision-maker could estimate a reliability of a result following the combination of expert opinion.

Assessment of Credibility

Suppose we had a prior opinion about the credibility of experts. Expert opinion with small credibility means that the opinion is less reliable. Such opinion should be handled carefully in the combination process. In order to incorporate the credibility of expert opinion in a manner with the DST, this analysis uses the following approach :

Let w_i denote a weighting factor regarding the credibility of an expert i. Then, all belief functions are reduced by a factor w_i, and the amount of bbm lost by this process is reallocated to [0,1], which corresponds to the amount of belief allocated to none of any elements, i. e., m(Ω). The meaning of such an allocation can be understood that the portion $1-w_i$ of expert opinion represents the amount of knowledge he or she cannot judge, (i. e., we assume that an expert i does not know the real value). This process is called discounting expert opinion. In the framework of a fuzzy probability, the portion of $1-w_i$ is allocated to [0,1] and the shape of fuzzy probability is changed.

Examples

Suppose that we combine the following two TFPs (Triangular Fuzzy Probabilities) :

$$\tilde{P}_1(x) = [0.2,\ 0.4,\ 0.6]$$

$$\tilde{P}_2(x) = [0.3,\ 0.5,\ 0.7] \qquad (6)$$

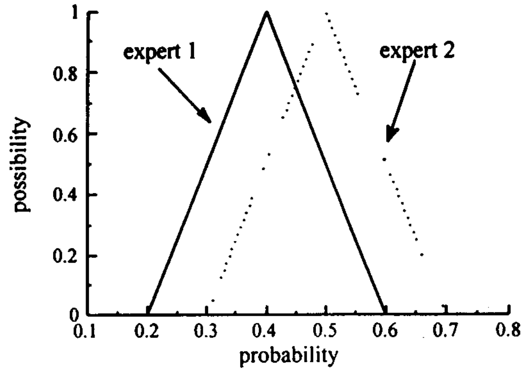

Fig. 3. Two TFPs in the Example

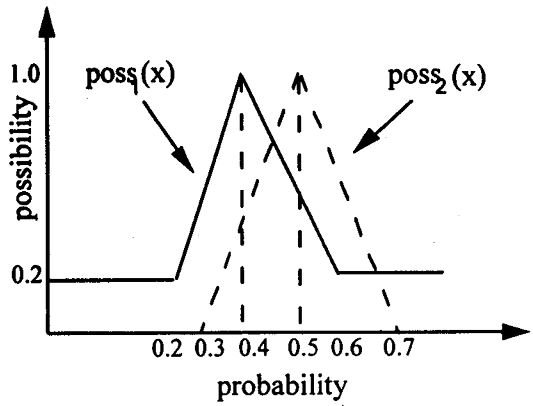

Fig. 5. Two Fuzzy Probabilities for Discounting Expert Opinion

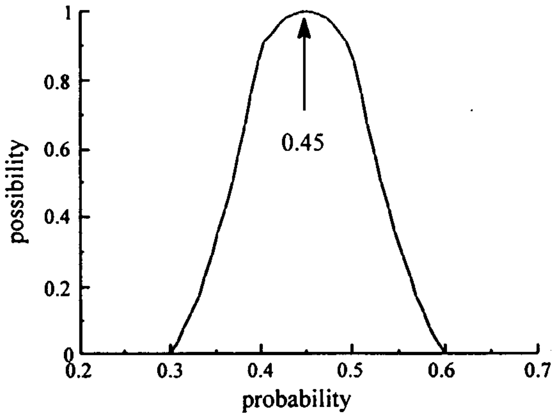

Fig. 4. A Combined Result from Two Equally-Treated Opinions

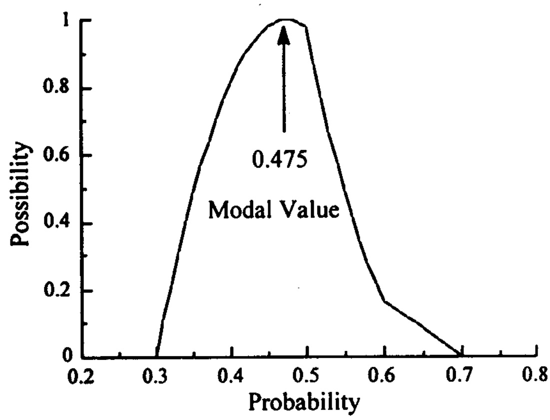

Fig. 6. Result of the Example for Non-Equally Treated Opinions

The TFPs given by Eq. (6) are depicted in Figure 3. The proposed method results in a modal value as 0.45 and the agreement between two experts, i. e., the value A, as 0.875 for the case of equally treated opinions. Figure 4 depicts the combined results in terms of a possibility distribution.

Consider the problem of non-equally treated opinions. We have a prior opinion regarding the credibility of two experts in the first example. Assume that the second expert is fully reliable, but the first expert is only 80% reliable. Hence, we assign different weighting factors to the opinions. (i. e., $w_1=0.8$ and $w_2=1.0$). In this case, the shape of the first fuzzy probability is changed and the new fuzzy probabilities are depicted in Figure 5. Figure 6 shows results obtained after a combination. The value A is computed as 0.9, and a modal value of the result is obtained as $x=0.475$. It shows that the result reflects the second expert's opinion more than that of the first case (i. e., the modal value of combined result shifts to an upper-value region) and the value A also increases. It is a natural conclusion when we combine two opinion for the example of discounting expert opinion.

Another example shows the effect of value A. Let's consider the following two TFPs :

$$\tilde{P}_1(x)=[0.2,\ \ 0.4,\ \ 0.6]$$

$$\widetilde{P}_2(x)=[0.5, \quad 0.7, \quad 0.9] \tag{7}$$

Applying the proposed method results in 0.55 as a modal value. However, the value A between two experts is calculated as 0.375 and seems to be low. It may show that there exists little agreement between two experts regarding the problem they deal with. In this case, we would think about reliability of the result obtained since there exists a considerable conflict between two experts.

The main problem in the case of conflicting information is how to interpret the conflict. There may be at least two situations which may explain why the conflict occurs :

a) One of the sources of knowledge is not reliable.
b) The sources are not addressing the same issue (inconsistency).

In the first case, we could solve the problem by assigning credibility weighting to each expert's opinion. The second problem may occur simply because it is meaningless to combine the information they provide. Therefore, it is necessary to know whether there exists a significant conflict between knowledge sources before using combined information. In the process of combining expert opinion, reliability of obtained result should be measured since expert opinion may be usually obtained under incomplete knowledge and limited data. Hence, we must consider a sound way of combining expert opinion and a validity of obtained result as well. From this point of view, the proposed method could be a promising since this provides a measure which indicates a reliability of the obtained result.

2.3. Propagation

The 'Extension Principle' can be used to propagate fuzzy probabilities through a system[2]. However, the implementation of the solution procedure is not trivial using this approach. The reason is that the solution procedure corresponds to a nonlinear programming problem which is very complex, except for the simplest mapping functions. A simple approach is to use the discretization technique on the variable domain. However, this technique would fail and lead to irregular and fuzzier results because the min-max operation on fuzzy sets can lead to irregular membership functions. Hence, in the present study, the calculational procedure is implemented by the method proposed by Dong and Wong[8]. This algorithm is based on the discretization technique on the possibility measure or membership value domain, instead of on the variable domain, and an interval analysis. This method provides a discrete but exact solution in a very efficient and simple manner.

3. The Selected Case Study

One of the most uncertain but important probabilities associated with evaluation of the BWR drywell flooding strategy is the conditional probability of vessel failure given the presence of water surrounding the vessel lower head. While the probability is expected to be lower than the dry case, it is very hard to estimate the probability corresponding to the issue since the physical processes are extremely complex and difficult to model. Major uncertainty is associated with the behavior of molten debris, i. e., the interaction with the bottom head of the vessel, the heat removal process through the vessel wall to the water in the pool, and the thermal stress due to a large temperature gradient across the vessel. Expert opinion can be used to the assessment of the probability associated with this kind of complicated physical phenomenon.

The objective of the section is to illustrate the handling of uncertain value regarding the selected issue using the proposed methodology.

3.1. The Experts Rationale

Experts should demonstrate experience through publications, hands-on experience, and managing research in the area related to issue selected. Also, the expert should be willing to have his or her opinion elicited with method to be used. The method employed in the study is designed to obtain subjective estimates of the unknown physical probability.

One expert, Expert A, was chosen in the field of heat transfer. He is a leading researcher in the heat transfer analysis field. He has also performed stress analyses for the reactor vessel and has authored or co-authored more than 4 papers on the behavior of a molten core on the vessel bottom head in PWRs and BWRs. Expert A pointed out that the assessment of the probability is very difficult because there is limited knowledge regarding the behavior of a molten core on the reactor bottom head and the heat transfer process in the presence of water outside. Also, he mentioned that there is a possibility of penetration failure due to the interaction of the molten core and the guide tubes. He based his conclusions on the analysis of stress due to thermal gradients across the reactor vessel without considering the interaction of the molten core and the guide tubes. In other words, he only considered global failure due to thermal stresss along the reactor vessel shell. He concluded that the probability of BWR vessel failure is smaller that of a PWR in the flooded case. Hence, he arrived at the probability of 0.5 as a best-estimate value and 0.1 and 0.6 as two extremes for the issue.

In order to obtain another expert's opinion for the issue, we selected a second expert, Expert B. He holds a Ph. D degree in the field of heat and mass transfer. His research has been focused mostly on the thermal behavior of the reactor vessel for the case of flooding cavity. He has also performed a stress analysis of the rector vessel lower head for both a BWR and a PWR. Expert B pointed out that the probability of BWR vessel failure under this circumstance is very difficult to estimate because there exists limited knowledge regarding the motion of the molten core, the heat removal process through the vessel wall, and the thermal stress due to a large temperature gradient across the vessel. However, based on his thermal and stress analyses he concluded that BWR vessel failure under this circumstance might be possible. Hence, he provided his best-estimate probability as 0.4. However, due to uncertainty in his judgement, he placed the lower bound value at 0.2 and upper bound value at 0.6.

Based on the two opinions from Expert A and B, we combine the following two TFPs for the issue:

Expert A: P(vessel failure for flooded case) = [0.1, 0.5, 0.6]

Expert B: P(vessel failure for flooded case) = [0.2, 0.4, 0.6]

The two TFPs for this issue are depicted in Figure 7.

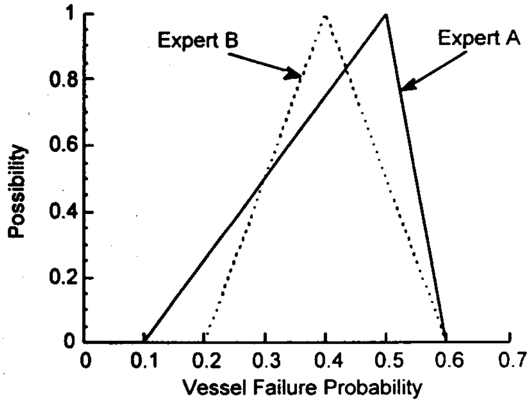

Fig. 7. Two Triangular Fuzzy Probabilities from Experts for the BWR Vessel Failure Issue

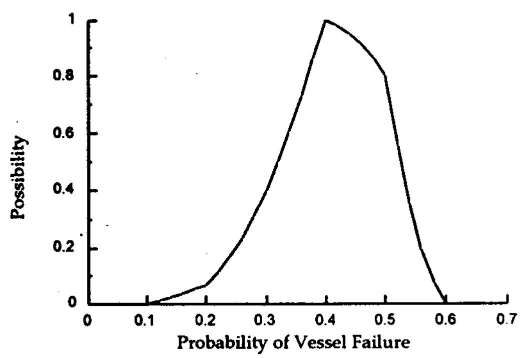

Fig. 8. Result of Combining Two Fuzzy Probabilities for the BWR Vessel Failure Issue

3.2. Assessment of Credibility

We had a prior opinion regarding the credibility of the two experts. That is, the opinion of Expert A is more credible than that of B, since Expert A has more working experience in the field. However, assigning credibility weighting factors in a real situation is a complicated subject because we need to consider each expert's biases and dependencies. This aspect of expert elicitation is beyond the scope of this study. More details regarding this issue can be found in Chhibber[9].

This analysis assumes the following weighting factors for combining two TFPs obtained from the selected experts :

$w_A = 1.0$ and $w_B = 0.8$

3.3. Results

The combined result using the weighting factors above is shown in Figure 8. The value of agreement, A, can be calculated as 0.95 in this case. This shows that there exists a considerable agreement between the two experts regarding the issue, i. e., the BWR vessel failure probability in the flooded case. A modal value is obtained as 0.4. The result for the modal value seems to be strange since we think that a more reliable opinion obtained from expert A (i. e., a modal value as 0.5) should lead to a modal value with a result close to 0.5. However, the result is natural. The reason is that expert A puts more emphasis on the lower value region (from 0.1 to 0.5), and combining it and the opinion from Expert B which also reflects a similar pattern results in locating the modal value (a most possible value) of the results at 0.4.

4. Conclusions

The treatment of expert opinion has became an important issue in risk assessment. It involves three basic problems : how to find a sound representation for expert opinion, how to combine these opinions, and how to propagate them. The fact that expert opinion elicitation process involves an uncertainty to be explicitly evaluated needs to distinguish between two major types of uncertainty : uncertainty due to stochastic variability and uncertainty due to lack of knowledge. The second type of uncertainty is quite different from the stochastic variability. The one involves a case when the end point is a fixed but unknown value due to the imprecision in the analysists knowledge about models, their parameters, and/or their predictions. It is vagueness or imprecision in an analysis, or stated value. In this case, the subjective confidence interval can be used for the unknown value. The distribution used represents a range of "degrees of belief" that the true but unknown value is equal to or less than any value selected from the distribution. Eventually as we gain more information and more knowledge, the uncertainty would decrease or would not exist.

The present study has presented an expert opinion elicitation process to handle the uncertainty due to lack of knowledge under fuzzy set theory. While the conventional approaches of point-valued probabilities may be used, the use of fuzzy probability can adequately reflect the imprecision of an expert's judgmental uncertainty under incomplete knowledge and limited data. The study has also demonstrated a method of combining fuzzy probabilities in a manner consistent with the Dempster-Shafer's Theory (DST). The presented method has provided a simple and sound way to combine fuzzy probabilities and a measure which indicates the validity of the result obtained as well. Finally, the proposed method has been applied to obtain the probability of a vessel failure for the flooded case in the Peach Bottom BWR nuclear power plant.

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