A Comparison of Flow Efficiency between To/From Ratio Method and Minimal Backward-Flow Model in a Linear Machine Layout

-선형기계배치에서 To/From 방법과 최소역흐름 모형의 효율비교-

Won, You-Dong*
원 유 동

요 지

제조셀에서의 기계배치 문제는 매우 중요한 문제 중의 하나이다. 제조셀에서의 기계배치 형태는 크게 선형배치 방법과 네트워크형배치 방법의 두 가지가 있다. 본 논문에서는 선형배치 방법에 대하여 검토한다. 선형배치를 위한 가장 일반적인 방법은 to/from 비율에 의한 방법이다. 본 논문에서는 최소역흐름모형을 제시하여 to/from 비율에 의한 방법과 비교한다. 비교 결과 최소역흐름모형에 의한 선형기계배치의 경우가 to/from 비율에 의한 방법보다 총역흐름 이동거리를 상당히 감소시키는 것으로 나타났다.

1. Introduction

Cellular manufacturing system was designed to cope with multi-product, small-lot-sized production. When a cell has been created(by whatever method) for a family of parts, the further problem of arranging machines within the cell remains. There are two basic types of machine layout: linear oriented layout and network oriented layout. [Figure 1] is an illustrative example of two types.(Lee, 1991)

Although modern material handling equipment is intelligent and flexible enough for us to have a more complex layout, the linear layout is still a popular design choice. Linear layouts exist in various forms such as a U-shape, a straight line, and a circular loop. Although a linear layout may not be the optimal design choice, the desire for simplicity in the resultant layout and material handling problems makes it a popular choice.(Burbidge, 1991)

In a linear layout, four types of flow movements can be observed shown in [Figure 2] (Aneke and Carrie, 1986): repeat, in-sequence, bypassing and backward-flow. Among these flow movements, in-sequence movement is the most desirable because of its unidirectional movement. Backward-flow is the least desirable since it complicates flow the most. Therefore, the main goal of flow analysis is to minimize the total amount of backward-flow movements.

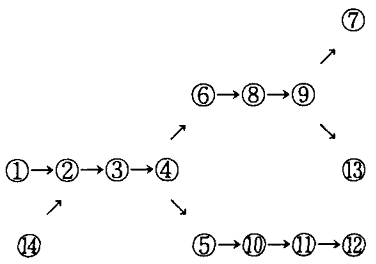
* 경남대학교 경상대학 경영학부

(a)

Product	Demand quantity	Operation sequence
A	20	2-3-4-6-8-9-7
B	10	14-2-3-4-5-10-11-12
C	15	2-4-6-8-9-13
D	10	1-2-3-5-11-12

(b) ①→⑭→②→③→④→⑥→⑧→⑨→⑬→⑦→⑤→⑩→⑪→⑫

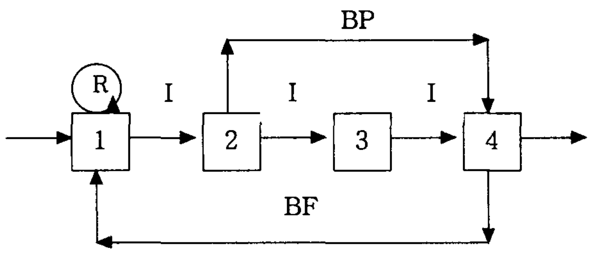
(c)



[Figure 1] (a) Product information for the illustrative example
(b) A possible linear oriented layout
(c) A possible network oriented layout

Although modern material handling equipment is intelligent and flexible enough for us to have a more complex layout, the linear layout is still a popular design choice. Linear layouts exist in various forms such as a U-shape, a straight line, and a circular loop. Although a linear layout may not be the optimal design choice, the desire for simplicity in the resultant layout and material handling problems makes it a popular choice.(Burbidge, 1991)

In a linear layout, four types of flow movements can be observed shown in Figure 2 (Aneke and Carrie, 1986): repeat, in-sequence, bypassing and backward-flow. Among these flow movements, in-sequence movement is the most desirable because of its unidirectional movement. Backward-flow is the least desirable since it complicates flow the most. Therefore, the main goal of flow analysis is to minimize the total amount of backward-flow movements.



[Figure 2] Four different flow movements within a linear layout. R=repeat; I=in sequence; BP=bypassing; BF=backward-flow.

2. Review of Linear Layout Analysis

Several methods for linear machine layout analysis have been proposed. Most of the previous methods are to allow duplicate machines in a flow line. This type of problem was first sutdied by Carrie(1995). Recently, Lee(1991) proposed a method that builds the flow line based on the sequence similarities of products. However, the solution of those methods depends on personal experience and skill.

The most common method for linear machine layout analysis in real situation is the to/from ratio method utilzing to/from ratios between machines. The to/from ratio method is a heuristic method, but it is very easy to implement. The to/from ratio method follows the following steps:

(1) Construct the from-to chart between machines.
(2) Calculate the to/from ratio for all machines.
(3) Arrange the machines in order based on increasing to/from ratio.

In this paper, a linear programming model, minimal backward-flow model, is presented to minimize the total amount of backward-flows, and then, the efficiency of minimal backward-flow model is compared with the to/from ratio method.

3. Minimal Backward-Flow Model

3.1 Model Description

The basic principle of determining the sequence of machines is to minimize the total amount of backward-flows in a manufacturing cell. In this paper, a linear programming model, minimal backward-flow model, is proposed. In the minimal backward-flow model, the following assumptions are adopted:

(1) Only one machine of any type is allowed in a flow line.
(2) The cost of material flows is proportional to the number of parts and the distance of flows.
(3) All machines are regarded as a point, and the distance between adjacent machines is '1', the unit distance.

The distance between the initial input point of parts, vertical reference line(vrl), and the first machine in sequence is also regarded as the unit distance, '1'. In the minimal backward-flow model, the following notations and definitions will be used throughout the paper.

n = the number of machines in a manufacturing cell
m = the number of items of parts to be produced in a manufacturing cell
d_j = the demand of part j, $j=1,2,\ldots\ldots, m$.
x_i = the distance between vrl and machine i, $i=1,2,\ldots\ldots, n$.

The variable x_i is the decision variable and its actual meaning would be the location of machine i in a machine sequence. Then, the minimal backward-flow model utilizing linear programming is stated as follows:

$$\min \sum_{h=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{m} d_j \mid x_h - x_i \mid, \quad h=1,2,\ldots\ldots,n. \quad i=1,2,\ldots\ldots,n. \quad j=1,2,\ldots\ldots,m. \qquad (1)$$

st.

$$\mid x_h - x_i \mid \geq 1, \quad h=1,2,\ldots\ldots,n. \quad i=1,2,\ldots\ldots,n. \quad h \neq i. \qquad (2)$$

$$x_i = 1,2,\ldots\ldots,n. \qquad (3)$$

In the above model, equation (1) is the objective function, which means the total travel distance of all the parts to be produced in a manufacturing cell. Constraint (2) ensures that no two machines in the layout overlap. Equation (3) is decision variables. In the objective function, the total travel distance is the sum of forward-flow distances and backward-flow distances in a machine sequence. Since the forward-flow distances in a machine sequence doses not affect material flows in a manufacturing cell, the material flows are best simplified when the sum of backward-flow distances of all the parts in a manufacturing cell is minimized. Therefore, the objective function should be mathematically transformed in order to minimize the sum of backward-flow distances of all the parts in a manufacturing cell. This is the reason why the name of the model presented in this paper is the minimal backward-flow model.

3.2 Solution Procedures

The minimal backward-flow model is not a standard form of linear programming. Therefore, it should be technically transformed into a standard form of linear programming. Let f_{hi} be a weighted frequency considering demand d_j, and define

$$z_{hi} = max(x_h - x_i, o) \qquad (4)$$

Then, the equivalent zero-one integer programming of minimal backward-flow model is restated as follows:

$$\min \sum_{h=1}^{n}\sum_{i=1}^{n} f_{hi} z_{hi} \qquad (5)$$

st.

$$x_h - x_i \geq 1 - M\omega_{hi}, \quad h=1,2,\ldots\ldots,n. \quad i=h+1,h+2,\ldots\ldots,n, h\neq i. \qquad (6)$$

$$-x_h + x_i \geq 1 - M(1-\omega_{hi}), \quad h=1,2,\ldots\ldots,n. \quad i=h+1,h+2,\ldots\ldots,n. h\neq i. \qquad (7)$$

$$z_{hi} \geq x_h - x_i, \quad h=1,2,\ldots\ldots,n. \quad i=h+1,h+2,\ldots\ldots,n. \qquad (8)$$

$$x_i = \sum_{j=1}^{n} j \cdot y_{ij}, \quad i=1,2,\ldots\ldots,n. \tag{9}$$

$$\sum_{j=1}^{n} y_{ij} = 1, \quad i=1,2,\ldots\ldots,n. \tag{10}$$

$$x_i, z_{hi} \geq 0, \quad \omega_{hi}, y_{ij} = 0 \text{ or } 1 \tag{11}$$

As a result, if the number of machines in a manufacturing cell is *n*, the number of variables and constraints would be $n(5n-1)/2$ and $2n^2$, respectively.

In order to solve the minimal backward-flow model, a new computer program with PASCAL utilizing LINDO package was drawn up. The program requires only the demands and the process route numbers of all the parts to be produced in a manufacturing cell, and then, the computer program provides the optimal linear machine sequence. The program can handle up to 40 machines and 400 items of parts. Since it is recommended that the number of machines in a manufacturing cell be within 20 machines(Logendran, 1990), the program is sufficient to find out the optimal linear machine sequence in real situations.

4. Comparison of Flow Efficiency

4.1 Numerical Example

Suppose we have five parts, P1, P2,....., and P5, to be completely processed with four machines, machine 1, 2, 3, and 4. The demands and the process route numbers of each part are given in [Table 1]. In this paper, two solutions by the to/from ratio method and the minimal backward-flow model are compared.

[Table 1] Data for Each Part

Part	Process	Demand
P1	1→3→4	10
P2	2→1→2→1→2	20
P3	3→4→1	15
P4	3→4→2→1	35
P5	3→2→4→2→1	25

4.2 To/From Ratio Method

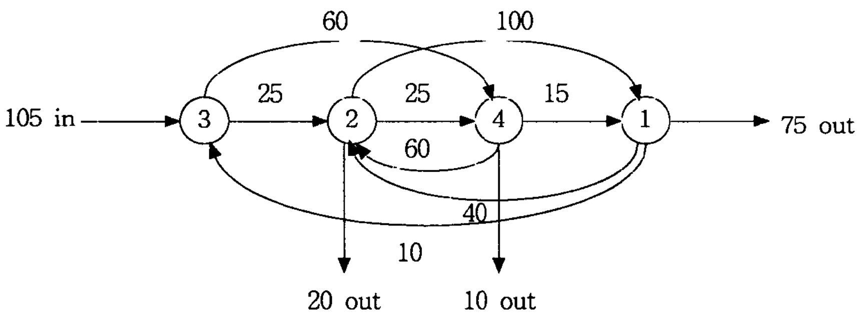
At first, in order to find the solution by the to/from ratio method, a from-to chart between machines should be completed. From the [Table 1], a from-to chart is given in [Table 2]. From the [Table 2], to/from ratios are calculated as shown in [Table 3]. As a result, the optimal machine sequence is obtained: machine 3 - machine 2 - machine 4 - machine 1. [Figure 3] shows the materials flow pattern by the to/from ratio method. The total backward-flow distance is 170.

[Table 2] From-To Chart

from \ to	1	2	3	4	"From" Sums
1	0	40	10	0	50
2	100	0	0	25	125
3	0	25	0	60	85
4	15	60	0	0	75
"To" Sums	115	125	10	85	

[Table 3] To/From Ratio

Machine	To	From	To/From Ratio
1	115	50	2.30
2	125	125	1.00
3	10	85	0.12
4	85	75	1.13



Total Backward-Flow Distance = $60 \times 1 + 40 \times 2 + 10 \times 3$ = 170

[Figure 3] Materials Flow Pattern by the To/From Ratio Method

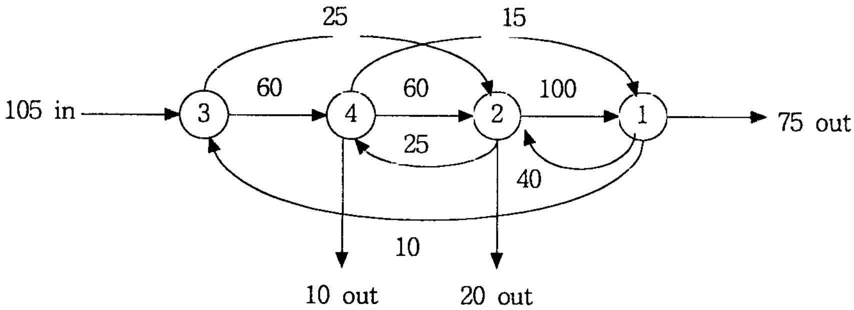
4.3 Minimal Backward-Flow Model

The new computer program is used to get an optimal solution by minimal backward-flow model. [Table 4] shows the computer output for example problem. According to [Table 4], the optimal machine sequence is machine 3, machine 4, machine 2, and machine 1. The result is slightly different from the solution by the to/from ratio method. [Figure 4] shows the materials flow pattern by the minimal backward-flow model. The total backward-flow distance is 95.

[Table 4] Computer Output for Example Problem

	95.000000	1.0000000	F	0.10000000E+31
W1_2	0.00000000E+00	0.00000000E+00	I	1.0000000
W1_3	0.00000000E+00	0.00000000E+00	I	1.0000000
W1_4	0.00000000E+00	0.00000000E+00	I	1.0000000
W2_3	0.00000000E+00	0.00000000E+00	I	1.0000000
W2_4	0.00000000E+00	0.00000000E+00	I	1.0000000
W3_4	1.0000000	0.00000000E+00	I	1.0000000
Y1_1	0.00000000E+00	50.000000	I	1.0000000
Y1_2	0.00000000E+00	100.00000	I	1.0000000
Y1_3	0.00000000E+00	150.00000	I	1.0000000
Y1_4	1.0000000	200.00000	I	1.0000000
Y2_1	0.00000000E+00	-15.000000	I	1.0000000
Y2_2	0.00000000E+00	-30.000000	I	1.0000000
Y2_3	1.0000000	-45.000000	I	1.0000000
Y2_4	0.00000000E+00	-60.000000	I	1.0000000
Y3_1	1.0000000	-10.000000	I	1.0000000
Y3_2	0.00000000E+00	-20.000000	I	1.0000000
Y3_3	0.00000000E+00	-30.000000	I	1.0000000
Y3_4	0.00000000E+00	-40.000000	I	1.0000000
Y4_1	0.00000000E+00	-25.000000	I	1.0000000
Y4_2	1.0000000	-50.000000	I	1.0000000
Y4_3	0.00000000E+00	-75.000000	I	1.0000000
Y4_4	0.00000000E+00	-100.00000	I	1.0000000
X1	4.0000000	0.00000000E+00	C	0.10000000E+31
X2	3.0000000	0.00000000E+00	C	0.10000000E+31
X3	1.0000000	0.00000000E+00	C	0.10000000E+31
X4	2.0000000	0.00000000E+00	C	0.10000000E+31
Z1_2	1.0000000	0.00000000E+00	C	0.10000000E+31
Z1_3	3.0000000	0.00000000E+00	C	0.10000000E+31
Z1_4	2.0000000	0.00000000E+00	C	0.10000000E+31
Z2_1	0.00000000E+00	100.00000	C	0.10000000E+31
Z2_3	2.0000000	0.00000000E+00	C	0.10000000E+31
Z2_4	1.0000000	0.00000000E+00	C	0.10000000E+31
Z3_1	0.00000000E+00	0.00000000E+00	C	0.10000000E+31
Z3_2	0.00000000E+00	25.000000	C	0.10000000E+31
Z3_4	0.00000000E+00	60.000000	C	0.10000000E+31
Z4_1	0.00000000E+00	15.000000	C	0.10000000E+31
Z4_2	0.00000000E+00	60.000000	C	0.10000000E+31
Z4_3	1.0000000	0.00000000E+00	C	0.10000000E+31

From X1=4, X2=3, X3=1, X4=2, the optimal machine sequence is 3-4-2-1.



Total Backward-Flow Distance = $25\times1+40\times1+10\times3$ = 95

[Figure 4] Materials Flow Pattern by the Minimal Backward-Flow Model

4.4 Discussion

The minimal backward-flow model stresses the machine sequence only so that the total backward-flow distance was substantially reduced. However, In the minimal backward-flow model, the number of variables and constraints are rapidly increased. Although this fact is the most weak point of the minimal backward-flow model, it is not a big problem to implement the minimal backward-flow model in practice. Since the usual number of machines in a manufacturing cell in real situation is 5 to 10, the optimal solution can be obtained by personal computers.

5. Conclusion

In this paper, the materials flow efficiency of minimal backward-flow model was compared with the to/from ratio method. The total backward-flow distance by the minimal backward-flow model is substantially less than the case of to/from ratio method.

There are two major advantages of minimal backward-flow model. First, the minimal backward-flow model does not need additional costs and efforts because it uses universally available data in a factory. Second, because of the simplification of the model, it is easy to implement. Among the assumptions adopted in the minimal backward-flow model, the first assumption of the model is still debated because an alternative process route or machine is a usual case in reality. In order to solve this problem, fuzzy set theory should be adopted.

References

[1] Aneke, N.A.G. and Carrie, A.S., "A Design Technique for the Layout of Multi-product Flow Lines," International Journal of Production Research, Vol.24, pp471-481, 1986,

[2] Burbidge, J.L., "Production Flow Analysis for Planning Group Technology", Journal of Operations Management, Vol.10, pp5-27, 1991.

[3] Carrie, A.S., "Layout of Multi-product Lines", International Journal of Production Research, Vol.13, pp541-557, 1975.

[4] Chow, W.S. and Hawaleshka, O., "An Efficient Algorithm for Solving Machine Chaining Problem in Cellular Manufacturing", Computers in Industrial Engineering, Vol. 22(1), pp. 95-100, 1992

[5] Lee, C.E.C., An Integrated Methodology for the Analysis and Design of Cellular Flexible Assembly Systems, Ph.D. Dissertation, Purdue University, West Lafayette, IN, U.S.A., 1991.

[6] Logendran, R., "A Workload Based Model for Minimizing Total Intercell and Intracell Moves in Cellular Manufacturing", International Journal of Production Research, Vol.28, pp 913-925, 1990.

[7] Salkin, H.M., Integer Programming, Addison-Wesley, pp. 210-231, 1975.

[8] Tanchoco, J.M.A., Material Flow Systems in Manufacturing, Chapman & Hall, pp. 84-86, 1994.

[9] Won, Y.K., Machine-Part Grouping in Group Technology Manufacturing Systems, Ph.D. Dissertation, Korea Advanced Institute of Science and Technology, 1994.