Qualitative Evaluation of Quality with Hierarchical Structure Using Fuzzy Inference

- 퍼지추론에 의한 계층구조를 가진 품질의 정성적 평가-

Kim, Jeong Man*

김 정 만

요 지

제품의 정성적 품질평가에서, 제품의 최종품질을 구성하는 다수의 특성에 대한 만족도가 언어로써 표현되어 소비자의 구매행동이란 의사결정으로 표출되는데, 이러한 주관적 평가에는 평가의 애매함(fuzziness)이 수반되므로 품질의 평가구조를 합리적으로 파악하기 위해서는 애매함의 존재를 고려에 넣지 않으면 안된다. 다수의 품질특성이 계층적(hierarchical)인 구조로 연결되어 최상위 품질특성으로 구성되며, 특성간의 중요도(relative importances)가 계층별로 결정되는 경우, 이들 개개의 특성에 대한 만족도의 평가로부터 어떤 구조적인 관계를 통해 그 제품에 대한 종합평가가 이루어지나, 개개의 특성에 대한 평가가 애매한 이상 최종 결과인 종합적 만족도도 애매한 것으로 된다. 즉, 평가모델의 구조도 평가의 패턴도 퍼지화되므로 이러한 평가에서 퍼지이론의 응용에 따른 효과를 가장 크게 기대할 수 있는 퍼지추론모델을 이용하여 계층간, 품질특성간의 퍼지관계와 특성의 중요도 및 언어변수(linguistic variables)의 형태로 주어지는 입력정보로써 품질구조를 명확히 하고, 패턴인식(pattern recognition)의 개념을 이용하여 평가자의 제품에 대한 평가결과를 언어로써 표현한다.

1. Introduction

In the quality evaluation of product with qualitative characteristics, there are two major problems, one is ambiguity that is followed by use of qualitative and subjective expression of language and the other is uncertainty that is followed by relation of elements involved in the object of quality evaluation. Generally, because there is a close correlation between the degree of satisfaction of product and the decision making of purchasing behavior in the quality evaluation of consumer, and the result of quality evaluation of consumer can be expressed as linguistic variables involving fuzziness of the degree of satisfaction, we have to consider the existence of fuzziness in order to understand the desire of consumer for purchasing products.

In the case that many elements of quality with qualitative characteristics are connected to the highest rank of characteristic of quality with hierarchical structure and the relative importance among these elements are decided hierarchically, the overall evaluation is

* Department of Industrial Engineering, Kyungil University

perfomed from the degree of satisfaction of each element through some structural relations, but as the result of evaluation of each element is fuzzy as that of overall evaluation is fuzzy too. That is, because the structures of evaluation pattern are fuzzified, in this paper, we evaluate the overall structure of quality of product by means of fuzzy relation among elements and the relative importances among elements and input informations that are given as the form of linguistic variables using the fuzzy inference model in order to maximize the effectiveness followed by application of fuzzy theory. We can take two methods of fuzzy theory that are used in overall evaluation of which items are connected with hierarchical structure, one is HFI(hierarchical fuzzy integral) suggested by Buckley and Ishizuka the other is the method suggested by Shiraishi[2][3][8]. But, it is said that HFI can not be easily applied to the pattern evaluation of similarity because HFI deal with only the evaluation on the assumption of λ-fuzzy measure[11]. And, in the method of Shiraishi, the input information of items has to be given in every rank because the weight of items is not determined hierarchically[8]. Accordingly, in this paper, we deal with the structure of fuzzy model that the weight of items is determined hierarchically and input information is given to only the lowest rank in order to evaluate the similarity between the predetermined pattern and the result of evaluation.

2. Fuzzy Evaluation Model

When we interprete sensory evaluation in a narrow sense, the object of that is appearence such as shape and color of product, but, in a broad sense, various kinds of characteristics of quality such as convenience of use and importance of additional function are involved in the object of evaluation[4]. Therefore, when we interprete that quality evaluation in a narrow sense is depended upon that of a broad one, it is said that various kinds of elements are included in the object of overall quality evaluation. As the result of quality evaluation of consumer is expressed with linguistic variables implying the degree of satisfaction in which fuzziness is implicated, usually, the overall degree of satisfaction gained from the final result of evaluation becomes fuzzy too. Accordingly, when the structure of evaluation extending from each characteristics to overall evaluation is formed hierarchically, we define the relation of elements gained from relative importances(weights) hierarchically as fuzzy structure, and we evaluate this hierarchical structure of quality.

There are several difficult problems that can not be solved with empirical fuzzy rules and usual equations using binary logics. To solve these problems, the application of model applying fuzzy theory would be useful. Many fuzzy models such as fuzzy inference model, fuzzy integral and fuzzy linear additive model are applicable for the modeling of structure of quality evaluation. In fuzzy integral model, the object of evaluation is divided into several items, and the evaluation is carried out using the evaluators and weights of each item. But, fuzzy integral model is not appropriate to construct models that have to reflects the evaluators and weights on each quality characteristic as fuzzy thing because it represents the evaluators and weights in ordinary numbers[11]. And, as in the fuzzy linear additive model, ordinary number is replaced with fuzzy number, if we represent the evaluators of multiple items in fuzzy number, also the overall evaluator is obtained as fuzzy number according to the addition of weight. In the above facts, we know that these two models are inappropriate to deal with various kinds of fuzziness on evaluation of product, because only the structure of model in fuzzy integral model, and only the pattern of evaluation in fuzzy linear additive model is fuzzified. On the contrary, in fuzzy inference model, we can expect the maximum effectiveness of application of fuzzy theory on sensory evaluation because the structure of model and the pattern of evaluation are fuzzified all together.

3. Structure of Fuzzy Inference Model

When the multiple items of evaluation with qualitative characteristics are connected to overall evaluation according to hierarchical structure and each item is evaluated by linguistic variables that express the degree of satisfaction such as 「good」 and 「satisfactory」, fuzzy inference model regularizes the relation of parameters with 「implication→conclusion」 such as the form of 「If x is A then y is B」 and these rules are replaced with fuzzy relation.

Thus, if A' of a new implication is established then B' of new conclusion is derived as the shape of fuzzy set. Now, if we let two kinds of parameter G(the degree of goodness of items) and S(the degree of satisfaction of items) and establish multiple rules on these parameters, then we can get a matrix $\boldsymbol{A}$ of fuzzy relation on event x and y using Cartesian product as below.

$$\boldsymbol{Rxy} = G \times S \quad (\ \times : \text{Cartesian product in fuzzy operation})$$

Also, if there are multiple relations between x and y then we can obtain the equation as below.

$$\boldsymbol{Rxy} = (\ G \times S) \cup (\ G' \times S') \cup \cdots$$

Actually, these fuzzy relations are established from a questionaire made by the usage of linguistic variables for experts. In case that an event is related to multiple events of a lower rank, it is necessary to give consideration to informations transmitted from these events.

For the above reason, we use the paired-comparison method suggested by Saaty and give the weights to fuzzy relation $\boldsymbol{Rxy}$[1][7][9].

If we let w_k, w_l denote the evalator of certain objects k and l(k, $l=1, 2, \cdots, n$: n is the number of elements), then the weights e_{kl} between element k and element l is

$$e_{kl} = w_k / w_l \quad (k,\ l = 1,\ 2, \cdots, n\ ,\ k \neq l)$$

and then, we can get matrix $\boldsymbol{E}$ of each element j having element e_{kl}.

$$\boldsymbol{E} = \begin{pmatrix} 1 & e_{12} & e_{13} & \cdots & e_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & \cdots & 1 \end{pmatrix} \qquad (1)$$

But, there are several inconsistencies in these relations under certain circumstances because the elements of matrix $\boldsymbol{E}$ are obtained from only the result of paired-comparison. In order to preserve the consistency of comparison, we consider the valid weight w_i according to eigenvalue approach with following equation.

$$\boldsymbol{Ew} = n\boldsymbol{w}, \quad \boldsymbol{w} = (w_1,\ w_2, \cdots, w_n)$$

But, in this paper, these weights are calculated in every rank, and fuzzy relation $\boldsymbol{R}_i$ of each item i is got by multiplying the weight of each element by fuzzy relation $\boldsymbol{R}$, and we evaluate the hierarchical quality structure of product by the fuzzy inference using fuzzy relations gained from the above operation.

For example, if the informations that are followed by linguistic variables such as I_1, I_2 are put in item ①, ② and a hierarchical structure among multiple items is given as Figure 1, then the information transmitted to item A is calculated by equation (2) considering the weight w_1 and w_2 .

$$I_A=\{(\boldsymbol{R}_{1A}\wedge w_1)\circ I_1\}\vee\{(\boldsymbol{R}_{2A}\wedge w_2)\circ I_2\} \qquad (2)$$

Also, the information of item B is

$$I_B=\{(\boldsymbol{R}_{3B}\wedge w_3)\circ I_3\}\vee\{(\boldsymbol{R}_{4B}\wedge w_4)\circ I_4\}\vee\{(\boldsymbol{R}_{5B}\wedge w_5)\circ I_5\}. \qquad (3)$$

Where ∘ is the composition of min-max operation[5].

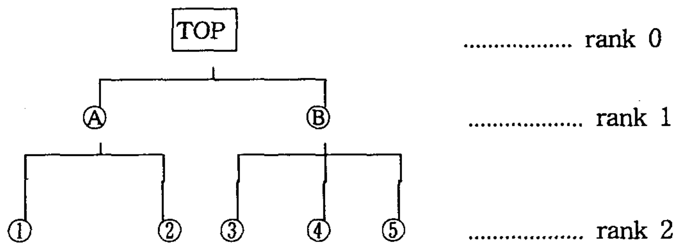

Figure 1. Hierarchical structure of items of evaluation

Finally, if the informations of I_1, I_2, I_3, I_4, I_5 are put in items ①, ②, ③, ④, and ⑤ at the same time, then information I_{TOP} of the top rank is gained from equation (4).

$$\begin{pmatrix} I_{A1} \\ I_{A2} \\ I_{B3} \\ I_{B4} \\ I_{B5} \end{pmatrix} = \begin{pmatrix} w_1 & 0 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 & 0 \\ 0 & 0 & w_3 & 0 & 0 \\ 0 & 0 & 0 & w_4 & 0 \\ 0 & 0 & 0 & 0 & w_5 \end{pmatrix} \begin{pmatrix} R_{1A} & 0 & 0 & 0 & 0 \\ 0 & R_{2A} & 0 & 0 & 0 \\ 0 & 0 & R_{3B} & 0 & 0 \\ 0 & 0 & 0 & R_{4B} & 0 \\ 0 & 0 & 0 & 0 & R_{5B} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix}$$

$$I_A=I_{A1}+I_{A2}$$

$$I_B=I_{B3}+I_{B4}+I_{B5} \qquad (4)$$

$$I_{TOP}=\begin{pmatrix} w_A & 0 \\ 0 & w_B \end{pmatrix}\begin{pmatrix} R_{ATOP} & 0 \\ 0 & R_{BTOP} \end{pmatrix}\begin{pmatrix} I_A \\ I_B \end{pmatrix}$$

Where symbol + represents the sum of fuzzy operation.

Besides, we have to deal with the problem that how to evaluate membership function obtained from overall evaluation. In this case, for example, we establish several typical patterns as Figure 2 dipicted previously, and evaluate the degree of satisfaction in accordance with similarity between the results of evaluation and typical patterns[6].

Now, suppose that membership function $\mu_i(u)$ represents a typical pattern and membership

function $\mu_j(u)$ is obtained from evaluation, then the similarity between $\mu_i(u)$ and $\mu_j(u)$ is given by equation (5)[5].

$$V_{ij}=\int_0^1 |\ \mu_i(u)-\mu_j(u)\ |\ du \qquad (5)$$

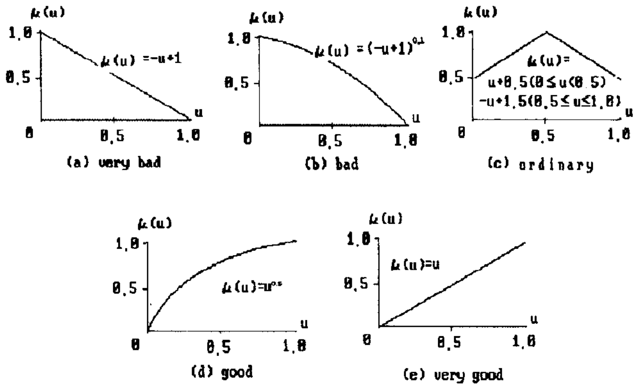

Figure 2. Classification of typical patterns

4. Operation Using Application of Fuzzy Inference

In order to the overall evaluation of quality of product that is consisted of multiple items of evaluation, we assume that there are six items of object of evaluation such as additional function, operational function and so on, and these items are connected with 3 ranks of hierarchical structure each other as shown in Figure 3.

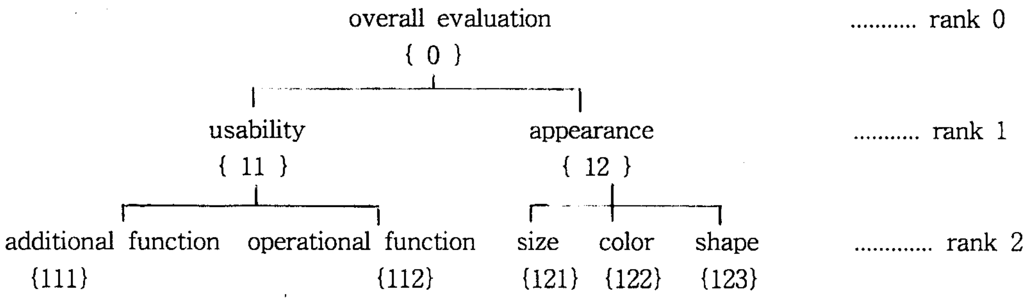

Figure 3. Hierarchical structure of qualitative characteristics
({ } : number of characteristics)

When we let parameter G denote the degree of goodness, this value is continuous number conceptually, but we divide it into 11scales for the convenience. But in this case, we use the intermediate value g_6 for the easiness of evaluation. That is, g_1 is the worst element and g_{11} is the best one. Also if we let the proposition of items such as 「good」 「ordinary」 「bad」

Table 1. Membership function of elements of preposition G and S

element		g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
G	μ_{Gg}	0.00	0.00	0.00	0.00	0.00	0.08	0.32	0.68	0.92	0.98	1.00
	μ_{Gn}	0.00	0.00	0.06	0.50	0.94	1.00	0.50	0.06	0.00	0.00	0.00
	μ_{Gb}	1.00	0.92	0.68	0.32	0.08	0.02	0.00	0.00	0.00	0.00	0.00
S	μ_{Sg}	0.00	0.00	0.01	0.07	0.20	0.28	0.62	0.80	0.93	0.97	1.00
	μ_{Sn}	0.20	0.43	0.56	0.65	0.69	0.70	0.65	0.56	0.43	0.28	0.20
	μ_{Sb}	1.00	0.93	0.80	0.62	0.38	0.28	0.07	0.01	0.00	0.00	0.00
element		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}

denote G of fuzzy subset G_g, G_n, G_b represented as μ_{Gg}, μ_{Gn}, μ_{Gb} , and let parameter S denote the degree of satisfaction of product then the universe of discourse S is

$$S=\{s_1, s_2, \cdots, s_{11}\}.$$

Where s_1 is 「the worst satisfactory」 and s_{11} is 「the most satisfactory」, and the proposition of these elements such as 「unsatisfactory」 「more or less good」 is represented by fuzzy subset S_g, S_n, S_b with the membership function μ_{Sg}, μ_{Sn}, μ_{Sb} which is shown in Table 1.

Now, we define three kinds of rules such as below using membership function of proposition G and S in Table 1.

「G : good → S : satisfactory」
「G : ordinary → S : more or less good」
「G : bad → S : unsatisfactory」

There are infinite numbers of rule of these parameters, in this case, but we have only three kinds of these because it is the peculiarity of fuzzy inference of which fuzziness can be performed with only a small number of rules. Accordingly, in order to transform these rules into fuzzy relations, we calculate the Cartesian product of G_g and S_g, G_n and S_n, G_b and S_b, and we obtain the fuzzy relation $\boldsymbol{R}_g$, $\boldsymbol{R}_n$, $\boldsymbol{R}_b$ of these rules. Also, we can get general fuzzy relation $\boldsymbol{R}$ of G and S using logic disjunction. That is, if the implication $G(G_g, G_n, G_b)$ and conclusion $S(S_g, S_n, S_b)$ are given, then we can now make the three kinds of rules as below.

for If x is G_g then y is S_g,

$$\boldsymbol{R}_g=G_g \times S_g$$

for If x is G_n then y is S_n,

$$\boldsymbol{R}_n=G_n \times S_n$$

for If x is G_b then y is S_b,

$$\boldsymbol{R}_b=G_b \times S_b$$

And general fuzzy relation $\boldsymbol{R}$ of G and S is represented in $G+S$ as shown in Table 2.

Table 2. $\boldsymbol{R}=G+S$

	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{11}
g_1	1.00	0.93	0.80	0.62	0.38	0.28	0.07	0.01	0.00	0.00	0.00
g_2	0.92	0.92	0.80	0.62	0.38	0.28	0.07	0.01	0.00	0.00	0.00
g_3	0.68	0.68	0.68	0.62	0.38	0.28	0.07	0.01	0.06	0.06	0.06
g_4	0.50	0.43	0.50	0.50	0.50	0.50	0.50	0.50	0.43	0.28	0.20
g_5	0.94	0.43	0.56	0.65	0.69	0.70	0.65	0.56	0.43	0.28	0.20
g_6	1.00	0.43	0.56	0.65	0.69	0.70	0.50	0.56	0.43	0.28	0.20
g_7	0.50	0.43	0.50	0.50	0.50	0.50	0.50	0.50	0.43	0.32	0.32
g_8	0.68	0.06	0.06	0.07	0.20	0.28	0.62	0.68	0.68	0.68	0.68
g_9	0.92	0.00	0.01	0.07	0.20	0.28	0.62	0.80	0.92	0.92	0.92
g_{10}	0.98	0.00	0.01	0.07	0.20	0.28	0.62	0.80	0.93	0.97	0.98
g_{11}	1.00	0.00	0.00	0.07	0.20	0.28	0.62	0.80	0.93	0.97	1.00

Now we can get the weights of each item by means of paired-comparison method from equation (1) because the relations of items that are connected to the top event are consistituted with hierarchical structure represented in Figure 2, and we can obtain the fuzzy

Table 3. Weights of each rank

rank 0	overall evaluation				
rank 1	usability		appearance		
	1.0		0.5		
rank 2	additional function	operational function	size	color	shape
	0.25	1.0	0.38	0.44	1.0

relation $\boldsymbol{R}_i$ of item i multiplying the weights by fuzzy relation $\boldsymbol{R}$. Then, the weights gained from this process is dipicted in Table 3. When we use these weights obtained from Table 3, the fuzzy relation of additional function $\boldsymbol{R}_{(111)}$ in rank 2 is $\boldsymbol{R}_{(111)}=0.25\times\boldsymbol{R}$ and that of operational function $\boldsymbol{R}_{(112)}$ is $\boldsymbol{R}_{(112)}=1.0\times\boldsymbol{R}$. And if we let the fuzzy relation and implication of each item denote membership function μ_{Gi}, then we can get the conclusion N_i on item i according to min-max composition. Also, when if we represent the degree of goodness of items in evaluator of seven scales of evaluation, then these evaluators are transformed into membership function μ_{Gg}, μ_{Gn}, μ_{Gb} and μ_{Gg}^2, $\mu_{Gg}^{1/2}$, $\mu_{Gb}^{1/2}$[10]. And the implication G_i and the membership functions that are given to five items of product A, B are described in Table 4.

Table 4. Evaluators of items of products

product / character	A		B	
	evaluator	$\mu_{G(i)}$	evaluator	$\mu_{G(i)}$
additional function	very good	μ_{Gg}^2	ordinary	μ_{Gn}
operational function	somewhat bad	$\mu_{Gb}^{1/2}$	very good	μ_{Gg}^2
size	ordinary	μ_{Gn}	bad	μ_{Gb}
color	very good	μ_{Gg}^2	ordinary	μ_{Gn}
shape	bad	μ_{Gb}	somewhat bad	$\mu_{Gb}^{1/2}$

Therefore, the input information $G_{(111)}$ of additional function for rank 2 of product A is obtained as membership function in Table 5 according to Table 1 and Table 4.

Table 5. Input information of additional function(μ_{Gg}^2)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	0.00	0.00	0.00	0.00	0.00	0.01	0.10	0.46	0.85	0.96	1.00

In this case, fuzzy relation $\boldsymbol{R}_{(111)}$ is represented $0.25 \times \boldsymbol{R}$ and conclusion of this element is derived from the min-max composition of $\boldsymbol{R}_{(111)}$ and $G_{(111)}$ as shown in Table 6.

Table 6. Conclusion of additional function ($S'_{(111)}$)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	0.25	0.10	0.10	0.10	0.10	0.28	0.16	0.20	0.23	0.24	0.25

And, as the weights of operational function that is the second item on rank 2 of product A is 1.0, we can easily obtain the fuzzy relation $\boldsymbol{R}_{(112)}$ with $1.0 \times \boldsymbol{R}$. And, as the evaluator of this item is 「somewhat bad」 shown as Table 4, the input information $G_{(112)}$ of this item is obtained from Table 7 with membership function.

Table 7. Input information of operational function($\mu_{Gb}{}^{1/2}$)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	1.0	0.96	0.82	0.57	0.28	0.14	0.00	0.00	0.00	0.00	0.00

From the min-max composition of input information in Table 7 and $\boldsymbol{R}_{(112)}$. , we can obtain the conclusion $S'_{(112)}$ of operational function in Table 8.

Table 8. Conclusion of operational function($S'_{(112)}$)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	1.00	0.93	0.80	0.62	0.50	0.50	0.50	0.50	0.43	0.20	0.20

Subsequently, in order to get the input information of usability on rank 1, if we have the composition $\mu_{S'(111)} \vee \mu_{S'(112)}$ of $S'_{(111)}$ in Table 6 and $S'_{(112)}$ in Table 8, then the result of this composition becomes the input information of the item of usability shown as Table 9.

Table 9. Input information of usability($\mu_{S'(111)} \vee \mu_{S'(112)}$)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	1.00	0.93	0.80	0.62	0.50	0.50	0.50	0.50	0.43	0.24	0.25

According to the operation of $\mu_{S'(121)} \vee \mu_{S'(122)} \vee \mu_{S'(123)}$, the conclusion S of items on rank 2
such as size, color and shape that are the lower items of appearance on rank 1 is easily obtained, and this conclusion becomes the input information to the item of appearance on rank 1.

Table 10. Input information of appearance($\mu_{S'(121)} \vee \mu_{S'(122)} \vee \mu_{S'(123)}$)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	1.00	0.92	0.80	0.62	0.38	0.32	0.32	0.35	0.4	0.42	0.44

By use of the above informations, we can get the fuzzy relation $\boldsymbol{R}_{(11)}$ of the item of usability and $\boldsymbol{R}_{(12)}$ of the item of appearance multiplying $\boldsymbol{R}$ by the weight 1.0 of the item of usability, and we obtain the conclusion $S'_{(11)}$, $S'_{(12)}$ of these items by means of min-max operation of the input information in Table 9 and Table 10. Finally, we determine the final conclusion S in rank 0 choosing the maximum membership function of $S'_{(11)}$ and $S'_{(12)}$ as dipicted in Table 11.

Table 11. Conclusion of overall evaluation(S)

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	1.00	0.93	0.80	0.62	0.50	0.50	0.50	0.50	0.50	0.50	0.50

Accordingly, we can represent the similarity $V(a)$ between the result of overall evaluation in Table 11 and typical pattern by use of equation (5) as below.

$V(a)=1.977$
$V(b)=1.518$
$V(c)=3.645$
$V(d)=3.920$
$V(e)=4.125$

That is, we know that product A is most similar to the typical pattern (b) and the evaluation of this product can be expressed as 「bad」. Also, if the input information of product B in rank 2 is given in table 4, then the menbership functions of the overall evaluation are obtained in Table 12,

Table 12. Conclusion of overall evaluation of procuct B

element	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
$\mu(g)$	1.00	0.93	0.80	0.62	0.50	0.50	0.62	0.80	0.93	0.97	1.00

and we know that the similarity between the product B and typical pattern is

$V(a)=3.797$
$V(b)=3.194$
$V(c)=4.139$
$V(d)=2.184$
$V(e)=2.791$.

That is, the result of evaluation of product B is the most similar to the pattern (d) and expressed as 「good」.

5. Conclusion

In order to apply fuzzy theory to the sensory evaluation, we dealt with hierarchical structure of qualitative characteristics of quality as the objects of evaluation and the processes in which the relative importances among items of evaluation given as hierarchically and input informations of evaluator gained as the form of linguistic variables are transmitted to the highest rank through hierarchical structure. Finally, an interpretation of these results of evaluation is performed.

The approaches performed in this paper are not sufficient to express the degree of completion and to express the reliability of fuzzy model from a practical point of view. But, in a stage of the quality design, if the fuzzy relations of items are given by an expert then it is said that these approaches are able to have the advantages that can easily opposed to the various kinds of desire and strengthen the adaptability for variation of the elements of quality with a view of diversifying input variables.

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