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New Calculating method for the Compression strength of Corrugated Fiberboard Box

- Concentrated on New Kellicutt Equation -

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1. Establishing of New Constants for Kellicutt's Equation

< Introduction >

While there are several kinds of equation for estimating compressive strength of corrugated fiberboard box, I would like to point out a problem about K. Q. Kellicutt's equation and propose a solution to it.

< K. Q. Kellicutt's equation >

According to "Basic Design Data for the Use of Fiberboard in Shipping Containers", No. D1911 issued at the date of November, 1951 (but actually in September, 1958):

$$P = P_x \left\{ \frac{(aX_2)^2}{(Z/4)^2} \right\}^{1/3} ZJ$$

P : Compressive strength of box (lb)
P_x : Composite ring crush value (lb/in)
aX_2 : Flute constant A F -- 8.36
B F -- 5.00
C F -- 6.10
Z : Perimeter of box (in)
J : Box factor

Table 1 : Tentative box factors for A-, B-, and C-flute boxes

Source of boxes	Type of manufacturer's joint	Box factors(J) for boxes with flutes vertical in side walls*		
		Flute		
		A	B	C
Laboratory made from commerciall material	Taped	0.717	0.752	0.717
	Stapled	———	0.622	———
Commercially made	Taped	0.677	0.597	0.667
	Stapled	———	0.564	———

* Box factors for boxes with flutes horizontal in side walls have not been determined.

< Kellicutt - Kawabata's equation >

In 1969, Kawabata converted the Kellicutt's equation to a simplified form and reported it under the name of K-K's equation.

$$P = R_x \, \beta (L+W)^{1/3}$$

P : Compressive strength of box (kgf)
R_x : Total ring crush value (kgf/6in)
β : Flute constant A F -- 0.944, B F -- 0.772, C F -- 0.880
L : Box length (cm) W : Box width (cm)

Kawabata established new flute constants β by integrating the flute constant (aX_2) and box factor (J) of Kellicutt's equation.

< Induction of K-K's equation realized by simplifying K.Q. Kellicutt's equation>

$$P = P_x \left\{ \frac{(aX_2)^2}{(Z/4)^2} \right\}^{1/3} ZJ = P_x \{(4aX_2/Z)^2\}^{1/3} ZJ$$
$$= P_x \{[4aX_2/2(L+W)]^2\}^{1/3} 2(L+W)J$$
$$= P_x \{[2aX_2/(L+W)]^2\}^{1/3} 2(L+W)J$$
$$= P_x \{(2aX_2)^2 (L+W)^3/(L+W)^2\}^{1/3} 2J = P_x \{(2aX_2)^2 (L+W)\}^{1/3} 2J$$

If you put $P_x = R_x/6$ in this equation and also perform conversion between (in) and (cm), you obtain:

$$P = R_x \{(2aX_2)^2 (L+W)/2.54\}^{1/3} 2J/6$$
$$= R_x \{(2aX_2)^2/2.54\}^{1/3} J/3(L+W)^{1/3} = R_x \, \beta (L+W)^{1/3}$$

Kellicutt's constant (aX_2)and box factor (J) were integrated into the constant β of K-K's equation.

$\beta = \{(2aX_2)^2/2.54\}^{1/3} J/3$, A F -- 0.9425, B F -- 0.7711, C F -- 0.8804

< Kellicutt's constant of AB F >

Kellicutt's equation included A F, B F, C F and Solid constants only and no constants of double wall corrugated fiberboard were publicized.

The constant β of respective flutes obtained by reading (P/P_x) and (Z) from the drawing (Fig. 4) given on page 192 of "Practical Dictionary of Corrugated Fiberboard" issued in 1967 and calculating with the equation $\beta = (P/P_x)/6(Z/2)^{1/3}$ was A F --0.94, B F --0.77, C F --0.88, AB F --1.21.

Fig. 4 : Strength calculation diagram of each flutes

In his "An Introduction to Corrugated Fiberboard Packaging Technology" (1985), Igarashi calculated the constants of double wall corrugated fiberboard in Kellicutt s equation. Namely, supposing the flute constant (aX_2) of AB F as 13.36 by adding 5.00 of B F to 8.36 of A F, he measured the compressive strength of a large number of boxes, counted back the box factor (J) of the boxes and decided it as 0.55.

By calculating the constant β of AB F on the basis of this flute constant (aX_2) (13.36) and the box factor (J) of boxes (0.55), we obtained

$$\beta = [(2aX_2)^2 / 2.54]^{1/3} J/3 = 1.2009.$$

If you plot the relationship between the constant β in the K-K's equation and the thickness of sheet h(cm) (Table 2), you obtain (Fig. 5). As shown in the drawing, a linear relation is established between the constant β and the thickness of sheet h.

Table 2 : Constant β and thickness of sheet (h)

	A F	B F	C F	AB F
h (cm)	0.5	0.3	0.4	0.8
β	0.9425	0.7711	0.8804	1.2009

Fig. 5 : Relationship between thickness of sheet h and constant β in the K-K's equation

< Proposal for modification of Kellicutt's constants >

Why, in Kellicutt's calculation expression of compressive strength of corrugated boxes, is the box factor (J) of the boxes small only for A F while the values of B F and C F are the same?

The gradient of the trend line obtained from the relational chart (Fig. 5) between thickness of corrugated sheet (h) and constant β in the K-K's equation was 1/2. Namely, the constant β obtained by integrating (aX_2) and (J) is proportional to the square root of the thickness of corrugated sheet (h). In other words, the constant β could be expressed approximately with the following expression:

$\beta = 1.3386h^{1/2}$ h: Thickness of corrugated sheet (cm)

The value of the constant β determined by this expression was about the same for A F and AB F but was slightly lower for B F and C F, as shown in Table 3.

Table 3 : Thickness of sheet h and constant β in K-K´s equation

Flute	h^{cm}	aX_2	J	α	β	$1.3386h^{1/2}$
A F	0.5	8.36	0.59	1.523	0.9425	0.9465
B F	0.3	5.00	0.68	1.361	0.7711	0.7332
C F	0.4	6.10	0.68	1.447	0.8804	0.8466
AB F	0.8	13.36	0.55	2.884	1.2009	1.1973

Next, by using the value of the constant in the K-K´s equation $\beta = 1.3386\ h^{1/2}$, we changed the Kellicutt´s constant (aX_2) and box factor (J) while keeping the take-up ratio α fixed. In that case, (aX_2) was calculated with $aX_2 = 9.1287\ h^{1/2}$ made to be proportional to $h^{1/2}$ with reference to 5.00 of B F, while (J) was calculated with $J = 0.7922\ h^{1/6}$ made to be proportional to $h^{1/6}$ with reference to 0.68 of C F. Those new constant and box factor are consistent constants proportional to the thickness of the respective flutes, as shown in Fig. 6.

Table 4 : New constant and box factor of Kellicutt and new constants of K-K´s equation of different kinds of flute

Flute	h^{cm}	aX_2	J	α	β →→→	β
E F	0.2	4.08	0.61	1.270	0.5986	0.60
B F	0.3	5.00	0.65	1.361	0.7371	0.74
C F	0.4	5.77	0.68	1.447	0.8484	0.85
A F	0.5	6.46	0.70	1.523	0.9416	0.94
BB F	0.6	7.07	0.73	2.722	1.0368	1.04
BC F	0.7	7.64	0.75	2.808	1.1199	1.12
AB F	0.8	8.17	0.76	2.884	1.1956	1.20
AC F	0.9	8.66	0.78	2.970	1.2699	1.27
AA F	1.0	9.13	0.79	3.046	1.3386	1.34

< Further simplification of K-K´s equation >

Because the constant β of K-K´s equation is proportional to the square root of the thickness (h) of corrugated fiber-board sheet, the constant β of K-K´s equation can also be substituted by a thickness (h) of sheet by simplification.

$$P = 1.3386 R_x h^{1/2} (L+W)^{1/3}$$

R_x : Total ring crush value (kgf/6in)
h : Thickness of corrugated sheet (cm)

This enabled estimation of compressive strength also for AA F and BC F, thus increasing the utility value of the equation.

Fig. 6 : Relationship between sheet thickness and new constant

< Conclusion >

By using the new constant (aX_2), and box factor (J) for Kellicutt´s equation, it becomes possible to estimate the compressive strength of boxes made of AB F, BC F, AA F, etc. Moreover, those new constants (aX_2), and box factor (J) can also be used for the original equation of Kellicutt. They are usable as they are even in in, lb unit system used in the United States, etc.

Furthermore, because, in K-K´s equation easy to calculate, the constant β is also proportional to the square root of the thickness (h), further simplification has been made, making it possible to estimate the compressive strength with single constant regardless of the kind of flute.

$$P = 1.3386 R_x h^{1/2} (L+W)^{1/3}$$

2. Simplest Equation of Predicting Compressive Strength for Corrugated Box

< Introduction >

This paper is intended to mention characteristics and relation of K. Q. Kellicutt´s equation to R. C. McKee´s equation which is most frequently used in the United States and propose an approximate estimation formula realized by simplification of R. C. McKee´s equation.

< R. C. McKee´s equation >

$$P_{Mc} = 5.87 P_m h^{0.5} Z^{0.5}$$

A characteristic of R. C. McKee´s equation is that, though it was originally a very complicated equation, it has been converted into a very simple expression easy for calculation instead by application of various new ideas.

< Relationship with K. Q. Kellicutt's equation >

McKee s equation can be transformed into the following expression by changing its units:

McKee's equation -- $P_{Mc} = 1.660\ P_m h^{1/2} (L+W)^{1/2}$

P : Compressive strength of box (kgf)

P_m : Edge crush value of corrugated fiberboard (kgf/50mm) (JIS Z 0401)

K-K's equation -- $P_{KK} = 1.3386 R_x h^{1/2} (L+W)^{1/3}$

R_x : Total ring crush value (kgf/6in)

L : Length of box (cm) W : Width of box (cm)

h : Thickness of corrugated fiberboard sheet (cm)

By comparing the two equations given above, we can see that the differences between McKee's equation easy to calculate and K-K's equation are the difference of indexes of exponential function (1/2 or 1/3) and that of substitute characteristic for compressive strength (edge crush value or ring crush value). This does not present any marked difference in the slope between indexes 1/2 and 1/3 in a certain range of (L+W) (30cm~150cm) as shown in Fig. 3. Therefore, if a constant for conversion is set, conversion of indexes can be made by approximation.

Fig. 3 : Difference between indexes 1/2 and 1/3

< Induction of McKee's type Kellicutt's equation >

While the cubic root of perimeter is used in Kellicutt's equation, McKee's equation uses the square root of perimeter. To convert cubic root into square root, we prepared the expression $\sqrt[3]{A}=\sqrt{A}/k$ and solved it as follows:

$$k=\sqrt{A}/\sqrt[3]{A}=\sqrt[6]{A},\quad A=k^6\ ,\ (k=2,\ A=64)$$

When (L+W) of K-K's equation is in the neighborhood of 64 cm, by multiplying the constant of Kellicutt's equation with 1/2 by approximation, the cubic root of (L+W) can be converted into a form of square root. Namely, you obtain M-K's equation, which is of the same type as that of McKee's equation. In other words, it becomes approximately equal to McKee's equation.

$P = R_x\ \beta (L+W)^{1/3}$ ------ $P_{Ke} = 1.3386 R_x\ h^{1/2}\ (L+W)^{1/3}$ ----K-K´s equation
$P = R_x\ \beta /2(L+W)^{1/2}$ ---- $P_{Ke} = 0.6693 R_x\ h^{1/2}\ (L+W)^{1/2}$ ----M-K´s equation
$P_{Mc} = 1.660\ P_m\ h^{1/2}\ (L+W)^{1/2}$ ----McKee s equation

< Approximate simplification of constant β of K-K´s equation >

On the other hand, to observe the constant β used in K-K´s equation more carefully, we can see that $\beta = 1.3386\ h^{1/2}$ can be converted by approximation into $\beta = 1.3333\ h^{1/2}$ i.e. $\beta = (4/3)\sqrt{h}$, $1/2\beta = (2/3)\sqrt{h}$.

Table 1 : Table of constants of flute

Flute	A	B	C	AB
α	1.523	1.361	1.447	2.884
β	0.942	0.772	0.880	1.203
$1/2\beta$	0.471	0.386	0.440	0.602
New β	0.94	0.74	0.85	1.20
New $1/2\beta$	0.47	0.37	0.43	0.60
h^{cm}	0.5	0.3	0.4	0.8
$\sqrt{h}$	0.707	0.548	0.632	0.894
$(4/3)\sqrt{h}$	0.943	0.730	0.843	1.193
$(2/3)\sqrt{h}$	0.471	0.365	0.422	0.596

As shown in Table 1, we can see that the new $1/2\beta$ is approximately equal to $(2/3)\sqrt{h}$. Therefore, K-K´s equation can be simplified as follows:

K-K´s equation $P = (4/3) R_x\ h^{1/2}\ (L+W)^{1/3}$

M-K´s equation can also be simplified in the same way.

M-K´s equation $P = (2/3) R_x\ h^{1/2}\ (L+W)^{1/2}$

$$P_{Ke} = (2/3) R_x\ [(L+W)h]^{1/2}$$

P_{Ke}: Compressive strength of corrugated fiberboard box (kgf)
R_x : Total ring crush value of liner (kgf/6in)
L : Length of box (cm), W : Width of box (cm)
h : Thickness of corrugated fiberboard sheet (cm)

The simplified comressive strength estimation formula obtained this way provides approximately the same result as the value calculated with Kellicutt´s equation. However, the value by this simplified expression becomes larger than that of Kellicutt's equation if (L+W) is extremely large.

< New compressive strength estimation formula >

According to data of Finnboad Company of Finland, the total ring crush value of corrugated fiberboard increased by 10% is close to the vertical compressive strength (edge crush value). Namely, the compressive strength increases by 10% as the liner is transformed into corrugated fiberboard.

Edge crush value (kgf/cm) =1.1 ×Total ring crush value (kgf/cm)

If this relation is introduced into McKee's equation, the compressive strength can be expressed with the following expression:

$$P = 1.66 \times 1.1 \times R_x/6 \times 5/2.54\ [(L+W)h]^{1/2}$$
$$= 0.5991 R_x\ [(L+W)h]^{1/2}$$

$$P_{Mc} = 0.6 R_x\ [(L+W)h]^{1/2}$$

P_{Mc}: Compressive strength of corrugated fiberboard box (kgf)
R_x : Total ring crush value of liner (kgf/6in)
L : Length of box (cm), W : Width of box (cm)
h : Thickness of corrugated fiberboard sheet (cm)

By comparing this with M-K's equation given above, (2/3)=0.6666 and the equation can be changed as follows:

(M-K's equation) ------ $P_{Ke} = (0.6666) R_x\ [(L+W)h]^{1/2}$
(McKee's equation) ---- $P_{Mc} = (0.6) R_x \cdot [(L+W)h]^{1/2}$

Namely, those two equations are in exactly the same form except that the constant 2/3 changed to 3/5. And yet, this value of 10% smaller than the constant of Kellicutt's equation. The value 0.625 i.e. 5/8 between those constants was adopted as constant by the new compressive strength estimation formula K-M-K's equation.

$$P_{KMK} = (5/8) R_x\ [(L+W)h]^{1/2}$$

< Conclusion >

McKee's equation is essentially an estimation formula of the same form as that of Kellicutt's equation. We proposed a new estimation formula easy to calculate while leaving good functions convenient for the selection of liner. Another characteristic of this expression is that it can provide an intermediate estimated value of Kellicutt's equation.

	Constant
Kellicutt	2/3 ----- 0.6666
McKee	3/5 ----- 0.6
K-M-K	5/8 ----- 0.625

3. Precise Equation for Predicting Compressive Strength of Corrugated Box

< Introduction >

By carefully studying Murray Wolf's equation intended for improvement of McKee's equation, further improvement in the accuracy of estimation seemed hopeful.

For the purpose of such improvement in the accuracy of estimation, we propose a new expression realized by further improving Wolf's equation.

< Murray Wolf's equation >

To obtain an estimation formula closer to measured value, Wolf adopted the following expression of strength without changing the indexes of perimeter and length:

$$S = kZ^{0.5}(pA - qA^2 + 1)/D^{0.04107} \quad \text{---------------- (6)}$$

If you put as $S' = S\ D^{0.04107}/Z^{0.5}$ by using a new variable value S', the expression (6) can be simplified as follows:

$$S' = k(pA - qA^2 + 1) \quad \text{---------------------------- (7)}$$

To calculate p, q, K of this expression, we obtain p = 0.3228, q = 0.1217 and K = 108.98.

By putting those values in the expression (6), the final results are given as follows:

$$S = 108.89Z^{0.5}(0.3228A - 0.1217A^2 + 1)/D^{0.04107} \text{ -------- } (8)$$

By differentiating the expression (8) regarding A and putting it as 0, you obtain the following formula:

$$dS/dA = 108.89Z^{0.5}(0.3228 - 0.2434A)/D^{0.04107} = 0 \text{ ---- } (9)$$

By solving this expression, you get A = 1.33. It indicates that the maximum compressive strength of the box is produced when the aspect ratio is 1.33.

< Estimation formula closer to measured value >

Wolf's equation is said to be an equation of the highest estimation accuracy today. However, it is only when the aspect ratio is 1.8 that the expression (8) indicated in Fig. 1 crosses with the line of test value, and the estimated value is smaller than the measured value at a smaller aspect ratio but larger at a larger aspect ratio.

By reading the numerical value of change in compressive strength by aspect ratio from the relational chart (Fig. 2) between aspect ratio and compressive strength, you obtain the results of Table 1.

Table 1 : Relation between aspect ratio and compressive strength

L/W	0.94	1.00	1.20	1.37	1.54	1.74	1.80	2.00
P	92.50	95.05	100.0	(max.)	100.0	95.05	92.50	85.13

To determine a quadratic equation of approximation from this Table,

$$P \propto 125.58A - 45.83A^2 + 15.35$$

$$P \propto (8.2A - 3A^2 + 1)15.35 \text{ --------------- } (10)$$

This relational will be converted at an aspect ratio of 1.8 where the estimated value agrees with the test value.

First, if you put Wolf's "quadratic equation of A" $(0.3228A - 0.1217A^2 + 1)$ as f(A) and put the new quadratic equation $(8.2A - 3.0A^2 + 1)$ as F(A), they can be expressed with the following formulas respectively:

$$f(A) = (0.3228A - 0.1217A^2 + 1), \quad F(A) = (8.2A - 3.0A^2 + 1)$$

Because Wolf's equation agrees with the test value when the aspect ratio is

1.8, you obtain the following by using a conversion coefficient K:

$$f(1.8) = F(1.8)K, \quad K = f(1.8)/F(1.8) = 1.18673/6.04 = 1/5.0896$$

By multiplying the constant 108.896 of the expression (8) with this conversion coefficient K, you can get the new coefficient 108.89/5.0896 = 21.3946.

Moreover, a new conversion coefficient can be obtained also when you read the strength 723.4 at A=1.2 from Fig. 1
and convert it to the strength 100 at A=1.2 from Fig. 2.

$$15.35 \times 723.4/100 = KZ^{0.5}/D^{0.04107},$$

$$K = 15.35 \times 7.234 \times D^{0.04107}/Z^{0.5} = 111.04 \times (4.8in)^{0.04107}/(30in)^{0.5}$$

$$= 21.6223$$

The average value of the new conversion coefficients obtained by the two methods if 21.50.

$$S = 21.50Z^{0.5}(8.2A - 3.0A^2 + 1)/D^{0.04107} \quad \text{---------} \quad (11)$$

By differentiating the expression (11) regarding A and putting it as 0, you obtain the following formula:

$$dS/dA = 21.50Z^{0.5}(8.2 - 6.0A)/D^{0.04107} = 0 \quad \text{-------} \quad (12)$$

By solving this equation, you obtain A=1.367, which also agrees with the conclusion by Mirasol and gives the maximum compressive strength.

In the range of an aspect ratio of 1.0 to 2.0, an estimated value of higher accuracy is obtained with the expression (11) than with the expression (8).

< Wolf's compressive strength estimation formula (in, lb system) >

Wolf improved McKee's equation and publicized his new equation on "Package Engineering" magazine (February, 1974).

$$S = 5.2426P_m Z^{1/2} h^{1/2}(0.3228A - 0.1217A^2 + 1)/D^{0.041} \quad \text{---------} \quad (13)$$

By multiplying the constant 5.2426 of expression (13) with
K = f(1.8)/F(1.8) = 1/5.0896, you obtain 1.03.

$$S = 1.03\, P_m Z^{1/2} h^{1/2}(8.2A - 3.0A^2 + 1)/D^{0.041} \quad \text{----------------} \quad (14)$$

S : Compressive strength of corrugated fiberboard box (lbs)
P_m : End crush value (lbs/in)
Z : Perimeter of box (in)
h : Thickness of corrugated fiberboard sheet (in)
A : Aspect ratio of box (L/W)
D : Depth of box (in)

< Wolf's compressive strength estimation formula (CGS unit system) >

$$S = 1.1772 P_m Z^{1/2} h^{1/2} (0.3228A - 0.1217A^2 + 1)/D^{0.041} \quad \text{---------} \quad (15)$$

S : Compressive strength of corrugated fiberboard box (kgf)
P_m : End crush value (kgf/5cm)
Z : Perimeter of box (cm)
h : Thickness of corrugated fiberboard sheet (cm)
A : Aspect ratio of box (L/W)
D : Depth of box (cm)

Also for this expression, you obtain the following expression by converting it into a quadratic function by the said method:

$$S = 0.2313 P_m Z^{1/2} h^{1/2} (8.2A - 3.0A^2 + 1)/D^{0.041} \quad \text{-----} \quad (16)$$ ---Equation ①

If we study the correlation between the measured compressive strength of box with the estimated strength obtained by using the above equation, a good correlation is shown within a certain range (A = 1~2) as indicated in Fig. 3. However, the correlation rapidly deteriorates when A exceeds 2. So, by fixing F(A) at F(2) = 5.4 in the range of A ≧ 2, we obtained the expression (17). The expression (17) will be renamed as equation ③.

$$S = 1.2490 P_m Z^{1/2} h^{1/2} / D^{0.041} \quad \text{------------------------} \quad (17)$$ ---Equation ③

< Influence of depth (D) of box >

The relationship between depth (D) and compressive strength of box is expressed with $D^{-0.041}$, but the compressive strength suddenly increases and the estimation formula drops in accuracy if D becomes smaller than 30 cm. For that reason, we tried to further make a correction, i.e.:

By limiting the application of the equation to:
$S \propto 3.07\ D^{-0.33}$ in the case where the depth of box is under 30 cm ($D<30$) and to $S \propto 1.15\ D^{-0.041}$ in the case where the depth of box is 30 cm or over ($D \geq 30$) and converting the equation ① and equation ③ to constants respectively, you obtain:

$$S = 0.6175 P_m Z^{1/2} h^{1/2} (8.2A - 3.0A^2 + 1)/D^{0.33} \quad \text{---- (18)}$$ ---Equation ②

and

$$S = 3.3343\ P_m Z^{1/2} h^{1/2} / D^{0.33} \quad \text{------------------------ (19)}$$ ---Equation ④

< Limitation of scope of application and correlation with measured value >

Table 2 indicates the scope of application of equations ①, ②, ③, ④ obtained by modifying Wolf's compressive strength estimation formula.

Table 2 :

	D≧30	D<30
A<2	①	②
A≧2	③	④

By limiting this scope of application, we can get a chart with high correlation between measured value and estimated value.

< Conclusion >

We intended to improve the accuracy of estimation on the basis of Wolf's compressive strength estimation formula. However, as we aim at improving the accuracy of estimation, the estimation formula becomes more and more complicated and requires a lot of operations such as limitation of scope of application, etc.

Accumulation of such efforts led us to establishment of an estimation formula of high accuracy, but an increase of the measured value again produces new deviations.

This fact indicates that there is not much difference between the estimated value of a complicated estimation formula and the estimated value of a simplified estimation formula easy to estimate, in the package design where a safety coefficient of 3 to 4 times is taken into account.